Combining the above results,

$$p_d V_d = p_a V_a \left(\frac{T_c}{T_a}\right)^{\gamma} r^{\gamma-\gamma^2}$$

Subsitution of the above results into Eq. (20.4) gives

$$e = 1 - \frac{5}{7}\left[\frac{\left(\frac{T_c}{T_a}\right)^{\gamma} r^{\gamma-\gamma^2} - 1}{\left(\frac{T_c}{T_a}\right) - r^{\gamma-1}}\right]$$

$$= 1 - \frac{1}{1.4}\left[\frac{(5.002) r^{-0.56} - 1}{(3.167) - r^{0.40}}\right],$$

where $\frac{T_c}{T_a} = 3.167, \gamma = 1.4$ have been used. Substitution of $r = 21.0$ yields $e = 0.708 = 70.8\%$.

21.1: $m_{\text{lead}} = 8.00$ g and charge $= -3.20 \times 10^{-9}$ C

a) $n_e = \dfrac{-3.20 \times 10^{-9}\ \text{C}}{-1.6 \times 10^{-19}\ \text{C}} = 2.0 \times 10^{10}.$

b) $n_{\text{lead}} = N_A \times \dfrac{8.00\ \text{g}}{207} = 2.33 \times 10^{22}$ and $\dfrac{n_e}{n_{\text{lead}}} = 8.58 \times 10^{-13}.$

21.2: current $= 20{,}000$ C/s and $t = 100\ \mu\text{s} = 10^{-4}$ s

$Q = It = 2.00$ C

$n_e = \dfrac{Q}{1.60 \times 10^{-19}\ \text{C}} = 1.25 \times 10^{19}.$

21.3: The mass is primarily protons and neutrons of $m = 1.67 \times 10^{-27}$ kg, so:

$n_{\text{p and n}} = \dfrac{70.0\ \text{kg}}{1.67 \times 10^{-27}\ \text{kg}} = 4.19 \times 10^{28}$

About one-half are protons, so $n_p = 2.10 \times 10^{28} = n_e$ and the charge on the electrons is given by: $Q = (1.60 \times 10^{-19}\ \text{C}) \times (2.10 \times 10^{28}) = 3.35 \times 10^9$ C.

21.4: Mass of gold $= 17.7$ g and the atomic weight of gold is 197 g/mol. So the number of atoms $N_A \times \text{mol} = (6.02 \times 10^{23}) \times \left(\frac{17.7\ \text{g}}{197\ \text{g/mol}}\right) = 5.41 \times 10^{22}$.

a) $n_p = 79 \times 5.41 \times 10^{22} = 4.27 \times 10^{24}$

$q = n_p \times 1.60 \times 10^{-19}\ \text{C} = 6.83 \times 10^5\ \text{C}$

b) $n_e = n_p = 4.27 \times 10^{24}$.

21.5: $1.80 \text{ mol} = 1.80 \times 6.02 \times 10^{23}$ H atoms $= 1.08 \times 10^{24}$ electrons.

$\text{charge} = -1.08 \times 10^{24} \times 1.60 \times 10^{-19} \text{ C} = -1.73 \times 10^{5} \text{ C}.$

21.6: First find the total charge on the spheres:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \Rightarrow q = \sqrt{4\pi\varepsilon_0 F r^2} = \sqrt{4\pi\varepsilon_0 (4.57 \times 10^{-21})(0.2)^2} = 1.43 \times 10^{-16} \text{ C}$$

And therefore, the total number of electrons required is

$$n = q/e = 1.43 \times 10^{-16} \text{ C}/1.60 \times 10^{-19} \text{ C} = 890.$$

21.7: a) Using Coulomb's Law for equal charges, we find:

$$F = 0.220 \text{ N} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(0.150 \text{ m})^2} \Rightarrow q = \sqrt{5.5 \times 10^{-13} \text{ C}^2} = 7.42 \times 10^{-7} \text{ C}.$$

b) When one charge is four times the other, we have:

$$F = 0.220 \text{ N} = \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{(0.150 \text{ m})^2} \Rightarrow q = \sqrt{1.375 \times 10^{-13} \text{ C}^2} = 3.71 \times 10^{-7} \text{ C}$$

So one charge is 3.71×10^{-7} C, and the other is 1.484×10^{-6} C.

21.8: a) The total number of electrons on each sphere equals the number of protons.

$$n_e = n_p = 13 \times N_A \times \frac{0.0250 \text{ kg}}{0.026982 \text{ kg/mol}} = 7.25 \times 10^{24}.$$

b) For a force of 1.00×10^4 N to act between the spheres,

$$F = 10^4 \text{ N} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \Rightarrow q = \sqrt{4\pi\varepsilon_0 (10^4 \text{ N})(0.08 \text{ m})^2} = 8.43 \times 10^{-4} \text{ C}.$$

$$\Rightarrow n'_e = q/e = 5.27 \times 10^{15}$$

c) n'_e is 7.27×10^{-10} of the total number.

21.9: The force of gravity must equal the electric force.

$$mg = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \Rightarrow r^2 = \frac{1}{4\pi\varepsilon_0}\frac{(1.60 \times 10^{-19} \text{ C})^2}{(9.11 \times 10^{-31} \text{ kg})(9.8 \text{ m/s})} = 25.8 \text{ m}^2 \Rightarrow r = 5.08 \text{ m}.$$

21.10: a) Rubbing the glass rod removes electrons from it, since it becomes positive.

$$7.50 \text{ nC} = (7.50\times10^{-9} \text{ C})\,(6.25\times10^{18} \text{ electrons/C}) = 4.69\times10^{10} \text{ electrons}$$

$$(4.69\times10^{10} \text{ electrons})\,(9.11\times10^{-31} \text{ kg/electron}) = 4.27\times10^{-20} \text{ kg}.$$

The rods mass decreases by 4.27×10^{-20} kg.

b) The number of electrons transferred is the same, but they are *added* to the mass of the
plastic rod, which increases by 4.27×10^{-20} kg.

21.11: $\vec{F}_2$ is in the $+x$-direction, so $\vec{F}_1$ must be in the $-x$-direction and q_1 is positive.

$$F_1 = F_2, \qquad k\frac{q_1 q_3}{r_{13}^2} = k\frac{|q_2| q_3}{r_{23}^2}$$

$$q_1 = (0.0200/0.0400)^2 |q_2| = 0.750 \text{ nC}$$

21.12: a) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r^2} \Rightarrow 0.200 \text{ N} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(0.550\times10^{-6} \text{ C})q_2}{(0.30 \text{ m})^2}$

$$\Rightarrow q_2 = +3.64\times10^{-6} \text{ C}.$$

b) $F = 0.200$ N, and is attractive.

21.13: Since the charges are equal in sign the force is repulsive and of magnitude:

$$F = \frac{kq^2}{r^2} = \frac{(3.50\times10^{-6} \text{ C})^2}{4\pi\varepsilon_0 (0.800 \text{ m})^2} = 0.172 \text{ N}$$

21.14: We only need the y-components, and each charge contributes equally.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(2.0\times10^{-6} \text{ C})(4\times10^{-6} \text{ C})}{(0.500 \text{ m})^2}\sin\alpha = 0.173 \text{ N (since } \sin\alpha = 0.6).$$

Therefore, the total force is $2F = 0.35$ N, downward.

21.15: $\vec{F}_2$ and $\vec{F}_3$ are both in the $+x$-direction.

$$F_2 = k\frac{|q_1 q_2|}{r_{12}^2} = 6.749\times10^{-5} \text{ N}, \quad F_3 = k\frac{|q_1 q_3|}{r_{13}^2} = 1.124\times10^{-4} \text{ N}$$

$F = F_2 + F_3 = 1.8\times10^{-4}$ N, in the $+x$-direction.

21.16: $F_{21} = \dfrac{(9\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\,(20.\times10^{-6} \text{ C})\,(2.0\times10^{-6} \text{ C})}{(0.60\text{m})^2} = 0.100 \text{ N}$

F_{Q1} is equal and opposite to F_{1Q} (Ex. 21.4), so

$$(F_{Q1})_x = -0.23\text{N}$$
$$(F_{Q1})_y = 0.17\text{ N}$$

Overall:

$$F_x = -0.23\text{ N}$$
$$F_y = 0.100\text{ N} + 0.17\text{ N} = 0.27\text{ N}$$

The magnitude of the total force is $\sqrt{(0.23\text{ N})^2 + (0.27\text{ N})^2} = 0.35\text{ N}$. The direction of the force, as measured from the $+y$ axis is

$$\theta = \tan^{-1}\frac{0.23}{0.27} = 40°$$

21.17: $\vec{F}_2$ is in the $+x-$direction.

$$F_2 = k\frac{|q_1 q_2|}{r_{12}^2} = 3.37\text{ N},\quad \text{so } F_{2x} = +3.37\text{ N}$$
$$F_x = F_{2x} + F_{3x} \text{ and } F_x = -7.00\text{ N}$$
$$F_{3x} = F_x - F_{2x} = -7.00\text{ N} - 3.37\text{ N} = -10.37\text{ N}$$

For F_{3x} to be negative, q_3 must be on the $-x$-axis.

$$F_3 = k\frac{|q_1 q_3|}{x^2},\ \text{so } |x| = \sqrt{\frac{k|q_1 q_3|}{F_3}} = 0.144\text{ m},\ \text{so } x = -0.144\text{ m}$$

21.18: The charge q_3 must be to the right of the origin; otherwise both q_2 and q_3 would exert forces in the $+x$ direction. Calculating the magnitude of the two forces:

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2} = \frac{(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-6}\text{ C})(5.00\times10^{-6}\text{ C})}{(0.200\text{ m})^2}$$
$$= 3.375\text{ N in the} + x\text{ direction.}$$

$$F_{31} = \frac{(9\times10^9\text{ N}\cdot\text{m}^3/\text{C}^2)\,(3.00\times10^{-6}\text{ C})\,(8.00\times10^{-6}\text{ C})}{r_{13}^2}$$
$$= \frac{0.216\text{ N}\cdot\text{m}^2}{r_{13}^2}\ \text{in the} - x\text{ direction}$$

We need $F_{21} - F_{31} = -7.00\text{ N}$:

$$3.375\text{ N} - \frac{0.216\text{ N}\cdot\text{m}^2}{r_{13}^2} = -7.00\text{ N}$$

$$r_{13}^2 = \frac{0.216\ \text{N}\cdot\text{m}^2}{3.375\ \text{N} + 7.00\ \text{N}} = 0.0208\ \text{m}^2$$

$$r_{13} = 0.144\ \text{m to the right of the origin}$$

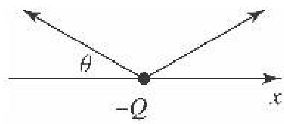
21.19: $\vec{F} = \vec{F}_1 + \vec{F}_2$ and $F = F_2 + F_1$ since they are acting in the same direction at $y = -0.400$ m so,

$$F = \frac{1}{4\pi\varepsilon_0}(5.00\times10^{-9}\ \text{C})\left(\frac{1.50\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} + \frac{3.20\times10^{-9}\ \text{C}}{(0.400\ \text{m})^2}\right) = 2.59\times10^{-6}\ \text{N downward.}$$

21.20: $\vec{F} = \vec{F}_1 + \vec{F}_2$ and $F = F_1 - F_2$ since they are acting in opposite directions at $x = 0$ so,

$$F = \frac{1}{4\pi\varepsilon_0}(6.00\times10^{-9}\ \text{C})\left(\frac{4.00\times10^{-9}\ \text{C}}{(0.200\ \text{m})^2} + \frac{5.00\times10^{-9}\ \text{C}}{(0.300\ \text{m})^2}\right) = 2.4\times10^{-6}\ \text{N to the right.}$$

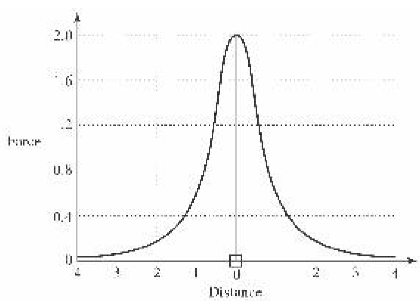
21.21: a)



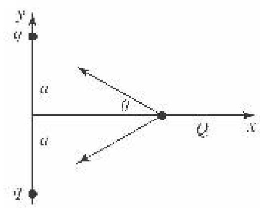
b) $F_x = 0,\ F_y = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{(a^2+x^2)}\sin\theta\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{2qQa}{(a^2+x^2)^{3/2}}$

c) At $x = 0$, $F_y = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2qQ}{a^2}$ in the $+y$ direction.

d)



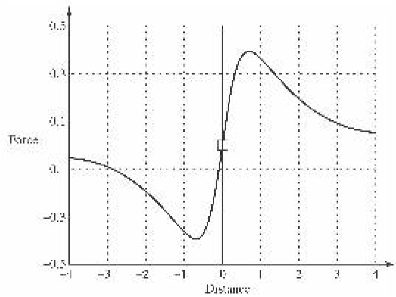
21.22: a)



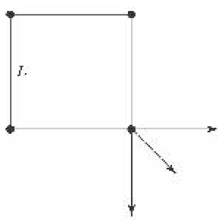
b) $F_x = -2\frac{1}{4\pi\varepsilon_0}\frac{qQ}{(a^2+x^2)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{-2qQx}{(a^2+x^2)^{3/2}}, F_y = 0$

c) At $x = 0$, $F = 0$.

d)



21.23:



b) $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2L^2} + \sqrt{2}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{L^2} = \left(1+2\sqrt{2}\right)\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2L^2}$ at an angle of 45° below the positive x-axis

21.24: a) $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\frac{1}{4\pi\varepsilon_0}\frac{(3.00\times10^{-9}\text{ C})}{(0.250\text{ m})^2} = 432\text{ N/C}$, down toward the particle.

b) $E = 12.00\text{N/C} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \Rightarrow r = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{(3.00\times10^{-9}\text{ C})}{(12.0\text{ N/C})}} = 1.50\text{ m.}$

21.25: Let +*x*-direction be to the right. Find a_x :

$v_{0x} = +1.50 \times 10^3\ \text{m/s}, v_x = -1.50 \times 10^3\ \text{m/s}, t = 2.65 \times 10^{-6}\ \text{s}, a_x = ?$

$v_x = v_{0x} + a_x t$ gives $a_x = -1.132 \times 10^9\ \text{m/s}^2$

$F_x = ma_x = -7.516 \times 10^{-18}\ \text{N}$

$\vec{F}$ is to the left $(-x$ - direction$)$, charge is positive, so $\vec{E}$ is to the left.

$E = F/q = (7.516 \times 10^{-18}\ \text{N})/[(2)(1.602 \times 10^{-19}\ \text{C})] = 23.5\ \text{N/C}$

21.26: (a) $x = \frac{1}{2}at^2$

$$a = \frac{2x}{t^2} = \frac{2(4.50\ \text{m})}{(3.00 \times 10^{-6}\ \text{s})^2} = 1.00 \times 10^{12}\ \text{m/s}^2$$

$$E = \frac{F}{q} = \frac{ma}{q} = \frac{(9.11 \times 10^{-31}\ \text{kg})\ (1.00 \times 10^{12}\ \text{m/s}^2)}{1.6 \times 10^{-19}\ \text{C}}$$

$= 5.69\ \text{N/C}$

The force is up, so the electric field must be *downward* since the electron is negative.
(b) The electron's acceleration is $\sim\ 10^{11}$ g, so gravity must be negligibly small compared to the electrical force.

21.27: a) $|q|E = mg \Rightarrow |q| = \dfrac{(0.00145\ \text{kg})\ (9.8\ \text{m/s}^2)}{650\ \text{N/C}} = 2.19 \times 10^{-5}\ \text{C}$, sign is negative.

b) $qE = mg \Rightarrow E = \dfrac{(1.67 \times 10^{-27}\ \text{kg})\ (9.8\ \text{m/s}^2)}{1.60 \times 10^{-19}\ \text{C}} = 1.02 \times 10^{-7}$ N/C, upward.

21.28: a) $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(26 \times 1.60 \times 10^{-19}\ \text{C})}{(6.00 \times 10^{-10}\ \text{m})^2} = 1.04 \times 10^{11}\ \text{N/C}.$

b) $E_{\text{proton}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(1.60 \times 10^{-19}\ \text{C})}{(5.29 \times 10^{-11}\ \text{m})^2} = 5.15 \times 10^{11}\ \text{N/C}.$

21.29: a) $q = -55.0 \times 10^{-6}$ C, and F is downward with magnitude 6.20×10^{-9} N. Therefore, $E = F/q = 1.13 \times 10^{-4}\ \text{N/C}$, upward.

b) If a copper nucleus is placed at that point, it feels an upward force of magnitude $F = qE = (29) \cdot 1.6 \times 10^{-19}\text{C} \cdot 1.13 \times 10^{-4}\ \text{N/C} = 5.24 \times 10^{-22}\ \text{N}.$

21.30: a) The electric field of the Earth points toward the ground, so a NEGATIVE charge will hover above the surface.

$$mg = qE \Rightarrow q = -\frac{(60.0\text{ kg})(9.8\text{ m/s}^2)}{150\text{ N/C}} = -3.92\text{ C}.$$

b) $F = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q^2}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(3.92\text{ C})^2}{(100.00\text{ m})^2} = 1.38\times10^7\text{ N}$. The magnitude of the charge is too great for practical use.

21.31: a) Passing between the charged plates the electron feels a force upward, and just misses the top plate. The distance it travels in the y-direction is 0.005 m. Time of flight $= t = \frac{0.0200\text{ m}}{1.60\times10^6\text{ m/s}} = 1.25\times10^{-8}\text{ s}$ and initial y-velocity is zero. Now,

$y = v_{0y}t + \frac{1}{2}at^2$ so $0.005\text{ m} = \frac{1}{2}a(1.25\times10^{-8}\text{ s})^2 \Rightarrow a = 6.40\times10^{13}\text{ m/s}^2$. But also

$a = \frac{F}{m} = \frac{eE}{m_e} \Rightarrow E = \frac{(9.11\times10^{-31}\text{ kg})(6.40\times10^{13}\text{ m/s}^2)}{1.60\times10^{-19}\text{ C}} = 364\text{ N/C}.$

b) Since the proton is more massive, it will accelerate less, and NOT hit the plates. To find the vertical displacement when it exits the plates, we use the kinematic equations again:

$$y = \frac{1}{2}at^2 = \frac{1}{2}\frac{eE}{m_p}(1.25\times10^{-8}\text{ s})^2 = 2.73\times10^{-6}\text{ m}.$$

c) As mention in b), the proton will not hit one of the plates because although the electric force felt by the proton is the same as the electron felt, a smaller acceleration results for the more massive proton.

d) The acceleration produced by the electric force is much greater than g; it is reasonable to ignore gravity.

21.32: a)

$$\vec{E}_1 = \frac{q_1}{4\pi\varepsilon_0 r_1^2}\hat{j} = \frac{(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(-5.00\times10^{-9}\text{ C})}{(0.0400\text{ m})^2} = (-2.813\times10^4\text{ N/C})\hat{j}$$

$$\left|\vec{E}_2\right| = \frac{q_2}{r_2^2} = \frac{(9\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(3.00\times10^{-9}\text{ C})}{(0.0300\text{m})^2 + (0.0400\text{ m})^2} = 1.08\times10^4\text{ N/C}$$

The angle of $\vec{E}_2$, measured from the x-axis, is $180 - \tan^{-1}\left(\frac{4.00\text{ cm}}{3.00\text{ cm}}\right) = 126.9°$ Thus

$$\begin{aligned}\vec{E}_2 &= (1.080\times10^4\text{ N/C})(\hat{i}\cos126.9° + \hat{j}\sin126.9°)\\ &= (-6.485\times10^3\text{ N/C})\hat{i} + (8.64\times10^3\text{ N/C})\hat{j}\end{aligned}$$

b) The resultant field is

$$\vec{E}_1 + \vec{E}_2 = (-6.485\times10^3\ \text{N/C})\,\hat{i} + (-2.813\times10^4\ \text{N/C} + 8.64\times10^3\ \text{N/C})\,\hat{j}$$
$$= (-6.485\times10^3\ \text{N/C})\,\hat{i} - (1.95\times10^4\ \text{N/C})\,\hat{j}$$

21.33: Let $+x$ be to the right and $+y$ be downward.

Use the horizontal motion to find the time when the electron emerges from the field:

$x - x_0 = 0.0200$ m, $a_x = 0$, $v_{0x} = 1.60\times10^6$ m/s, $t = ?$

$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = 1.25\times10^{-8}$ s

$v_x = 1.60\times10^6$ m/s

$y - y_0 = 0.0050$ m, $v_{0y} = 0$, $t = 1.25\times10^{-8}$ s, $v_y = ?$

$y - y_0 = \left(\dfrac{v_{0y} + v_y}{2}\right)t$ gives $v_y = 8.00\times10^5$ m/s

$v = \sqrt{v_x^2 + v_y^2} = 1.79\times10^6$ m/s

21.34: a) $\vec{E} = -11\ \text{N/C}\,\hat{i} + 14\ \text{N/C}\,\hat{j}$, so $E = \sqrt{(-11)^2 + (14)^2} = 17.8$ N/C.

$\theta = \tan^{-1}(-14/11) = -51.8°$, so $\theta = 128°$ counterclockwise from the x-axis

b) $\vec{F} = \vec{E}q$ so $F = (17.8\ \text{N/C})(2.5\times10^{-9}\ \text{C}) = 4.45\times10^{-8}$ N, i) at $-52°$ (repulsive) ii) at $+128°$ (repulsive).

21.35: a) $F_g = m_e g = (9.11\times10^{-31}\ \text{kg})(9.8\ \text{m/s}^2) = 8.93\times10^{-30}$ N. $F_e = eE = (1.60\times10^{-19}\ \text{C})(1.00\times10^4\ \text{N/C}) = 1.60\times10^{-15}$ N. Yes, ok to neglect F_g because $F_e >> F_g$.

b) $E = 10^4\ \text{N/C} \Rightarrow F_e = 1.6\times10^{-15}\ \text{N} = mg \Rightarrow m = 1.63\times10^{-16}$ kg

$\Rightarrow m = 1.79\times10^{14}\, m_e$.

c) No. The field is uniform.

21.36: a) $x = \dfrac{1}{2}at^2 = \dfrac{1}{2}\dfrac{eE}{m_p}t^2 \Rightarrow E = \dfrac{2(0.0160\ \text{m})(1.67\times10^{-27}\ \text{kg})}{(1.60\times10^{-19}\ \text{C})(1.50\times10^{-6}\ s)^2} = 148$ N/C.

b) $v = v_0 + at = \dfrac{eE}{m_p}t = 2.13\times10^4$ m/s.

21.37: a) $\tan^{-1}\left(\frac{-1.35}{0}\right) = -\frac{\pi}{2}, = \hat{r} - \hat{j}$ b) $\tan^{-1}\left(\frac{12}{.2}\right) = \frac{\pi}{4}, \hat{r} = \frac{\sqrt{2}}{2}\hat{i} + \frac{\sqrt{2}}{2}\hat{j}$

c) $\tan^{-1}\left(\frac{2.6}{+1.10}\right) = 1.97 \text{ radians} = 112.9°, \hat{r} = -0.39\hat{i} + 0.92\hat{j}$ (Second quadrant).

21.38: a) $E = 614 \text{ N/C}, F = qE = 9.82 \times 10^{-17}$ N.

b) $F = e^2 / 4\pi\varepsilon_0 (1.0 \times 10^{-10})^2 = 2.3 \times 10^{-8}$ N.

c) Part (b) >> Part (a), so the electron hardly notices the electric field. A person in the electric field should notice nothing if physiological effects are based solely on magnitude.

21.39: a) Let $+x$ be east.

$\vec{E}$ is west and q is negative, so $\vec{F}$ is east and the electron speeds up.

$F_x = |q|E = (1.602 \times 10^{-19} \text{ C})(1.50 \text{ V/m}) = 2.403 \times 10^{-19}\text{N}$

$a_x = F_x/m = (2.403 \times 10^{-19} \text{ N})/(9.109 \times 10^{-31}\text{kg}) = +2.638 \times 10^{11} \text{ m/s}^2$

$v_{0x} = +4.50 \times 10^5 \text{ m/s}, a_x = +2.638 \times 10^{11} \text{ m/s}^2, x - x_0 = 0.375 \text{ m}, v_x = ?$

$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$ gives $v_x = 6.33 \times 10^5 \text{ m/s}$

b) $q > 0$ so $\vec{F}$ is west and the proton slows down.

$F_x = -|q|E = -(1.602 \times 10^{-19} \text{ C})(1.50 \text{ V/m}) = -2.403 \times 10^{-19}$ N

$a_x = F_x/m = (-2.403 \times 10^{-19} \text{ N})/(1.673 \times 10^{-27}\text{kg}) = -1.436 \times 10^8 \text{ m/s}^2$

$v_{0x} = +1.90 \times 10^4 \text{ m/s}, a_x = -1.436 \times 10^8 \text{ m/s}^2, x - x_0 = 0.375 \text{ m}, v_x = ?$

$v^2{}_x = v^2{}_{0x} + 2a_x(x - x_0)$ gives $v_x = 1.59 \times 10^4 \text{ m/s}$

21.40: Point charges q_1 (0.500 nC) and q_2 (8.00 nC) are separated by $x = 1.20$ m. The electric field is zero when $E_1 = E_2 \Rightarrow \frac{kq_1}{r_1^2} = \frac{kq_2}{(1.20 - r_1)^2} \Rightarrow q_2 r_1^2 = q_1(1.2 - r_1)^2 =$

$q_1 r_1^2 - 2q_1(1.2)r_1 + 1.2^2 q_1 \Rightarrow (q_2 - q_1)r_1^2 + 2(1.2)q_1 r_1 - (1.2)^2 q_1 = 0$ or $7.5r_1^2 + 1.2r_1 - 0.72 = 0$

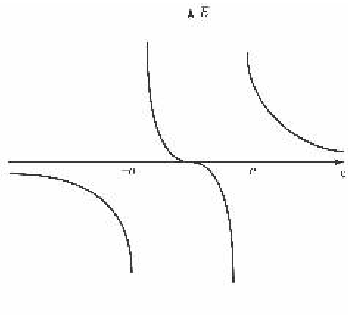
$r_1 = +0.24, -0.4$ $r_1 = 0.24$ is the point between.

21.41: Two positive charges, q, are on the x-axis a distance a from the origin.

a) Halfway between them, $E = 0$.

b) At any position x, $E = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right), & |x| < a \\ \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right), & x > a \\ \dfrac{-1}{4\pi\varepsilon_0}\left(\dfrac{q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right), & x < -a \end{cases}$

For graph, see below.



21.42: The point where the two fields cancel each other will have to be closer to the negative charge, because it is smaller. Also, it cant't be between the two, since the two fields would then act in the same direction. We could use Coulomb's law to calculate the actual values, but a simpler way is to note that the 8.00 nC charge is twice as large as the –4.00 nC Charge. The zero point will therefore have to be a factor of $\sqrt{2}$ farther from the 8.00 nC charge for the two fields to have equal magnitude. Calling x the distance from the –4.00 nC charge:

$$1.20 + x = \sqrt{2}x$$
$$x = 2.90 \text{ m}$$

21.43: a) Point charge q_1 (2.00 nC) is at the origin and q_2 (-5.00 nC) is at $x = 0.800$ m.

i) At $x = 0.200$ m, $E = \dfrac{k\,|q_1|}{(0.200\text{ m})^2} + \dfrac{k\,|q_2|}{(0.600\text{ m})^2} = 575\text{ N/C right.}$

ii) At $x = 1.20$ m, $E = \dfrac{k\,|q_2|}{(0.400\text{ m})^2} + \dfrac{k\,|q_1|}{(1.20\text{ m})^2} = 269\text{ N/C left.}$

iii) At $x = -0.200$ m, $E = \dfrac{k\,|q_1|}{(0.200\text{ m})^2} + \dfrac{k\,|q_2|}{(1.00\text{ m})^2} = 405\text{ N/C left.}$

b) $F = -eE$ i) $F = 1.6\times10^{-19}\text{ C}\cdot 575\text{ N/C} = 9.2\times10^{-17}\text{ N left}$, ii) $F =$

$1.6\times10^{-19}\ \text{C}\cdot 269\ \text{N/C} = 4.3\times10^{-17}\ \text{N right},\ \text{iii)}\ F = 1.6\times10^{-19}\cdot 405 = 6.48\times10^{-17}\text{N right}.$

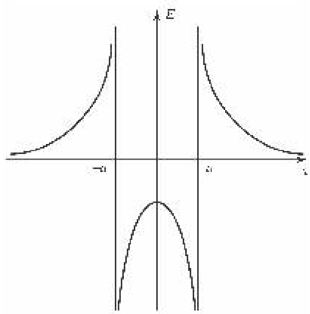
21.44: A positive and negative charge, of equal magnitude q, are on the x-axis, a distance a from the origin.

a) Halfway between them, $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{a^2}$, to the left.

b) At any position x, $E = \begin{cases} \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right), & |x| < a \\ \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{-q}{(a+x)^2} + \dfrac{q}{(a-x)^2}\right), & x > a \\ \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{-q}{(a+x)^2} - \dfrac{q}{(a-x)^2}\right), & x < -a \end{cases}$

with "+" to the right.

This is graphed below.



21.45: a) At the origin, $E = 0$.

b) At $x = 0.3$ m, $y = 0$:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}(6.00\times10^{-9}\ \text{C})\left(\frac{1}{(0.15\ \text{m})^2} + \frac{1}{(0.45\text{m})^2}\right)\hat{i} = 2667\hat{i}\ \text{N/C}.$$

c) At $x = 0.15$ m, $y = -0.4$ m:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}(6.00\times10^{-9}\ \text{C})\left(\frac{-1}{(0.4\ \text{m})^2}\hat{j} + \frac{1}{(0.5\text{m})^2}\frac{0.3}{0.5}\hat{i} - \frac{1}{(0.5\ \text{m})^2}\frac{0.4}{0.5}\hat{j}\right)$$

$\Rightarrow \vec{E} = (129.6\hat{i} - 510.3\hat{j})\ \text{N/C} \Rightarrow E = 526.5\ \text{N/C}$ and $\theta = 75.7^\circ$ down from the x-axis.

d) $x = 0,\ y = 0.2\ \text{m}: \vec{E} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2(6.00\times10^{-9}\ \text{C})\cdot\left(\dfrac{0.2}{0.25}\right)}{(0.25\ \text{m})^2} = 1382\hat{j}\ \text{N/C}$

21.46: Calculate in vector form the electric field for each charge, and add them.

$$\vec{E}_- = \frac{-1}{4\pi\varepsilon_0}\frac{(6.00\times10^{-9}\ \text{C})}{(0.6\ \text{m})^2}\hat{i} = -150\hat{i}\ \text{N/C}$$

$$\vec{E}_+ = \frac{-1}{4\pi\varepsilon_0}(4.00\times10^{-9}\ \text{C})\left(\frac{1}{(1.00\ \text{m})^2}(0.6)\hat{i} + \frac{1}{(1.00\ \text{m})^2}(0.8)\hat{j}\right) = 21.6\hat{i} + 28.8\hat{j}\ \text{N/C}$$

$\Rightarrow E = \sqrt{(128.4)^2 + (28.8)^2} = 131.6\ \text{N/C}$, at $\theta = \tan^{-1}\left(\dfrac{28.8}{128.4}\right) = 12.6^\circ$ up from $-x$ axis.

21.47: a) At the origin, $\vec{E} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{2(6.0\times10^{-9}\ \text{C})}{(0.15\ \text{m})^2}\hat{i} = -4800\hat{i}\ \text{N/C}$.

b) At $x = 0.3\ \text{m},\ y = 0$:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}(6.0\times10^{-9}\text{C})\left(\frac{1}{(0.15\ \text{m})^2} - \frac{1}{(0.45\ \text{m})^2}\right)\hat{i} = 2133\hat{i}\ \text{N/C}.$$

c) At $x = 0.15\ \text{m},\ y = -0.4\ \text{m}$:

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}(6.0\times10^{-9}\ \text{C})\left(\frac{-1}{(0.4\ \text{m})^2}\hat{j} - \frac{1}{(0.5\ \text{m})^2}\frac{0.3}{0.5}\hat{i} + \frac{1}{(0.5\ \text{m})^2}\frac{0.4}{0.5}\hat{j}\right)$$

$\Rightarrow \vec{E} = (-129.6\hat{i} - 164.5\hat{j})\ \text{N/C} \Rightarrow E = 209\ \text{N/C}$ and $\theta = 232°$ clockwise from $+x$-axis.

d) $x = 0, y = 0.2\ \text{m}: E_y = 0, \vec{E} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{2(6.00\times10^{-9}\ \text{C})\left(\frac{0.15}{0.25}\right)}{(0.25\ \text{m})^2} = -1037\hat{i}\ \text{N/C}$

21.48: For a long straight wire, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r} \Rightarrow r = \dfrac{1.5\times10^{-10}\ \text{C/m}}{2\pi\varepsilon_0(2.5\ \text{N/C})} = 1.08\ \text{m}$.

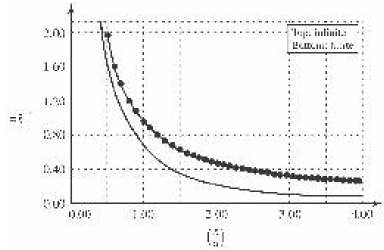
21.49: a) For a wire of length $2a$ centered at the origin and lying along the y-axis, the electric field is given by Eq. (21.10).

$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x\sqrt{x^2/a^2+1}}\hat{i}$$

b) For an infinite line of charge:

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 x}\hat{i}$$

Graphs of electric field versus position for both are shown below.



21.50: For a ring of charge, the electric field is given by Eq. (21.8).

a) $\vec{\boldsymbol{E}} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}\hat{\boldsymbol{i}}$ so with

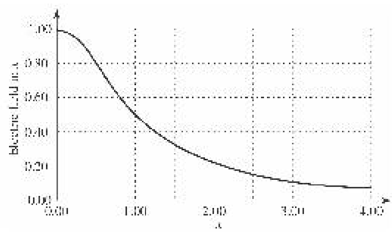
$Q = 0.125\times10^{-9}$ C, $a = 0.025$ m and $x = 0.4$ m $\Rightarrow \vec{\boldsymbol{E}} = 7.0\hat{\boldsymbol{i}}$ N/C.

b) $\vec{\boldsymbol{F}}_{\text{on ring}} = -\vec{\boldsymbol{F}}_{\text{on q}} = -q\,\vec{\boldsymbol{E}} = -(-2.50\times10^{-6}\text{ C})(7.0\hat{\boldsymbol{i}}\text{ N/C}) = 1.75\times10^{-5}\hat{\boldsymbol{i}}\text{ N}.$

21.51: For a uniformly charged disk, the electric field is given by Eq. (21.11):

$$\vec{\boldsymbol{E}} = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{R^2/x^2+1}}\right)\hat{\boldsymbol{i}}$$

The x-component of the electric field is shown below.



21.52: The earth's electric field is 150 N/C, directly downward. So,

$E = 150 = \dfrac{\sigma}{2\varepsilon_0} \Rightarrow \sigma = 300\varepsilon_0 = 2.66\times10^{-9}\text{ C/m}^2$, and is negative.

21.53: For an infinite plane sheet, E is constant and is given by $E = \dfrac{\sigma}{2\varepsilon_0}$ directed perpendicular to the surface.

$$\sigma = 2.5\times10^6\,\frac{e^-}{\text{cm}^2}\left(-1.6\times10^{-19}\frac{\text{C}}{e^-}\right)\cdot\left(\frac{100\text{ cm}}{1\text{ m}}\right)^2 = -4\ \times10^{-9}\,\frac{\text{C}}{\text{m}^2}$$

so $E = \dfrac{4 \times 10^{-9}\ \frac{\text{C}}{\text{m}^2}}{2\varepsilon_0} = 226\ \text{N/C}$ directed toward the surface.

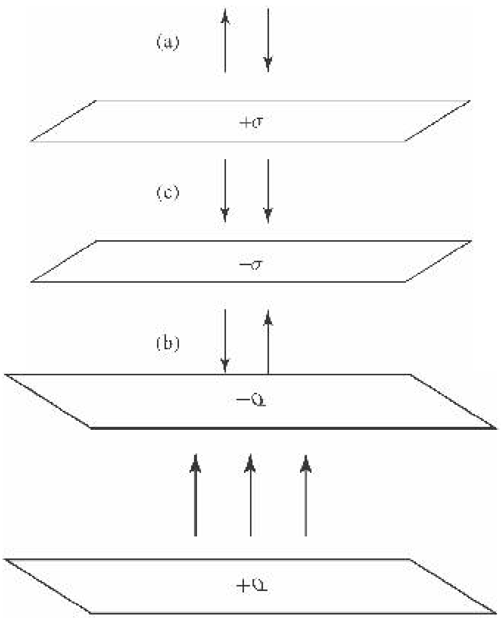
21.54: By superposition we can add the electric fields from two parallel sheets of charge.

a) $E = 0.$

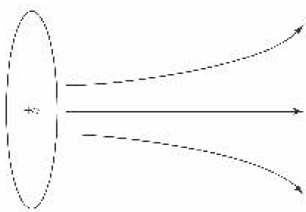
b) $E = 0.$

c) $E = 2\dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{\varepsilon_0}$, directed downward.

21.55:

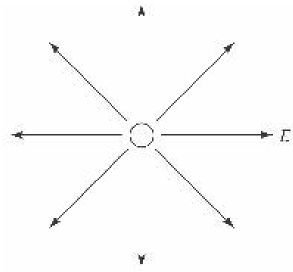


21.56: The field appears like that of a point charge a long way from the disk and an infinite plane close to the disk's center. The field is symmetrical on the right and left (not shown).

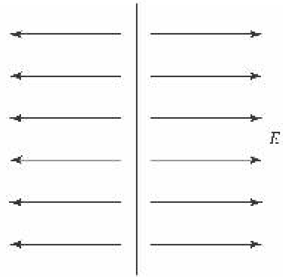


21.57: An infinite line of charge has a radial field in the plane through the wire, and constant in the plane of the wire, mirror-imaged about the wire:

Cross section through the wire:



Plane of the wire:



Length of vector does not depend on angle.

Length of vector gets shorter at points further away from wire.

21.58: a) Since field lines pass from positive charges and toward negative charges, we can deduce that the top charge is positive, middle is negative, and bottom is positive.

b) The electric field is the smallest on the horizontal line through the middle charge, at two positions on either side where the field lines are least dense. Here the y-components of the field are cancelled between the positive charges and the negative charge cancels the x-component of the field from the two positive charges.

21.59: a) $p = qd \Rightarrow (4.5 \times 10^{-9}\ \text{C})(0.0031\ \text{m}) = 1.4 \times 10^{-11}\ \text{C} \cdot \text{m}$, in the direction from and towards q_2.

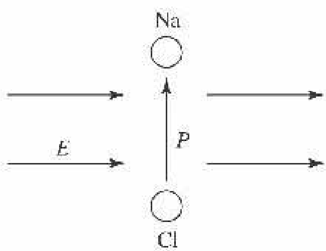
b) If $\vec{E}$ is at 36.9°, and the torque $\tau = pE \sin\phi$, then:

$$E = \frac{\tau}{p \sin\phi} = \frac{7.2 \times 10^{-9}\ \text{N} \cdot \text{m}}{(1.4 \times 10^{-11}\ \text{C} \cdot \text{m}) \sin 36.9°} = 856.5\ \text{N/C}.$$

21.60: a) $d = p/q = (8.9 \times 10^{-30}\ \text{C} \cdot \text{m})/(1.6 \times 10^{-19}\ \text{C}) = 5.56 \times 10^{-11}\ \text{m}.$

b) $\tau_{\text{max}} = pE = (8.9 \times 10^{-30}\ \text{C} \cdot \text{m})(6.0 \times 10^{5}\ \text{N/C}) = 5.34 \times 10^{-24}\ \text{N} \cdot \text{m}.$

Maximum torque:



21.61: a) Changing the orientation of a dipole from parallel to perpendicular yields:
$\Delta U = U_f - U_i = -(pE\cos 90° - pE\cos 0°) = +(5.0\times 10^{-30}\ \text{C}\cdot\text{m})(1.6\times 10^{6}\ \text{N/C}) =$
$+8\times 10^{-24}$ J.

b) $\frac{3}{2}kT = 8\times 10^{-24}\ \text{J} \Rightarrow T = \frac{2(8\times 10^{-24}\ \text{J})}{3(1.38\times 10^{-23}\ \text{J/K})} = 0.384\ \text{K}.$

21.62: $E_{\text{dipole}}(x) = \frac{p}{2\pi\varepsilon_0 x^3} \Rightarrow E_{\text{dipole}}(3.00\times 10^{-9}\ \text{m}) = \frac{6.17\times 10^{-30}\ \text{C}\cdot\text{m}}{2\pi\varepsilon_0(3.0\times 10^{-9}\ \text{m})^3} = 4.11$
$\times 10^{6}$ N/C. The electric force
$F = qE = (1.60\times 10^{-19}\ \text{C})(4.11\times 10^{6}\ \text{N/C}) = 6.58\times 10^{-13}$ N and is toward the water molecule (negative x-direction).

21.63: a) $\frac{1}{(y-d/2)^2} - \frac{1}{(y+d/2)^2} = \frac{(y+d/2)^2-(y-d/2)^2}{(y^2-d^2/4)^2} = \frac{2yd}{(y^2-d^2/4)^2}$

$\Rightarrow E_y = \frac{q}{4\pi\varepsilon_0}\frac{2yd}{(y^2-d^2/4)^2} = \frac{qd}{2\pi\varepsilon_0}\frac{y}{(y^2-d^2/4)^2} \approx \frac{p}{2\pi\varepsilon_0 y^3}.$

b) This also gives the correct expression for E_y since y appears in the full expression's denominator squared, so the signs carry through correctly.

21.64: a) The magnitude of the field the due to each charge is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{(d/2)^2+x^2)}\right),$$

where d is the distance between the two charges. The x-components of the forces due to the two charges are equal and oppositely directed and so cancel each other. The two fields have equal y-components, so:

$$E = 2E_y = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{(d/2)^2+x^2}\right)\sin\theta$$

where θ is the angle below the x-axis for both fields.

$$\sin\theta = \frac{d/2}{\sqrt{(d/2)^2+x^2}};$$

thus

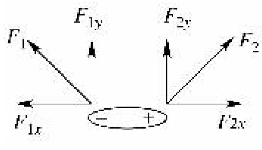
$$E_{\text{dipole}} = \left(\frac{2q}{4\pi\varepsilon_0}\right)\left(\frac{1}{(d/2)^2+x^2}\right)\left(\frac{d/2}{\sqrt{(d/2)^2+x^2}}\right) = \frac{qd}{4\pi\varepsilon_0((d/2)^2+x^2)^{3/2}}$$

The field is the $-y$ directions.

b) At large x, $x^2 >> (d/2)^2$, so the relationship reduces to the approximations

$$E_{\text{dipole}} \approx \frac{qd}{4\pi\varepsilon_0 x^3}$$

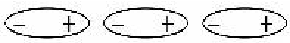
21.65:



The dipoles attract.

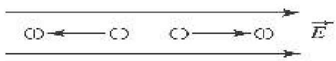
$F_x = F_{1x} + F_{2x} = 0, \quad F_y = F_{1y} + F_{2y} = 2F_{1y}$

b)



Opposite charges are closest so the dipoles attract.

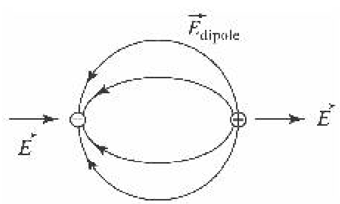
21.66: a)



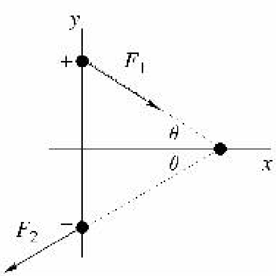
The torque is zero when $\vec{p}$ is aligned either in the *same* direction as $\vec{E}$ or in the *opposite* directions

b) The stable orientation is when $\vec{p}$ is aligned in the *same* direction as $\vec{E}$

c)



21.67:



$\sin\theta = 1.50/2.00$ so $\theta = 48.6°$

Opposite charges attract and like charges repel.

$F_x = F_{1x} + F_{2x} = 0$

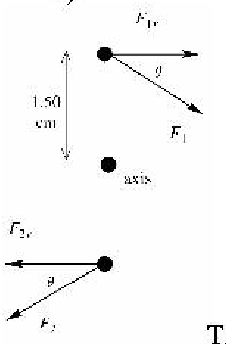
$$F_1 = k\frac{|qq'|}{r^2} = k\frac{(5.00\times10^{-6}\ \text{C})(10.0\times10^{-6}\ \text{C})}{(0.0200\ \text{m})^2} = 1.124\times10^3\ \text{N}$$

$$F_{1y} = -F_1\sin\theta = -842.6\ \text{N}$$

$$F_{2y} = -842.6\ \text{N so } F_y = F_{1y} + F_{2y} = -1680\ \text{N}$$

(in the direction from the $+5.00\text{-}\mu\text{C}$ charge toward the $-5.00\text{-}\mu\text{C}$ charge).

b)



The $y-$components have zero moment arm and therefore zero torque.

F_{1x} and F_{2x} both produce clockwise torques.

$$F_{1x} = F_1\cos\theta = 743.1\ \text{N}$$

$$\tau = 2(F_{1x})(0.0150\ \text{m}) = 22.3\ \text{N}\cdot\text{m, clockwise}$$

21.68: a) $\vec{F}_{13} = +\left|\frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}^2}\right|\cos\theta\hat{i} + \left|\frac{1}{4\pi\varepsilon_0}\frac{q_1q_3}{r_{13}^2}\right|\sin\theta\hat{j}$

$$\Rightarrow \vec{F}_{13} = +\frac{1}{4\pi\varepsilon_0}\frac{(5.00\ \text{nC})(6.00\ \text{nC})}{((9.00+16.0)\times10^{-4}\ \text{m})}\frac{4}{5}\hat{i} + \frac{1}{4\pi\varepsilon_0}\frac{(5.00\ \text{nC})(6.00\ \text{nC})}{((9.00+16.0)\times10^{-4}\text{m})}\frac{3}{5}\hat{j}$$

$$\Rightarrow \vec{F}_{13} = +(8.64\times10^{-5}\ \text{N})\hat{i} + (6.48\times10^{-5}\ \text{N})\hat{j}.$$

Similarly for the force from the other charge:

$$\vec{F}_{23} = \frac{-1}{4\pi\varepsilon_0}\frac{q_2q_3}{r_{23}^2}\hat{j} = \frac{-1}{4\pi\varepsilon_0}\frac{(2.00\ \text{nC})(6.00\ \text{nC})}{(0.0300\ \text{m})^2}\hat{j} = -(1.20\times10^{-4}\ N)\hat{j}$$

Therefore the two force components are:

$$F_x = 8.64\times10^{-5}\ \text{N} \qquad F_y = 6.48\times10^{-5} - 12.0\times10^{-5} = -5.52\times10^{-5}\text{N}$$

b) Thus, $F = \sqrt{F_x^2 + F_y^2} = \sqrt{(8.64\times10^{-5}\ \text{N})^2 + (-5.52\times10^{-5}\ \text{N})^2} = 1.03\times10^{-4}\ \text{N}$, and the angle is $\theta = \arctan(F_y/F_x) = 32.6$, below the x axis

21.69: a) $F_q = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(a+x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{qQ}{(a-x)^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\left(\frac{1}{(1+x/a)^2} - \frac{1}{(1-x/a)^2}\right)$

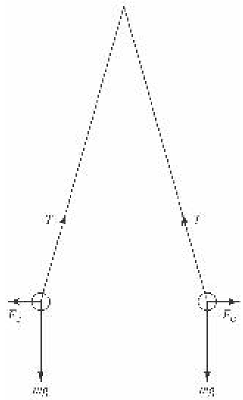
$\Rightarrow F_q \approx \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}(1-2\frac{x}{a}\ldots-(1+2\frac{x}{a}\ldots)) = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2}\left(-4\frac{x}{a}\right) = -\left(\frac{qQ}{\pi\varepsilon_0 a^3}\right)x$. But this is the equation of a simple harmonic oscillator, so:

$$\omega = 2\pi f = \sqrt{\frac{qQ}{m\pi\,\varepsilon_0 a^3}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{qQ}{m\pi\,\varepsilon_0 a^3}} = \sqrt{\frac{kqQ}{m\pi^2 a^3}}.$$

b) If the charge was placed on the y-axis there would be no restoring force if q and Q had the same sign. It would move straight out from the origin along the y-axis, since the x-components of force would cancel.

21.70: Examining the forces: $\sum F_x = T\sin\theta - F_e = 0$ and $\sum F_y = T\cos\theta - mg = 0$.
So $\frac{mg\sin\theta}{\cos\theta} = F_e = \frac{kq^2}{d^2}$ But $\tan\theta \approx \frac{d}{2L} \Rightarrow d^3 = \frac{2kq^2L}{mg} \Rightarrow d = \left(\frac{q^2L}{2\pi\varepsilon_0 mg}\right)^{1/3}$.

21.71: a)



b) Using the same force analysis as in problem **21.70,** we find:
$q^2 = 4\pi\varepsilon_0 d^2 mg\,\tan\theta$ and
$d = 2\cdot(1.2)\sin 25 \Rightarrow q = \sqrt{4\pi\varepsilon_0(2\cdot(1.2)\cdot\sin 25°)^2\tan 25°(0.015\text{ kg})(9.80\text{ m/s}^2)} \Rightarrow q =$ 2.79×10^{-6} C.

c) From Problem 21.70, $mg\,\tan\theta = \frac{kq^2}{d^2}$.

$$\sin\theta = \frac{d}{2L} \Rightarrow \tan\theta = \frac{kq^2}{mg(2L)^2\sin^2\theta} = \frac{(8.99\times10^9\text{ Nm}^2/\text{C})(2.79\times10^{-6}\text{ C})^2}{4(0.6\text{m})^2(0.015\text{ kg})(9.8\text{ m/s}^2)\sin^{-2}\theta}$$

Therefore $\tan\theta = \frac{0.331}{\sin^2\theta}$. Numerical solution of this transcendental equation leads to $\theta = 39.5°$.

21.72: a) Free body diagram as in **21.71.** Each charge still feels equal and opposite electric forces.

b) $T = mg/\cos 20° = 0.0834$ N so $F_e = T\sin 20° = 0.0285\text{ N} = \frac{kq_1q_2}{r_1^2}$. (Note: $r_1 = 2(0.500\text{ m})\sin 20° = 0.342$ m.)

c) From part (b), $q_1 q_2 = 3.71\times 10^{-13}\ \text{C}^2$.

d) The charges on the spheres are made equal by connecting them with a wire, but we still have $F_e = mg\tan\theta = 0.0453\ \text{N} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{r_2^2}$ where $Q = \frac{q_1+q_2}{2}$. But the separation r_2 is known: $r_2 = 2(0.500\ \text{m})\sin 30° = 0.500\ \text{m}$. Hence: $Q = \frac{q_1+q_2}{2} = \sqrt{4\pi\varepsilon_0 F_e r^2{}_2}$ $= 1.12\times 10^{-6}$ C. This equation, along with that from part (b), gives us two equations in q_1 and q_2. $q_1 + q_2 = 2.24\times 10^{-6}$ C and $q_1 q_2 = 3.70\times 10^{-13}\ \text{C}^2$. By elimination, substitution and after solving the resulting quadratic equation, we find: $q_1 = 2.06\times 10^{-6}$ C and $q_2 = 1.80\times 10^{-7}$ C.

21.73: a) $0.100\ \text{mol NaCl} \Rightarrow m_{\text{Na}} = (0.100\ \text{mol})(22.99\ \text{g/mol}) = 2.30\ \text{g}$

$$\Rightarrow m_{\text{Cl}} = (0.100\ \text{mol})(35.45\ \text{g/mol}) = 3.55\ \text{g}$$

Also the number of ions is $(0.100\ \text{mol})N_A = 6.02\times 10^{22}$ so the charge is: $q = (6.02\times 10^{22})(1.60\times 10^{-19}\ \text{C}) = 9630\ \text{C}$. The force between two such charges is:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(9630)^2}{(0.0200\ \text{m})^2} = 2.09\times 10^{21}\ \text{N}.$$

b) $a = F/m = (2.09\times 10^{21}\ \text{N})/(3.55\times 10^{-3}\ \text{kg}) = 5.89\times 10^{23}\ \text{m/s}^2$.

c) With such a large force between them, it does not seem reasonable to think the sodium and chlorine ions could be separated in this way.

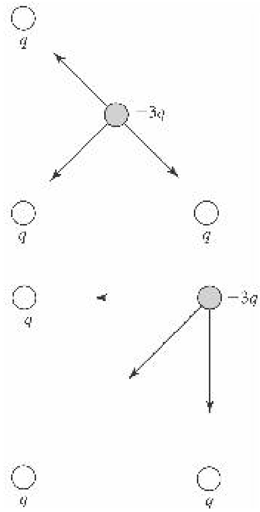
21.74: a) $F_3 = 4.0\times 10^{-6}\ \text{N} = \frac{kq_1q_3}{r_{13}^2} + \left|\frac{kq_2q_3}{r_{23}^2}\right| = kq_3\left(\frac{(2.5\times 10^{-9}\ \text{C})}{(-0.3\ \text{m})^2} + \frac{4.5\times 10^{-9}\ \text{C}}{(+0.2\ \text{m})^2}\right) \Rightarrow$

$$q_3 = \frac{4.0\times 10^{-6}\ \text{N}}{(1262\ \text{N/C})} = 3.2\ \text{nC}.$$

b) The force acts on the middle charge to the right.

c) The force equals zero if the two forces from the other charges cancel. Because of the magnitude and size of the charges, this can only occur to the left of the negative charge q_2. Then: $F_{13} = F_{23} \Rightarrow \frac{kq_1}{(0.300-x)^2} = \frac{kq_2}{(-0.200-x)^2}$ where x is the distance from the origin. Solving for x we find: $x = -1.76$ m. The other value of x was between the two charges and is not allowed.

21.75: a) $F = +\frac{1}{4\pi\varepsilon_0}\frac{q(3q)}{(L/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{6q^2}{L^2}$, toward the lower the left charge. The other two forces are equal and opposite.



b) The upper left charge and lower right charge have equal magnitude forces at right angles to each other, resulting in a total force of twice the force of one, directed toward the lower left charge. So, all the forces sum to:

$$F=\frac{1}{4\pi\varepsilon_0}\left(\frac{q(3q)\sqrt{2}}{L^2}+\frac{q(3q)}{(\sqrt{2}L)^2}\right)=\frac{q^2}{4\pi\varepsilon_0 L^2}\left(3\sqrt{2}+\frac{3}{2}\right)\text{N.}$$

21.76: a) $E(p)=\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{(y-a)^2}+\dfrac{q}{(y+a)^2}-\dfrac{2q}{y^2}\right).$

b) $E(p)=\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{y^2}((1-a/y)^{-2}+(1+a/y)^{-2}-2)$. Using the binomial expansion:

$$\Rightarrow E(p)\approx\frac{1}{4\pi\varepsilon_0}\frac{q}{y^2}\left(1+\frac{2a}{y}+\frac{3a^2}{y^2}+\cdots+1-\frac{2a}{y}+\frac{3a^2}{y^2}+\cdots-2\right)=\frac{1}{4\pi\varepsilon_0}\frac{6qa^2}{y^4}.$$

Note that a point charge drops off like $\dfrac{1}{y^2}$ and a dipole like $\dfrac{1}{y^3}$.

21.77: a) The field is all in the x-direction (the rest cancels). From the $+q$ charges:

$$E=\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2+x^2}\Rightarrow E_x=\frac{1}{4\pi\varepsilon_0}\frac{q}{a^2+x^2}\frac{x}{\sqrt{a^2+x^2}}=\frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2+x^2)^{3/2}}.$$

(Each $+q$ contributes this). From the $-2q$:

$$E_x=-\frac{1}{4\pi\varepsilon_0}\frac{2q}{x^2}\Rightarrow E_{\text{total}}=\frac{1}{4\pi\varepsilon_0}\left(\frac{2qx}{(a^2+x^2)^{3/2}}-\frac{2q}{x^2}\right)=\frac{1}{4\pi\varepsilon_0}\frac{2q}{x^2}((a^2/x^2+1)^{-3/2}-1).$$

b) $E_{\text{total}}\approx\dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{x^2}\left(1-\dfrac{3a^2}{2x^2}+\cdots-1\right)=\dfrac{1}{4\pi\varepsilon_0}\dfrac{3qa^2}{x^4}$, for $x\gg a$..

Note that a point charge drops off like $\frac{1}{x^2}$ and a dipole like $\frac{1}{x^3}$.

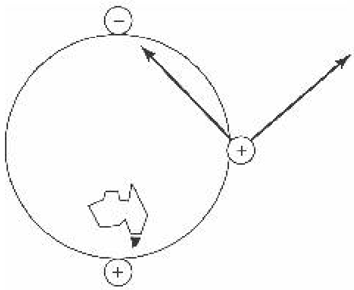
21.78: a) 20.0 g carbon $\Rightarrow \frac{20.0\ \text{g}}{12.0\ \text{g/mol}} = 1.67$ mol carbon $\Rightarrow 6(1.67) = 10.0$ mol electrons $\Rightarrow q = (10.0)N_A(1.60\times10^{-19}\text{C}) = 0.963\times10^{6}$ C. This much charge is placed at the earth's poles (negative at north, positive at south), leading to a force:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R_{\text{earth}})^2} = \frac{1}{4\pi\varepsilon_0}\frac{(0.963\times10^6\text{C})^2}{(1.276\times10^7\text{m})^2} = 5.13\times10^7\text{N}.$$

b) A positive charge at the equator of the same magnitude as above will feel a force in the south-to-north direction, perpendicular to the earth's surface:

$$F = 2\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2R_{\text{earth}})^2}\sin 45^\circ$$

$$\Rightarrow F = 2\frac{1}{4\pi\varepsilon_0}\frac{4}{\sqrt{2}}\frac{(0.963\times10^6\text{C})^2}{(1.276\times10^7\text{m})^2} = 1.44\times10^8\ \text{N}.$$

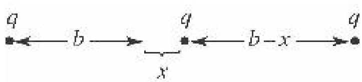


21.79: a) With the mass of the book about 1.0 kg, most of which is protons and neutrons, we find: #protons $= \frac{1}{2}(1.0\ \text{kg})/(1.67\times10^{-27}\ \text{kg}) = 3.0\times10^{26}$. Thus the charge difference present if the electron's charge was 99.999% of the proton's is $\Delta q = (3.0\times10^{26})(0.00001)(1.6\times10^{-19}\ \text{C}) = 480$ C.

b) $F = k(\Delta q)^2/r^2 = k(480\ \text{C})^2/(5.0\ \text{m})^2 = 8.3\times10^{13}$ N – repulsive. The acceleration $a = F/m = (8.3\times10^{13}\ \text{N})/(1\ \text{kg}) = 8.3\times10^{13}\ \text{m/s}^2$.

c) Thus even the slightest charge imbalance in matter would lead to explosive repulsion!

21.80: (a)



$$F_{net}(\text{on central charge}) = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(b-x)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(b+x)^2}$$

$$= \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{(b-x)^2} - \frac{1}{(b+x)^2}\right]$$

$$= \frac{q^2}{4\pi\varepsilon_0}\frac{(b+x)^2-(b-x)^2}{(b-x)^2(b+x)^2} = \frac{q^2}{4\pi\varepsilon_0}\frac{4bx}{(b-x)^2(b+x)^2}$$

For $x << b$, this expression becomes

$$F_{net} \approx \frac{q^2}{\pi\varepsilon_0}\frac{bx}{b^2b^2} = \frac{q^2}{\pi\varepsilon_0 b^3}x$$ Direction is opposite to x.

(b) $\sum F = ma : -\frac{-q^2}{\pi\varepsilon_0 b^3}x = m\frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} = -\left(\frac{q^2}{m\pi\varepsilon_0 b^3}\right)x$$

$$\omega = \sqrt{\frac{q^2}{m\pi\varepsilon_0 b^3}} = 2\pi\ f \rightarrow f = \frac{1}{2\pi}\sqrt{\frac{q^2}{m\pi\varepsilon_0 b^3}}$$

(c) $q = e, b = 4.0\times10^{-10}\,\text{m},\ m = 12\,\text{amu} = 12(1.66\times10^{-27}\ \text{kg})$

$$f = \frac{1}{2\pi}\sqrt{\frac{(1.6\times10^{-19}\ \text{C})^2}{12(1.66\times10^{-27}\ \text{kg})\pi(8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2)(4.0\times10^{-10}\ \text{m})^3}} = 4.28\times10^{12}\,\text{Hz}$$

21.81: a) $m = \rho V = \rho(\frac{4}{3}\pi r^3) = (8.9\times10^3\ \text{kg}/\text{m}^3)(\frac{4}{3}\pi)(1.00\times10^{-3}\ \text{m})^3 =$
3.728×10^{-5} kg

$n = m/M = (3.728\times10^{-5}\ \text{kg})(63.546\times10^{-3}\ \text{kg}/\text{mol}) = 5.867\times10^{-4}$ mol

$N = nN_A = 3.5\times10^{20}$ atoms

(b) $N_e = (29)(3.5\times10^{20}) = 1.015\times10^{22}$ electrons and protons

$q_{net} = eN_e - (0.99900)eN_e = (0.100\times10^{-2})(1.602\times10^{-19}\ \text{C})(1.015\times10^{22}) = 1.6\ \text{C}$

$$F = k\frac{q^2}{r^2} = k\frac{(1.6\ \text{C})^2}{(1.00\ \text{m})^2} = 2.3\times10^{10}\ \text{N}$$

21.82: First, the mass of the drop:

$$m = \rho V = (1000\ \text{kg}/\text{m}^3)\left(\frac{4\pi(15.0\times10^{-6}\ \text{m})^3}{3}\right) = 1.41\times10^{-11}\ \text{kg}.$$

Next, the time of flight: $t = D/v = 0.02/20 = 0.00100$ s and the acceleration :

$$d = \frac{1}{2}at^2 \Rightarrow a = \frac{2d}{t^2} = \frac{2(3.00\times10^{-4}\ \text{m})}{(0.001\ \text{s})^2} = 600\ \text{m}/\text{s}^2.$$

So:

$$a = F/m = qE/m \Rightarrow q = ma/E = \frac{(1.41\times10^{-11}\ \text{kg})(600\ \text{m/s}^2)}{8.00\times10^{4}\ \text{N/C}} = 1.06\times10^{-13}\ \text{C}.$$

21.83: $F_y = eE \qquad F_x = 0$

$$a_y = \frac{F_y}{m_p} = \frac{eE}{m_p} \quad a_x = 0$$

a) $$v_y^2 = v_{0y}^2 + 2a_y\Delta y = v_0^2\sin^2\alpha + \frac{2eE}{m_p}\Delta y \qquad |\Delta y| = h_{\max} \text{ when } v_y = 0$$

$$\Rightarrow h_{\max} = \frac{v_0^2 m_p \sin^2\alpha}{2eE}$$

b) $$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$t = t_{\text{orig}} \text{ when } \Delta y = 0$$

$$\Rightarrow 0 = \left(-v_0\sin\alpha + \frac{1}{2}a_y t_{\text{orig}}\right)t_{\text{orig}}$$

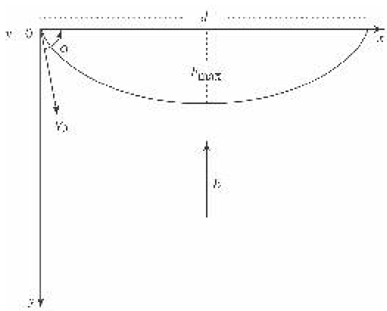
$$\text{so } t_{\text{orig}} = 0, \frac{2v_0\sin\alpha}{a_y}$$

or

$$t_{\text{orig}} = \frac{2v_0 m_p\sin\alpha}{eE}$$

$$d = v_{0x}t_{\text{orig}} = \frac{2v_0^2 m_p}{eE}\cos\alpha\sin\alpha$$

c)



d) $$h_{\max} = \frac{(4\times10^{5}\ \text{m/s})^2(1.67\times10^{-27}\text{kg})\sin^2 30°}{2(1.6\times10^{-19}\ \text{C})(500\ \text{N/C})} = 0.42\ \text{m}$$

$$d = \frac{2(4\times10^{5}\text{m/s})^2(1.67\times10^{27}\ \text{kg})\cos30°\sin30°}{(1.6\times10^{-19}\ \text{C})(500\ \text{N/C})} = 2.89\ \text{m}$$

21.84: a) $$E = 50\ \text{N/C} = \left|\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\right| + \left|\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\right| = \frac{1}{4\pi\varepsilon_0}\left(\left|\frac{q_1}{r_1^2}\right| + \left|\frac{q_2}{r_2^2}\right|\right) \Rightarrow q_2 =$$

$$r_2^2\left(4\pi\varepsilon_0 E-\left|\frac{q_1}{r_1^2}\right|\right)\Rightarrow q_2=(1.2\text{ m})^2\left(4\pi\varepsilon_0 50.0\text{ N/C}-\frac{(4.00\times10^{-9}\text{ C})}{(0.6\text{ m})^2}\right)=-8\times10^{-9}\text{ C.}$$

b) $$E=-50\text{ N/C}=\left|\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\right|+\left|\frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\right|=\frac{1}{4\pi\varepsilon_0}\left(\left|\frac{q_1}{r_1^2}\right|+\left|\frac{q_2}{r_2^2}\right|\right)\Rightarrow q_2=$$

$$r_2^2\left(\frac{-50}{k}-\left|\frac{q_1}{r_1^2}\right|\right)\Rightarrow q_2=(1.2\text{ m})^2\left(4\pi\varepsilon_0(-50.0)-\frac{(4.00\times10^{-9}\text{ C})}{(0.6\text{ m})^2}\right)=-24.0\times10^{-9}\text{ C.}$$

21.85: $$E=12.0\text{ N/C}=\frac{-k(16.0\text{ nC})}{(3.00\text{ m})^2}+\frac{k(12.0\text{ nC})}{(8.00\text{ m})^2}+\frac{kq}{(5.00\text{ m})^2}$$

$$\Rightarrow q=25.0\text{ m}^2\left(\frac{12}{k}+\frac{1.60\times10^{-8}\text{ C}}{9.0\text{ m}^2}-\frac{1.20\times10^{-8}\text{ C}}{64.00\text{ m}^2}\right)=+7.31\times10^{-8}\text{ C}=+73.1\text{ nC.}$$

21.86: a) On the x-axis: $$dE_x=\frac{1}{4\pi\varepsilon_0}\frac{dq}{(a+r)^2}\Rightarrow E_x=\frac{1}{4\pi\varepsilon_0}\int_0^a\frac{Qdx}{a(a+r-x)^2}=$$

$$\frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\left(\frac{1}{r}-\frac{1}{a+r}\right).$$ And $E_y=0$.

b) If $a+r=x$, then $$E=\frac{1}{4\pi\varepsilon_0}\frac{Q}{a}\left(\frac{1}{x-a}-\frac{1}{x}\right)\Rightarrow\vec{F}=q\vec{E}=\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a}\left(\frac{1}{x-a}-\frac{1}{x}\right)\hat{\imath}.$$

c) For $x>>a$, $$F=\frac{kqQ}{ax}((1-a/x)^{-1}-1)=\frac{kqQ}{ax}(1+a/x+\cdots-1)\approx\frac{kqQ}{x^2}\approx$$

$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$. (Note that for $x>>a$, $r=x-a\approx x$.) Charge distribution looks like a point from far away.

21.87: a) $$dE=\frac{k\,dq}{(x^2+y^2)}=\frac{kQ\,dy}{a(x^2+y^2)}$$ with $$dE_x=\frac{kQx\,dy}{a(x^2+y^2)^{3/2}}$$ and $dE_y=$

$\frac{-KQy\,dy}{a(x^2+y^2)^{3/2}}$. Thus:

$$E_x=\frac{1}{4\pi\varepsilon_0}\frac{Qx}{a}\int_0^a\frac{dy}{(x^2+y^2)^{3/2}}=\frac{1}{4\pi\varepsilon_0}\frac{Qx}{a}\frac{1}{(x^2+a^2)^{1/2}}\frac{a}{x^2}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{x(x^2+a^2)^{1/2}}$$

$$E_y=\frac{-1}{4\pi\varepsilon_0}\frac{Q}{a}\int_0^a\frac{ydy}{(x^2+y^2)^{3/2}}=\frac{-1}{4\pi\varepsilon_0}\frac{Q}{a}\left(\frac{1}{x}-\frac{1}{(x^2+a^2)^{1/2}}\right)$$

b) $F_x=-qE_x$ and $F_y=-qE_y$ where E_x and E_y are given in (a).

c) For $x >> a$, $F_y = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{ax}(1-(1+a^2/x^2)^{-1/2}) \approx \frac{1}{4\pi\varepsilon_0}\frac{qQ}{ax}\frac{a^2}{2x^2} = \frac{1}{4\pi\varepsilon_0}\frac{qQa}{2x^3}.$

Looks dipole-like in y-direction $F_x = -\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2}\left(1+\frac{a^2}{x^2}\right)^{-1/2} \approx \frac{qQ}{4\pi\varepsilon_0 x^2}.$

Looks point-like along x-direction

21.88: a) From *Eq.* (22.9), $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{i}$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{(-9.00\times10^{-9}\ C)}{(2.5\times10^{-3}\ \text{m})\sqrt{(2.5\times10^{-3}\ \text{m})^2+(0.025\ \text{m})^2}}\hat{i} = (-1.29\times10^6\ \text{N/C})\hat{i}.$$

(b) The electric field is less than that at the same distance from an infinite line of charge $(E_{a\to\infty} = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{x} = \frac{-1}{4\pi\varepsilon_0}\frac{2Q}{x2a} = -1.30\times10^6\ \text{N/C})$. This is because in the approximation, the terms left off were negative. $\frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x\left(1+\frac{x^2}{a^2}\right)^{1/2}} \approx \frac{\lambda}{2\pi\varepsilon_0 x}\left(1-\frac{x^2}{2a^2}+\cdots\right) =$

$E_{\text{Line}} \infty -$ (Higher order terms).

c) For a 1% difference, we need the next highest term in the expansion that was left off to be less than 0.01:

$$\frac{x^2}{2a^2} < 0.01 \Rightarrow x < a\sqrt{2(0.01)} = 0.025\text{m}\sqrt{2(0.01)} \Rightarrow x < 0.35\ \text{cm}.$$

21.89: (a) From Eq. (22.9), $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x\sqrt{x^2+a^2}}\hat{i}$

$$\Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{(-9.0\times10^{-9}\ \text{C})}{(0.100\ \text{m})\sqrt{(0.100\ \text{m})^2+(0.025\ \text{m})^2}} = (-7858\ \text{N/C})\hat{i}.$$

b) The electric field is less than that at the same distance from a point charge (8100 N/C). Since $E_{x\to\infty} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{x^2}\left(1-\frac{a^2}{2x^2}+\cdots\right) = E_{\text{point}} -$(Higher order terms).

c) For a 1% difference, we need the next highest term in the expansion that was left off to be less than 0.01:

$$\frac{a^2}{2x^2} \approx 0.01 \Rightarrow x \approx a\sqrt{1/(2(0.01))} = 0.025\sqrt{1/0.02} \Rightarrow x \approx 0.177 \text{ m.}$$

21.90: (a) On the axis,

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \left(\frac{R^2}{x^2}+1\right)^{-1/2}\right] = \frac{4.00 \text{ pC}/\pi(0.025 \text{ m})^2}{2\varepsilon_0}\left[1 - \left(\frac{(0.025 \text{ m})^2}{(0.0020 \text{ m})^2}+1\right)^{-1/2}\right]$$

$\Rightarrow E = 106 \text{ N/C}$, in the $+x$-direction.

b) The electric field is less than that of an infinite sheet $E_\infty = \dfrac{\sigma}{2\varepsilon_0} = 115 \text{ N/C}$.

c) Finite disk electric field can be expanded using the binomial theorem since the expansion terms are small: $\Rightarrow E \approx \dfrac{\sigma}{2\varepsilon_0}\left[1 - \dfrac{x}{R} + \dfrac{x^3}{2R^3} - \cdots\right]$ So the difference between the infinite sheet and finite disk goes like $\dfrac{x}{R}$. Thus:

$\Delta E(x = 0.20 \text{ cm}) \approx 0.2/2.5 = 0.08 = 8\%$ and $\Delta E(x = 0.40 \text{ cm})$
$\approx 0.4/2.5 = 0.16 = 16\%$.

21.91: (a) As in **22.72:** $E = \dfrac{\sigma}{2\varepsilon_0}\left[1 - \left(\dfrac{R^2}{x^2}+1\right)^{-1/2}\right]$

$$= \frac{4.00 \text{ pC}/\pi(0.025 \text{ m})^2}{2\varepsilon_0}\left[1 - \left(\frac{(0.025 \text{ m})^2}{(0.200 \text{ m})^2}+1\right)^{-1/2}\right] \Rightarrow E$$

$= 0.89 \text{ N/C}$ in the $+x$-direction.

b) $x >> R,\ E = \dfrac{\sigma}{2\varepsilon_0}[1-(1-R^2/2x^2 + 3R^4/8x^4 - \cdots)]$

$$\approx \frac{\sigma}{2\varepsilon_0}\frac{R^2}{2x^2} = \frac{\sigma\pi R^2}{4\pi\varepsilon_0 x^2} = \frac{Q}{4\pi\varepsilon_0 x^2}.$$

c) The electric field of (a) is less than that of the point charge (0.90 N/C) since the correction term that was omitted was negative.

d) From above $x = 0.2\text{ m}\dfrac{(0.9-0.89)}{0.89} = 0.01 = 1\%$. For $x = 0.1\text{ m}$

$$E_{\text{disk}} = 3.43\text{ N/C}$$
$$E_{\text{point}} = 3.6\text{ N/C}$$

so $\dfrac{(3.6-3.43)}{3.6} = 0.047 \approx 5\%$.

21.92: a) $f(x) = f(-x): \int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx = \int_{0}^{-a} f(-x)d(-x) +$

$\int_{0}^{a} f(x)dx$. Now replace $-x$ with y: $\Rightarrow \int_{-a}^{a} f(x)dx = \int_{0}^{a} f(y)d(y) + \int_{0}^{a} f(x)dx =$

$2\int_{0}^{a} f(x)dx$.

b) $g(x) = -g(-x): \int_{-a}^{a} g(x)dx = \int_{-a}^{0} g(x)dx + \int_{0}^{a} g(x)dx = -\int_{0}^{-a} -g(-x)(-d(-x)) +$

$\int_{0}^{a} g(x)dx$. Now replace $-x$ with y: $\Rightarrow \int_{-a}^{a} g(x)dx = -\int_{0}^{a} g(y)d(y) + \int_{0}^{a} g(x)dx = 0$.

c) The integrand in E_y for Example 21.11 is odd, so $E_y = 0$.

21.93: a) The y-components of the electric field cancel, and the x-components from both charges, as given in problem **21.87** is:

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{-2Q}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right) \Rightarrow \vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{-2Qq}{a}\left(\frac{1}{y} - \frac{1}{(y^2+a^2)^{1/2}}\right)\hat{i}.$$

If $y >> a$, $\vec{F} \approx \dfrac{1}{4\pi\varepsilon_0}\dfrac{-2Qq}{ay}(1-(1-a^2/2y^2+\cdots))\hat{i} = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Qqa}{y^3}$.

b) If the point charge is now on the x-axis the two charged parts of the rods provide different forces, though still along the x-axis (see problem **21.86**).

$$\vec{F}_+ = q\vec{E}_+ = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{1}{x}\right)\hat{i} \text{ and } \vec{F}_- = q\,\vec{E}_- = -\frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}\left(\frac{1}{x} - \frac{1}{x+a}\right)\hat{i}$$

So,

$$\vec{F} = \vec{F}_+ + \vec{F}_- = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{a}\left(\frac{1}{x-a} - \frac{2}{x} + \frac{1}{x+a}\right)\hat{i}$$

For $x >> a$, $\vec{F} \approx \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qq}{ax}\left(\left(1+\dfrac{a}{x}+\dfrac{a^2}{x^2}+\ldots\right) - 2 + \left(1-\dfrac{a}{x}+\dfrac{a^2}{x^2}-\ldots\right)\right)\hat{i} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2Qqa}{x^3}\hat{i}$.

21.94: The electric field in the x-direction cancels the left and right halves of the semicircle. The remaining y-component points in the negative y-direction. The charge per unit length of the semicircle is:

$$\lambda = \frac{Q}{\pi a} \text{ and } dE = \frac{k\lambda\, dl}{a^2} = \frac{k\lambda d\theta}{a} \text{ but } dE_y = dE \sin\theta = \frac{k\lambda \sin\theta\, d\theta}{a}.$$

So, $E_y = \dfrac{2k\lambda}{a}\displaystyle\int_0^{\pi/2} \sin\theta\, d\theta = \dfrac{2k\lambda}{a}[-\cos\theta]_0^{\pi/2} = \dfrac{2k\lambda}{a} = \dfrac{2kQ}{\pi a^2}$, downward.

21.95: By symmetry, $E_x = E_y$. For E_y, compared to problem **21.94**, the integral over the angle is halved but the charge density doubles—giving the same result. Thus,

$$E_x = E_y = \frac{2k\lambda}{a} = \frac{2kQ}{\pi a^2}.$$

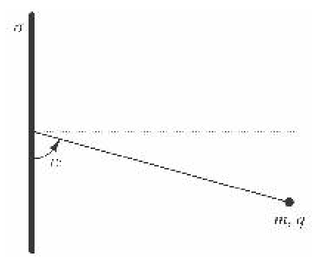
21.96:

$$\sum F_x = 0 \Rightarrow T\cos\alpha = mg \Rightarrow T = \frac{mg}{\cos\alpha}$$

$$\sum F_y = 0 \Rightarrow T\sin\alpha = \frac{q\sigma}{2\varepsilon_0} \Rightarrow T = \frac{q\sigma}{2\varepsilon_0 \sin\alpha}$$

$$\Rightarrow \frac{mg}{\cos\alpha} = \frac{q\sigma}{2\varepsilon_0\sin\alpha} \Rightarrow \tan\alpha = \frac{q\sigma}{2\varepsilon_0 mg}$$

$$\Rightarrow \alpha = \arctan\left(\frac{q\sigma}{2\varepsilon_0 mg}\right)$$



21.97: a) $qE = 10\, mg \Rightarrow \dfrac{m}{q} = \dfrac{E}{10\text{g}} = \dfrac{1.4\times10^5 \text{ N/C}}{10(9.8 \text{ m/s}^2)} = 1429 \text{ kg/C}.$

b) $1429 \dfrac{\text{kg}}{\text{C}} \cdot \dfrac{1 \text{ mol}}{12\times10^{-3}\text{ kg}} \cdot \dfrac{6.02\times10^{23}\text{ carbons}}{\text{mol}} \cdot \dfrac{1.6\times10^{-19}\text{ C}}{\text{excess } e^-} = 1.15\times10^{10} \dfrac{\text{carbons}}{\text{excess } e^-}.$

21.98: a) $E_x = E_y$, and $E_x = 2E_{\text{length of wire } a,\text{ charge } Q} = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{x\sqrt{x^2+\left(\frac{a}{2}\right)^2}}$, where

$$x = \frac{a}{2} \Rightarrow E_x = -\frac{\sqrt{2}Q}{\pi\varepsilon_0 a^2},\ E_y = -\frac{\sqrt{2}Q}{\pi\varepsilon_0 a^2}.$$

b) If all edges of the square had equal charge, the electric fields would cancel by symmetry at the center of the square.

21.99: a)

$$E(P) = -\frac{|\sigma_1|}{2\varepsilon_0} - \frac{|\sigma_2|}{2\varepsilon_0} + \frac{|\sigma_3|}{2\varepsilon_0} = -\frac{0.0200\ \text{C/m}^2}{2\varepsilon_0} - \frac{0.0100\ \text{C/m}^2}{2\varepsilon_0} + \frac{0.0200\ \text{C/m}^2}{2\varepsilon_0}$$

$$\Rightarrow E(P) = \frac{0.0100\ \text{C/m}^2}{2\varepsilon_0} = 5.65\times 10^8\ \text{N/C, in the} -x\text{-direction.}$$

b) $$E(R) = +\frac{|\sigma_1|}{2\varepsilon_0} - \frac{|\sigma_2|}{2\varepsilon_0} + \frac{|\sigma_3|}{2\varepsilon_0} = +\frac{0.0200\ \text{C/m}^2}{2\varepsilon_0} - \frac{0.0100\ \text{C/m}^2}{2\varepsilon_0} + \frac{0.0200\ \text{C/m}^2}{2\varepsilon_0}$$

$$\Rightarrow E(R) = \frac{0.0300\ \text{C/m}^2}{2\varepsilon_0} = 1.69\times 10^9\ \text{N/C, in the} +x\text{-direction.}$$

c) $$E(S) = +\frac{|\sigma_1|}{2\varepsilon_0} + \frac{|\sigma_2|}{2\varepsilon_0} + \frac{|\sigma_3|}{2\varepsilon_0} = +\frac{0.0200\ \text{C/m}^2}{2\varepsilon_0} + \frac{0.0100\ \text{C/m}^2}{2\varepsilon_0} + \frac{0.0200\ \text{C/m}^2}{2\varepsilon_0}$$

$$\Rightarrow E(S) = \frac{0.0500\ \text{C/m}^2}{2\varepsilon_0} = 2.82\times 10^9\ \text{N/C, in the} +x\text{-direction.}$$

d) $$E(T) = +\frac{|\sigma_1|}{2\varepsilon_0} + \frac{|\sigma_2|}{2\varepsilon_0} - \frac{|\sigma_3|}{2\varepsilon_0} = +\frac{0.0200\ \text{C/m}^2}{2\varepsilon_0} + \frac{0.0100\ \text{C/m}^2}{2\varepsilon_0} - \frac{0.0200\ \text{C/m}^2}{2\varepsilon_0}$$

$$\Rightarrow E(S) = \frac{0.0100\ \text{C/m}^2}{2\varepsilon_0} = 5.65\times 10^8\ \text{N/C, in the} +x\text{-direction.}$$

21.100: $$\frac{F_{\text{on I}}}{A} = \frac{qE_{\text{at I}}}{A} = \sigma_1\left(\frac{-|\sigma_2|+|\sigma_3|}{2\varepsilon_0}\right) = \frac{2.00\times 10^{-4}\ \text{C/m}^2}{2\varepsilon_0} = +1.13\times 10^7\ \text{N/m}.$$

$$\frac{F_{\text{on II}}}{A} = \frac{qE_{\text{at II}}}{A} = \sigma_2\left(\frac{+|\sigma_1|+|\sigma_3|}{2\varepsilon_0}\right) = \frac{4.00\times 10^{-4}\ \text{C/m}^2}{2\varepsilon_0} = +2.26\times 10^7\ \text{N/m}$$

$$\frac{F_{\text{on III}}}{A} = \frac{qE_{\text{at III}}}{A} = \sigma_3\left(\frac{+|\sigma_1|+|\sigma_2|}{2\varepsilon_0}\right) = \frac{-6.00\times 10^{-4}\ \text{C/m}^2}{2\varepsilon_0} = -3.39\times 10^7\ \text{N/m}$$

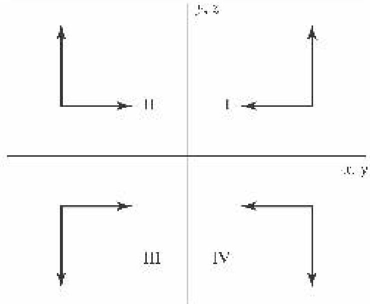
(Note that "+" means toward the right, and "–" is toward the left.)

21.101: By inspection the fields in the different regions are as shown below:

$$E_I = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\boldsymbol{i}} + \hat{\boldsymbol{k}}), \quad E_{II} = \left(\frac{\sigma}{2\varepsilon_0}\right)(+\hat{\boldsymbol{i}} + \hat{\boldsymbol{k}})$$

$$E_{III} = \left(\frac{\sigma}{2\varepsilon_0}\right)(+\hat{\boldsymbol{i}} - \hat{\boldsymbol{k}}), \; E_{IV} = \left(\frac{\sigma}{2\varepsilon_0}\right)(-\hat{\boldsymbol{i}} - \hat{\boldsymbol{k}})$$

$$\therefore \vec{\boldsymbol{E}} = \left(\frac{\sigma}{2\varepsilon_0}\right)\left(-\frac{|x|}{x}\hat{\boldsymbol{i}} + \frac{|z|}{z}\hat{\boldsymbol{k}}\right).$$



21.102: a) $Q = A\sigma = \pi(R_2^2 - R_2^2)\sigma$

b) Recall the electric field of a disk, Eq. (21.11): $E = \frac{\sigma}{2\varepsilon_0}\left[1 - 1/\sqrt{(R/x)^2 + 1}\right]$ So,

$$\vec{\boldsymbol{E}}(x) = \frac{\sigma}{2\varepsilon_0}\left(\left[1 - 1/\sqrt{(R_2/x)^2 + 1}\right] - \left[1 - 1/\sqrt{(R_1/x)^2 + 1}\right]\right)\frac{|x|}{x}\hat{\boldsymbol{i}} \Rightarrow E(x) = \frac{-\sigma}{2\varepsilon_0} \times$$

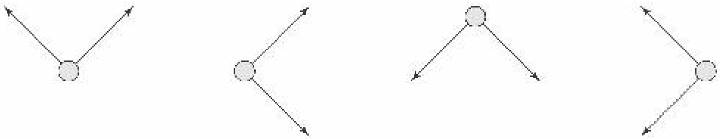
$$\left(1/\sqrt{(R_2/x)^2 + 1} - 1/\sqrt{(R_1/x)^2 + 1}\right)\frac{|x|}{x}\hat{\boldsymbol{i}}.$$

c) Note that $1/\sqrt{(R_1/x)^2 + 1} = \frac{|x|}{R_1}(1 + (x/R_1)^2)^{-1/2} \approx \frac{|x|}{R_1}\left(1 - \frac{(x/R_1)^2}{2} + \ldots\right)$

$\Rightarrow \vec{E}(x) = \frac{\sigma}{2\varepsilon_0}\left(\frac{x}{R_1} - \frac{x}{R_2}\right)\frac{x}{|x|}\hat{\boldsymbol{i}} = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\frac{x}{|x^2|}\hat{\boldsymbol{i}}$, and sufficiently close means that $(x/R_1)^2 << 1$.

d) $F = qE(x) = -\frac{q\sigma}{2\varepsilon_0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)x = m\ddot{x} \Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{q\sigma}{2\varepsilon_0 m}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}.$

21.103: a) The four possible force diagrams are:



Only the last picture can result in an electric field in the $-x$-direction.

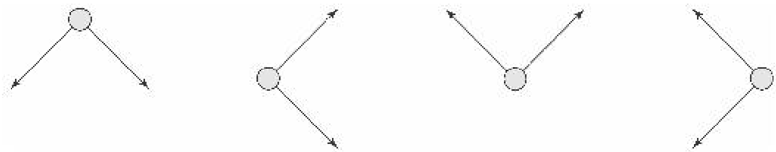
b) $q_1 = -2.00\ \mu\text{C},\ q_3 = +4.00,\ \mu\text{C},$ and $q_2 > 0.$

c) $E_y = 0 = \dfrac{1}{4\pi\varepsilon_0}\,\dfrac{q_1}{(0.0400\ \text{m})^2}\sin\theta_1 - \dfrac{1}{4\pi\varepsilon_0}\,\dfrac{q_2}{(0.0300\ \text{m})^2}\sin\theta_2$

$$\Rightarrow q_2 = \frac{9}{16}q_1\frac{\sin\theta_1}{\sin\theta_2} = \frac{9}{16}\ q_1\frac{3/5}{4/5} = \frac{27}{64}q_1 = 0.843\ \mu\text{C}.$$

d) $F_3 = q_3E_x = q_3\dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q_1}{0.0016}\,\dfrac{4}{5} + \dfrac{q_2}{0.0009}\,\dfrac{3}{5}\right) = 56.2\ \text{N}$

21.104: (a) The four possible diagrams are:



The first diagram is the only one in which the electric field must point in the negative y-direction.

b) $q_1 = -3.00\ \mu\text{C},$ and $q_2 < 0.$

c) $E_x = 0 = \dfrac{kq_1}{(0.050\ \text{m})^2}\,\dfrac{5}{13} - \dfrac{kq_2}{(0.120\ \text{m})^2}\,\dfrac{12}{13} \Rightarrow \dfrac{kq_2}{(0.120\ \text{m})^2} = \dfrac{kq_1}{(0.050\ \text{m})^2}\,\dfrac{5}{12}$

$$E = E_y = \frac{kq_1}{(0.050\ \text{m})^2}\,\frac{12}{13} + \frac{kq_2}{(0.120\ \text{m})^2}\,\frac{5}{13} = \frac{kq_1}{(0.05\ \text{m})^2}\left(\frac{12}{13} + \left(\frac{5}{12}\right)\left(\frac{5}{13}\right)\right)$$

$$\Rightarrow E = E_y = 1.17\times10^7\ \text{N/C}.$$

21.105: a) For a rod in general of length L, $E = \dfrac{kQ}{L}\left(\dfrac{1}{r} - \dfrac{1}{L+r}\right)$ and here $r = x + \dfrac{a}{2}$.

So, $E_{\text{left rod}} = \dfrac{kQ}{L}\left(\dfrac{1}{x+a/2} - \dfrac{1}{L+x+a/2}\right) = \dfrac{2kQ}{L}\left(\dfrac{1}{2x+a} - \dfrac{1}{2L+2x+a}\right).$

b) $dF = dq\ E \Rightarrow F = \int E\ dq = \int_{a/2}^{L+a/2}\dfrac{EQ}{L}\,dx = \dfrac{2kQ^2}{L^2}\int_{a/2}^{L+a/2}\times$

$$\left(\frac{1}{2x+a} - \frac{1}{2L+2x+a}\right)dx$$

$$\Rightarrow F = \frac{2kQ^2}{L^2}\frac{1}{2}\left([\ln(a+2x)]_{a/2}^{L+a/2} - [\ln(2L+2x+a)]_{a/2}^{L+a/2}\right)$$

$$\Rightarrow F = \frac{kQ^2}{L^2}\ln\left(\left(\frac{a+2L+a}{2a}\right)\left(\frac{2L+2a}{4L+2a}\right)\right) = \frac{kQ^2}{L^2}\ln\left(\frac{(a+L)^2}{a(a+2L)}\right).$$

c) For $a \gg L : F = \dfrac{kQ^2}{L^2}\ln\left(\dfrac{a^2(1+L/a)^2}{a^2(1+2L/a)}\right) = \dfrac{kQ^2}{L^2}(2\ln(1+2L/a) - \ln(1+2L/a))$

$$\Rightarrow F \approx \frac{kQ^2}{L^2}\left(2\left(\frac{L}{a} - \frac{L^2}{2a^2} + \cdots\right) - \left(\frac{2L}{a} - \frac{2L^2}{a^2} + \cdots\right)\right) \Rightarrow F \approx \frac{kQ^2}{a^2}.$$

22.1: a) $\Phi = \vec{E}\cdot\vec{A} = (14\ \text{N/C})(0.250\ \text{m}^2)\cos 60° = 1.75\ \text{Nm}^2/\text{C}.$

b) As long as the sheet is flat, its shape does not matter.

ci) The maximum flux occurs at an angle $\phi = 0°$ between the normal and field.

cii) The minimum flux occurs at an angle $\phi = 90°$ between the normal and field.

In part i), the paper is oriented to "capture" the most field lines whereas in ii) the area is oriented so that it "captures" no field lines.

22.2: a) $\Phi = \vec{E}\cdot\vec{A} = EA\cos\theta$ where $\vec{A} = A\hat{n}$

$\hat{n}_{S_1} = -\hat{j}$ (left)$\Phi_{S_1} = -(4\times10^3\ \text{N/C})(0.1\ \text{m})^2\cos(90-36.9°) = -24\ \text{N}\cdot\text{m}^2/\text{C}$

$\hat{n}_{S_2} = +\hat{k}$ (top)$\Phi_{S_2} = -(4\times10^3\ \text{N/C})(0.1\ \text{m})^2\cos 90° = 0$

$\hat{n}_{S_3} = +\hat{j}$ (right)$\Phi_{S_3} = +(4\times10^3\ \text{N/C})(0.1\ \text{m})^2\cos(90°-36.9°) = +24\ \text{N}\cdot\text{m}^2/\text{C}$

$\hat{n}_{S_4} = -\hat{k}$ (bottom)$\Phi_{S_4} = (4\times10^3\ \text{N/C})(0.1\ \text{m})^2\cos 90° = 0$

$\hat{n}_{S_5} = +\hat{i}$ (front)$\Phi_{S_5} = +(4\times10^3\ \text{N/C})(0.1\ \text{m})^2\cos 36.9° = 32\ \text{N}\cdot\text{m}^2/\text{C}$

$\hat{n}_{S_6} = -\hat{i}$ (back)$\Phi_{S_6} = -(4\times10^3\ \text{N/C})(0.1\ \text{m})^2\cos 36.9° = -32\ \text{N}\cdot\text{m}^2/\text{C}$

b) The total flux through the cube must be zero; any flux entering the cube must also leave it.

22.3: a) Given that $\vec{E} = -\text{B}\hat{i} + \text{C}\hat{j} - \text{D}\hat{k}$, $\Phi = \vec{E}\cdot\vec{A}$, edge length L, and

$$\hat{n}_{S_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E}\cdot A\hat{n}_{S_1} = -\text{CL}^2.$$

$$\hat{n}_{S_2} = +\hat{k} \Rightarrow \Phi_2 = \vec{E}\cdot A\hat{n}_{S_2} = -\text{DL}^2.$$

$$\hat{n}_{S_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E}\cdot A\hat{n}_{S_3} = +\text{CL}^2.$$

$$\hat{n}_{S_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E}\cdot A\hat{n}_{S_4} = +\text{DL}^2.$$

$$\hat{n}_{S_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E}\cdot A\hat{n}_{S_5} = -\text{BL}^2.$$

$$\hat{n}_{S_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E}\cdot A\hat{n}_{S_6} = +\text{BL}^2.$$

b) Total flux $= \sum_{i=1}^{6} \Phi_i = 0$

22.4: $\Phi = \vec{E} \cdot \vec{A} = (75.0\ \text{N/C})\ (0.240\ \text{m}^2) \cos 70° = 6.16\ \text{Nm}^2/\text{C}.$

22.5: a) $\Phi = \vec{E} \cdot \vec{A} = \frac{\lambda}{2\pi\varepsilon_0 r}(2\pi r l) = \frac{\lambda l}{\varepsilon_0} = \frac{(6.00\times10^{-6}\ \text{C/m})\,(0.400\ \text{m})}{\varepsilon_0} = 2.71\times10^5\ \text{Nm}^2/\text{C}.$

b) We would get the same flux as in (a) if the cylinder's radius was made larger—the field lines must still pass through the surface.

c) If the length was increased to $l = 0.800$ m, the flux would increase by a factor of two: $\Phi = 5.42 \times 10^5\ \text{Nm}^2/\text{C}.$

22.6: a) $\Phi_{S_1} = q_1/\varepsilon_0 = (4.00\times10^{-9}\ \text{C})/\varepsilon_0 = 452\ \text{Nm}^2/\text{C}.$

b) $\Phi_{S_2} = q_2/\varepsilon_0 = (-7.80\times10^{-9}\ \text{C})/\varepsilon_0 = -881\ \text{Nm}^2/\text{C}.$

c) $\Phi_{S_3} = (q_1 + q_2)/\varepsilon_0 = ((4.00 - 7.80)\times10^{-9}\ \text{C})/\varepsilon_0 = -429\ \text{Nm}^2/\text{C}.$

d) $\Phi_{S_4} = (q_1 + q_2)/\varepsilon_0 = ((4.00 + 2.40)\times10^{-9}\ \text{C})/\varepsilon_0 = 723\ \text{Nm}^2/\text{C}.$

e) $\Phi_{S_5} = (q_1 + q_2 + q_3)/\varepsilon_0 = ((4.00 - 7.80 + 2.40)\times10^{-9}\ \text{C})/\varepsilon_0 = -158\ \text{Nm}^2/\text{C}.$

f) All that matters for Gauss's law is the total amount of charge enclosed by the surface, not its distribution within the surface.

22.7: a) $\Phi = q/\varepsilon_0 = (-3.60\times10^{-6}\ \text{C})/\varepsilon_0 = -4.07\times10^5\ \text{Nm}^2/\text{C}.$

b) $\Phi = q/\varepsilon_0 \Rightarrow q = \varepsilon_0 \Phi = \varepsilon_0 (780\ \text{Nm}^2/\text{C}) = 6.90\times10^{-9}\ \text{C}.$

c) No. All that matters is the total charge enclosed by the cube, not the details of where the charge is located.

22.8: a) No charge enclosed so $\Phi = 0$

b)
$$\Phi = \frac{q_2}{\varepsilon_0} = \frac{-6.00\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2} = -678\ \text{Nm}^2/\text{C}.$$

c)
$$\Phi = \frac{q_1 + q_2}{\varepsilon_0} = \frac{(4.00 - 6.00)\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2} = -226\ \text{Nm}^2/\text{C}.$$

22.9: a) Since $\vec{E}$ is uniform, the flux through a closed surface must be zero. That is: $\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} = \frac{1}{\varepsilon_0}\int \rho dV = 0 \Rightarrow \int \rho dV = 0.$ But because we can choose any volume we want, ρ must be zero if the integral equals zero.

b) If there is no charge in a region of space, that does NOT mean that the electric field is uniform. Consider a closed volume close to, but not including, a point charge. The field diverges there, but there is no charge in that region.

22.10: a) If $\rho > 0$ and uniform, then q inside any closed surface is greater than zero. $\Rightarrow \Phi > 0 \Rightarrow \oint \vec{E} \cdot d\vec{A} > 0$ and so the electric field cannot be uniform, i.e., since an arbitrary surface of our choice encloses a non-zero amount of charge, E must depend on position.

b) However, inside a small bubble of zero density within the material with density ρ, the field CAN be uniform. All that is important is that there be zero flux through the surface of the bubble (since it encloses no charge). (See Exercise 22.61.)

22.11: $\Phi_{6\text{sides}} = q/\varepsilon_0 = (9.60 \times 10^{-6}\ \text{C})/\varepsilon_0 = 1.08 \times 10^6\ \text{Nm}^2/\text{C}$. But the box is symmetrical, so for one side, the flux is: $\Phi_{1\text{side}} = 1.80 \times 10^5\ \text{Nm}^2/\text{C}$.

b) No change. Charge enclosed is the same.

22.12: Since the cube is empty, there is no net charge enclosed in it. The net flux, according to Gauss's law, must be zero.

22.13: $\Phi_E = Q_{\text{encl}}/\varepsilon_0$

The flux through the sphere depends only on the charge within the sphere.

$Q_{\text{encl}} = \varepsilon_0 \Phi_E = \varepsilon_0 (360\ \text{N} \cdot \text{m}^2/\text{C}) = 3.19\ \text{nC}$

22.14: a) $E(r = 0.450\ \text{m} + 0.1\ \text{m}) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(2.50 \times 10^{-10}\ \text{C})}{(0.550\ \text{m})^2} = 7.44\ \text{N/C}.$

b) $\vec{E} = 0$ inside of a conductor or else free charges would move under the influence of forces, violating our electrostatic assumptions (i.e., that charges aren't moving).

22.15: a) $|E| = \dfrac{1}{4\pi\varepsilon_0}\dfrac{|q|}{r^2} \Rightarrow r = \sqrt{\dfrac{1}{4\pi\varepsilon_0}\left|\dfrac{q}{E}\right|} = \sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{(0.180 \times 10^{-6}\ \text{C})}{614\ \text{N/C}}} = 1.62\ \text{m}.$

b) As long as we are outside the sphere, the charge enclosed is constant and the sphere acts like a point charge.

22.16: a) $\Phi = EA = q/\varepsilon_0 \Rightarrow q = \varepsilon_0 EA = \varepsilon_0\ (1.40 \times 10^5\ \text{N/C})\ (0.0610\ \text{m}^2) = 7.56 \times 10^{-8}\ \text{C}.$

b) Double the surface area: $q = \varepsilon_0 (1.40 \times 10^5\ \text{N/C})\ (0.122\ \text{m}^2) = 1.51 \times 10^{-7}\ \text{C}.$

22.17: $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \Rightarrow q = 4\pi\varepsilon_0 Er^2 = 4\pi\varepsilon_0 (1150\ \text{N/C})\,(0.160\ \text{m})^2 = 3.27\times10^{-9}$ C. So the number of electrons is: $n_e = \frac{3.27\times10^{-9}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = 2.04\times10^{10}$.

22.18: Draw a cylindrical Gaussian surface with the line of charge as its axis. The cylinder has radius 0.400 m and is 0.0200 m long. The electric field is then 840 N/C at every point on the cylindrical surface and directed perpendicular to the surface. Thus

$$\oint \vec{E}\cdot d\vec{s} = (E)(A_{\text{cylinder}}) = (E)(2\pi rL)$$

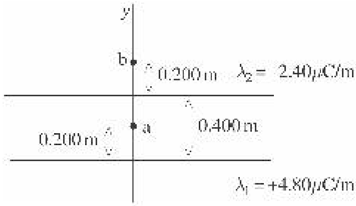
$$= (840\ \text{N/C})\,(2\pi)\,(0.400\ \text{m})\,(0.0200\ \text{m}) = 42.2\ \text{N}\cdot\text{m}^2/\text{C}$$

The field is parallel to the end caps of the cylinder, so for them $\oint \vec{E}\cdot d\vec{s} = 0$. From Gauss's law:

$$q = \varepsilon_0\Phi_E = (8.854\times10^{-12}\frac{\text{C}^2}{\text{N}\cdot\text{m}^2})\,(42.2\frac{\text{N}\cdot\text{m}^2}{\text{C}})$$

$$= 3.74\times10^{-10}\ \text{C}$$

22.19:



$$E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

a) At point a, $\vec{E}_1$ and $\vec{E}_2$ are in the $+y$-direction (toward negative charge, away from positive charge).

$$E_1 = (1/2\pi\varepsilon_0[(4.80\times10^{-6}\ \text{C/m})\ /(0.200\ \text{m})] = 4.314\times10^5\ \text{N/C}$$

$$E_2 = (1/2\pi\varepsilon_0[(2.40\times10^{-6}\ \text{C/m})\ /(0.200\ \text{m})] = 2.157\times10^5\ \text{N/C}$$

$$E = E_1 + E_2 = 6.47\times10^5\ \text{N/C, in the } y\text{-direction.}$$

b) At point b, $\vec{E}_1$ is in the $+y$-direction and $\vec{E}_2$ is in the $-y$-direction.

$$E_1 = (1/2\pi\varepsilon_0[(4.80\times10^{-6}\ \text{C/m})\ /(0.600\ \text{m})] = 1.438\times10^5\ \text{N/C}$$

$$E_2 = (1/2\pi\varepsilon_0[(2.80\times10^{-6}\ \text{C/m})\ /(0.200\ \text{m})] = 2.157\times10^5\ \text{N/C}$$

$$E = E_2 - E_1 = 7.2\times10^4\ \text{N/C, in the } -y\text{-direction.}$$

22.20: a) For points outside a uniform spherical charge distribution, all the charge can be considered to be concentrated at the center of the sphere. The field outside the sphere is thus inversely proportional to the square of the distance from the center. In this case:

$$E = (480\ \text{N/C})\left(\frac{0.200\ \text{cm}}{0.600\ \text{cm}}\right)^2 = 53\ \text{N/C}$$

b) For points outside a long cylindrically symmetrical charge distribution, the field is identical to that of a long line of charge:

$$E = \frac{\lambda}{2\pi\varepsilon_0 r},$$

that is, inversely proportional to the distance from the axis of the cylinder. In this case

$$E = (480\ \text{N/C})\left(\frac{0.200\ \text{cm}}{0.600\ \text{cm}}\right) = 160\ \text{N/C}$$

c) The field of an infinite sheet of charge is $E = \sigma/2\varepsilon_0$; i.e., it is independent of the distance from the sheet. Thus in this case $E = 480\ \text{N/C}$.

22.21: Outside each sphere the electric field is the same as if all the charge of the sphere were at its center, and the point where we are to calculate $\vec{E}$ is outside both spheres. $\vec{E}_1$ and $\vec{E}_2$ are both toward the sphere with negative charge.

$$E_1 = k\frac{|q_1|}{r_1^2} = k\frac{1.80\times10^{-6}\ \text{C}}{(0.250\ \text{m})^2} = 2.591\times10^5\ \text{N/C}$$

$$E_2 = k\frac{|q_2|}{r_2^2} = k\frac{3.80\times10^{-6}\ \text{C}}{(0.250\ \text{m})^2} = 5.471\times10^5\ \text{N/C}$$

$$E = E_1 + E_2 = 8.06\times10^5\ \text{N/C, toward the negatively charged sphere.}$$

22.22: For points outside the sphere, the field is identical to that of a point charge of the same total magnitude located at the center of the sphere. The total charge is given by charge density × volume:

$$q = (7.50 \text{ nC/m}^3)(\frac{4}{3}\pi)(0.150 \text{ m})^3 = 1.60 \times 10^{-10} \text{ C}$$

a) The field just outside the sphere is

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{(9\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\,(1.06\times10^{-10} \text{ C})}{(0.150 \text{ m})^2} = 42.4 \text{ N/C}$$

b) At a distance of 0.300 m from the center (double the sphere's radius) the field will be 1/4 as strong: 10.6 N/C

c) Inside the sphere, only the charge inside the radius in question affects the field. In this case, since the radius is half the sphere's radius, 1/8 of the total charge contributes to the field:

$$E = \frac{(9\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)\,(1/8)\,(1.06\times10^{-10} \text{ C})}{(0.075 \text{ m})^2} = 21.2 \text{ N/C}$$

22.23: The point is inside the sphere, so $E = kQr/R^3$ (Example 22.9)

$$Q = \frac{ER^3}{kr} = \frac{(950 \text{ N/C})\,(0.220 \text{ m})^3}{k(0.100 \text{ m})} = 10.2 \text{ nC}$$

22.24: a) Positive charge is attracted to the inner surface of the conductor by the charge in the cavity. Its magnitude is the same as the cavity charge: $q_{\text{inner}} = +6.00 \text{ nC}$, since $\vec{E} = 0$ inside a conductor.

b) On the outer surface the charge is a combination of the net charge on the conductor and the charge "left behind" when the $+6.00$ nC moved to the inner surface:

$q_{\text{tot}} = q_{\text{inner}} + q_{\text{outer}} \Rightarrow q_{\text{outer}} = q_{\text{tot}} - q_{\text{inner}} = 5.00 \text{ nC} - 6.00 \text{ nC} = -1.00 \text{ nC}.$

22.25: S_2 and S_3 enclose no charge, so the flux is zero, and electric field outside the plates is zero. For between the plates, S_1 shows that: $EA = q/\varepsilon_0 = \sigma A/\varepsilon_0 \Rightarrow E = \sigma/\varepsilon_0$.

22.26: a) At a distance of 0.1 mm from the center, the sheet appears "infinite," so:

$$\oint \vec{E}\cdot d\vec{A} = E\,2A = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{2\varepsilon_0 A} = \frac{7.50\times10^{-9} \text{ C}}{2\varepsilon_0(0.800 \text{ m})^2} = 662 \text{ N/C}.$$

b) At a distance of 100 m from the center, the sheet looks like a point, so:

$$E \approx \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(7.50\times10^{-9}\ \text{C})}{(100\ \text{m})^2} = 6.75\times10^{-3}\ \text{N/C}.$$

c) There would be no difference if the sheet was a conductor. The charge would automatically spread out evenly over both faces, giving it half the charge density on any as the insulator (σ :). $E_c = \frac{\sigma}{\varepsilon_0} = \frac{\sigma_i}{2\varepsilon_0}$ near one face. Unlike a conductor, the insulator *is* the charge density in some sense. Thus one shouldn't think of the charge as "spreading over each face" for an insulator. Far away, they both look like points with the same charge.

22.27: a) $$\sigma = \frac{Q}{A} = \frac{Q}{2\pi RL} \Rightarrow \frac{Q}{L} = \sigma 2\pi R = \lambda.$$

b) $$\oint \vec{E}\cdot d\vec{A} = E(2\pi rL) = \frac{Q}{\varepsilon_0} = \frac{\sigma 2\pi RL}{\varepsilon_0} \Rightarrow E = \frac{\sigma R}{r\varepsilon_0}.$$

c) But from (a), $\lambda = \sigma 2\pi R$, so $E = \frac{\lambda}{2\pi\varepsilon_0 r}$, same as an infinite line of charge.

22.28: All the σ's are absolute values.

(a) at $A: E_A = \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} + \frac{\sigma_4}{2\varepsilon_0} - \frac{\sigma_1}{2\varepsilon_0}$

$$E_A = \frac{1}{2\varepsilon_0}(\sigma_2 + \sigma_3 + \sigma_4 - \sigma_1)$$
$$= \frac{1}{2\varepsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2)$$
$$= 2.82\times10^5\ \text{N/C to the left.}$$

(b)

$$E_B = \frac{\sigma_1}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} + \frac{\sigma_4}{2\varepsilon_0} - \frac{\sigma_2}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(\sigma_1 + \sigma_3 + \sigma_4 - \sigma_2)$$
$$= \frac{1}{2\varepsilon_0}(6\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 + 4\ \mu\text{C/m}^2 - 5\ \mu\text{C/m}^2)$$
$$= 3.95\times10^5\ \text{N/C to the left.}$$

(c)

$$E_C = \frac{\sigma_2}{2\varepsilon_0} + \frac{\sigma_3}{2\varepsilon_0} - \frac{\sigma_4}{2\varepsilon_0} - \frac{\sigma_1}{2\varepsilon_0} = \frac{1}{2\varepsilon_0}(\sigma_2 + \sigma_3 - \sigma_4 - \sigma_1)$$
$$= \frac{1}{2\varepsilon_0}(5\ \mu\text{C/m}^2 + 2\ \mu\text{C/m}^2 - 4\ \mu\text{C/m}^2 - 6\ \mu\text{C/m}^2)$$
$$= 1.69\times10^5\ \text{N/C to the left}$$

22.29: a) Gauss's law says $+Q$ on inner surface, so $E = 0$ inside metal.

b) The outside surface of the sphere is grounded, so no excess charge.

c) Consider a Gaussian sphere with the $-Q$ charge at its center and radius less than the inner radius of the metal. This sphere encloses net charge $-Q$ so there is an electric field flux through it; there is electric field in the cavity.

d) In an electrostatic situation $E = 0$ inside a conductor. A Gaussian sphere with the $-Q$ charge at its center and radius greater than the outer radius of the metal encloses zero net charge (the $-Q$ charge and the $+Q$ on the inner surface of the metal) so there is no flux through it and $E = 0$ outside the metal.

e) No, $E = 0$ there. Yes, the charge has been shielded by the grounded conductor. There is nothing like positive and negative mass (the gravity force is always attractive), so this cannot be done for gravity.

22.30: Given $\vec{E} = (-5.00\ (\text{N/C})\cdot\text{m})x\hat{i} + (3.00\ (\text{N/C})\cdot\text{m})z\hat{k}$, edge length $L = 0.300\ \text{m}$, $L = 0.300\ \text{m}$, and $\hat{n}_{S_1} = -\hat{j} \Rightarrow \Phi_1 = \vec{E}\cdot\hat{n}_{S_1}A = 0.$

$\hat{n}_{S_1} = +\hat{k} \Rightarrow \Phi_2 = \vec{E}\cdot\hat{n}_{S_2}A = (3.00\ (\text{N/C})\cdot\text{m})(0.300\ \text{m})^2 z = (0.27\ (\text{N/C})\text{m})z =$
$(0.27\ (\text{N/C})\text{m})(0.300\ \text{m}) = 0.081\ (\text{N/C})\ \text{m}^2.$

$\hat{n}_{S_3} = +\hat{j} \Rightarrow \Phi_3 = \vec{E}\cdot\hat{n}_{S_3}A = 0.$

$\hat{n}_{S_4} = -\hat{k} \Rightarrow \Phi_4 = \vec{E}\cdot\hat{n}_{S_4}A = -(0.27\ (\text{N/C})\cdot\text{m})z = 0\ (z = 0).$

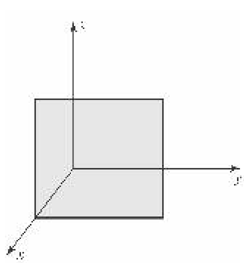
$\hat{n}_{S_5} = +\hat{i} \Rightarrow \Phi_5 = \vec{E}\cdot\hat{n}_{S_5}A = (-5.00\ (\text{N/C})\cdot\text{m})(0.300\ \text{m})^2 x = -(0.45(\text{N/C})\cdot\text{m})x$
$= -(0.45\ (\text{N/C})\cdot\text{m})(0.300\ \text{m}) = -(0.135\ (\text{N/C})\cdot\text{m}^2).$

$\hat{n}_{S_6} = -\hat{i} \Rightarrow \Phi_6 = \vec{E}\cdot\hat{n}_{S_6}A = +(0.45\ (\text{N/C})\cdot\text{m})x = 0\ (x = 0).$

b) Total flux:

$$\Phi = \Phi_2 + \Phi_5 = (0.081 - 0.135)\ (\text{N/C})\cdot\text{m}^2 = -0.054\ \text{Nm}^2/\text{C}$$
$$q = \varepsilon_0\Phi = -4.78\times10^{-13}\ \text{C}$$

22.31: a)



b) Imagine a charge q at the center of a cube of edge length $2L$. Then: $\Phi = q/\varepsilon_0$. Here the square is one 24th of the surface area of the imaginary cube, so it intercepts 1/24 of the flux. That is, $\Phi = q/24\varepsilon_0$.

22.32: a) $\Phi = EA = (125\ \text{N/C})(6.0\ \text{m}^2) = 750\ \text{N}\cdot\text{m}^2/\text{C}$.

b) Since the field is parallel to the surface, $\Phi = 0$.

c) Choose the Gaussian surface to equal the volume's surface. Then: 750 – $EA = q/\varepsilon_0 \Rightarrow E = \frac{1}{6.0\ \text{m}^2}(2.40\times10^{-8}\ \text{C}/\varepsilon_0 + 750) = 577\ \text{N/C}$, in the positive x-direction.

Since $q < 0$ we must have some net flux flowing *in* so $EA \to -\left| EA \right|$ on second face.

d) $q < 0$ but we have E pointing *away* from face I. This is due to an external field that does not affect the flux but affects the value of E.

22.33: To find the charge enclosed, we need the flux through the parallelepiped:

$$\Phi_1 = AE_1 \cos 60° = (0.0500\ \text{m})(0.0600\ \text{m})(2.50\times10^4\ \text{N/C})\cos 60° = 37.5\ \text{N}\cdot\text{m}^2/\text{C}$$

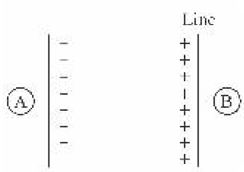
$$\Phi_2 = AE_2 \cos 120° = (0.0500\ \text{m})(0.0600\ \text{m})(7.00\times10^4\ \text{N/C})\cos 60° = -105\ \text{N}\cdot\text{m}^2/\text{C}$$

So the total flux is $\Phi = \Phi_1 + \Phi_2 = (37.5 - 105)\ \text{N}\cdot\text{m}^2/\text{C} = -67.5\ \text{N}\cdot\text{m}^2/\text{C}$, and

$q = \Phi\varepsilon_0 = (-67.5\ \text{N}\cdot\text{m}^2/\text{C})\varepsilon_0 = -5.97\times10^{-10}\ \text{C}$.

b) There must be a net charge (negative) in the parallelepiped since there is a net flux flowing into the surface. Also, there must be an external field or all lines would point toward the slab.

22.34:



The α particle feels no force where the net electric field is zero. The fields can cancel only in regions A and B.

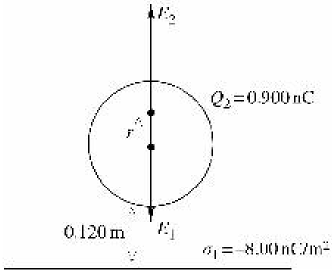
$$E_{line} = E_{sheet}$$

$$\frac{\lambda}{2\pi\varepsilon_0 r} = \frac{\sigma}{2\varepsilon_0}$$

$$r = \lambda/\pi\sigma = \frac{50\ \mu\text{C/m}}{\pi(100\ \mu\text{C/m}^2)} = 0.16\text{m} = 16\text{cm}$$

The fields cancel 16 cm from the line in regions A *and* B.

22.35:



The electric field $\vec{E}_1$ of the sheet of charge is toward the sheet, so the electric field $\vec{E}_2$ of the sphere must be away from the sheet. This is true above the center of the sphere. Let r be the distance above the center of the sphere for the point where the electric field is zero.

$$E_1 = E_2 \text{ so } \frac{\sigma_1}{2\varepsilon_0} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2 r}{R^3}$$

$$r = \frac{2\pi\sigma_1 R^3}{Q_2} = \frac{2\pi(8.00\times10^{-9}\ \text{C/m}^2)(0.120\ \text{m})^3}{0.900\times10^{-9}\ \text{C}} = 0.097\ \text{m}$$

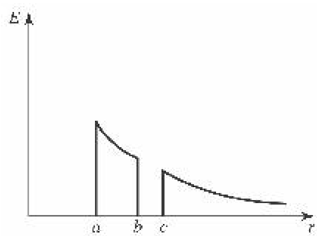
22.36: a) For $r < a$, $E = 0$, since no charge is enclosed.

For $a < r < b$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, since there is $+q$ inside a radius r.

For $b < r < c$, $E = 0$, since now the $-q$ cancels the inner $+q$.

For $r > c$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$, since again the total charge enclosed is $+q$.

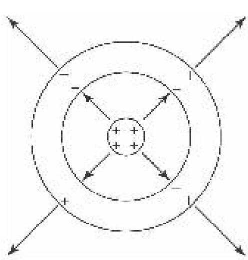
b)



c) Charge on inner shell surface is $-q$.

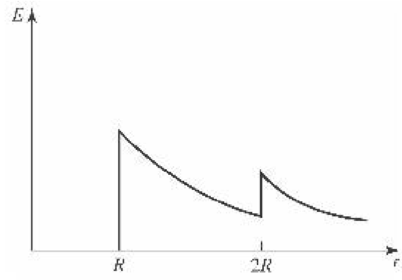
d) Charge on outer shell surface is $+q$.

e)



22.37: a) $r < R, E = 0$, since no charge is enclosed.

b) $R < r < 2R, E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, since charge enclosed is Q. $r > 2R, E = \frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$, since charge enclosed is $2Q$.



22.38: a) $r < a, E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}$, since the charge enclosed is Q.

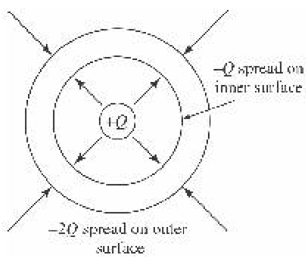
$a < r < b, E = 0$, since the $-Q$ on the inner surface of the shell cancels the $+Q$ at the center of the sphere.

$r > b, E = -\frac{1}{4\pi\epsilon_0}\frac{2Q}{r^2}$, since the total enclosed charge is $-2Q$.

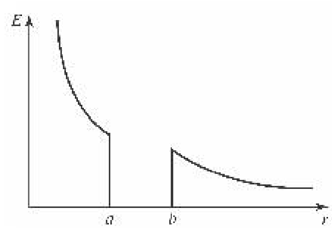
b) The surface charge density on inner surface: $\sigma = -\frac{Q}{4\pi a^2}$.

c) The surface charge density on the outer surface: $\sigma = -\frac{2Q}{4\pi b^2}$.

d)



e)



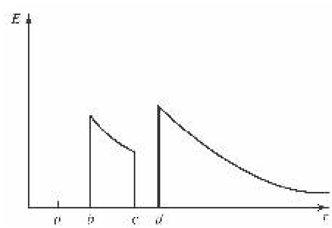
22.39: a)(i) $r < a, E = 0$, since $Q = 0$

(ii) $a < r < b, E = 0$, since $Q = 0$.

(iii) $b < r < c, E = \frac{1}{4\pi\epsilon_0}\frac{2q}{r^2}$, since $Q = +2q$.

(iv) $c < r < d$, $E = 0$, since $Q = 0$.

(v) $r > d$, $E = \frac{1}{4\pi\varepsilon_0}\frac{6q}{r^2}$, since $Q = +6q$.



b)(i) small shell inner: $Q = 0$

(ii) small shell outer: $Q = +2q$

(iii) large shell inner: $Q = -2q$

(iv) large shell outer: $Q = +6q$

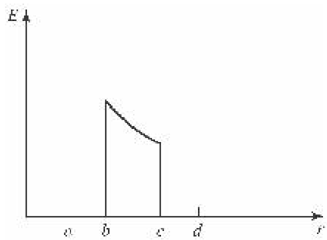
22.40: a)(i) $r < a$, $E = 0$, since the charge enclosed is zero.

(ii) $a < r < b, E = 0$, since the charge enclosed is zero.

(iii) $b < r < c, E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$, since charge enclosed is $+2q$.

(iv) $c < r < d$, $E = 0$, since the net charge enclosed is zero.

(v) $r > d$, $E = 0$, since the net charge enclosed is zero.



b)(i) small shell inner: $Q = 0$

(ii) small shell outer: $Q = +2q$

(iii) large shell inner: $Q = -2q$

(iv) large shell outer: $Q = 0$

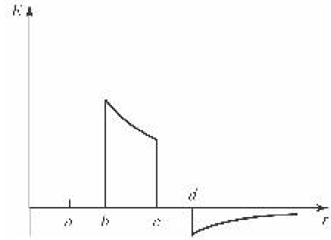
22.41: a)(i) $r < a$, $E = 0$, since charge enclosed is zero.

(ii) $a < r < b$, $E = 0$, since charge enclosed is zero.

(iii) $b < r < c$, $E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$, since charge enclosed is $+2q$.

(iv) $c < r < d$, $E = 0$, since charge enclosed is zero.

(v) $r > d, E = -\frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}$, since charge enclosed is $-2q$.



b)(i) small shell inner: $Q = 0$

(ii) small shell outer: $Q = +2q$

(iii) large shell inner: $Q = -2q$

(iv) large shell outer: $Q = -2q$

22.42: a) We need:

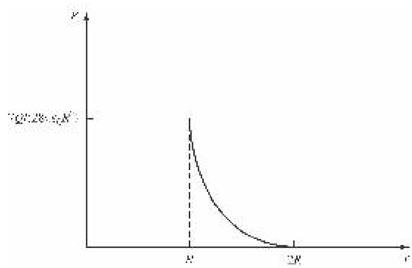
$$-Q = \frac{4\pi\rho}{3}((2R)^3 - R^3) \Rightarrow Q = \frac{-28\pi\rho R^3}{3} \Rightarrow \rho = -\frac{3Q}{28\pi R^3}.$$

b) $r < R, E = 0$ and $r > 2R, E = 0$, since the net charges are zero.

$$R < r < 2R, \Phi = E(4\pi r^2) = \frac{Q}{\varepsilon_0} + \frac{4\pi\rho}{3\varepsilon_0}(r^3 - R^3) \Rightarrow E = \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{\rho}{3\varepsilon_0 r^2}(r^3 - R^3).$$

Substituting ρ from (a) $E = \frac{2}{7\pi\varepsilon_0}\frac{Q}{r^2} - \frac{Qr}{28\pi\varepsilon_0 R^3}$.

c) We see a discontinuity in going from the conducting sphere to the insulator due to the thin surface charge of the conducting sphere—but we see a smooth transition from the uniform insulator to the outside.

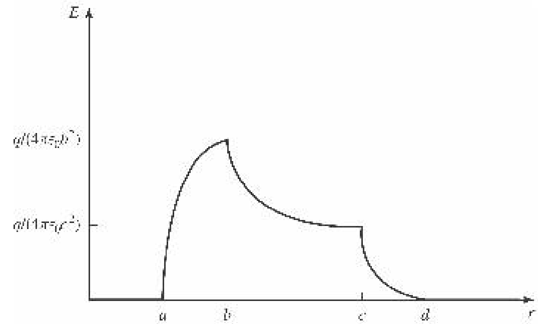


22.43: a) The sphere acts as a point charge on an external charge, so:
$F = qE = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{r^2}$, radially inward.

(b) If the point charge was inside the sphere (where there is no electric field) it would feel zero force.

22.44: a) $\rho_{\text{inner}} = \dfrac{+q}{V_b - V_a} = \dfrac{q}{\frac{4}{3}\pi b^3 - \frac{4}{3}\pi a^3} = \dfrac{3q}{4\pi}\left(\dfrac{1}{b^3 - a^3}\right)$

$$\rho_{\text{outer}} = \frac{-q}{V_d - V_c} = \frac{-q}{\frac{4}{3}\pi d^3 - \frac{4}{3}\pi c^3} = \frac{-3q}{4\pi}\left(\frac{1}{d^3 - c^3}\right)$$



b) (i) $r < a \oint \vec{E} \cdot d\vec{A} = 0 \Rightarrow E = 0.$

(ii) $a < r < b \oint \vec{E} \cdot d\vec{A} = \dfrac{1}{\varepsilon_0}\int \rho_{\text{inner}} dV \Rightarrow E 4\pi r^2 = \dfrac{4}{3\varepsilon_0}\pi(r^3 - a^3)\rho_{\text{inner}}$

$$E = \frac{1}{3\varepsilon_0}\rho_{\text{inner}}\frac{(r^3 - q^3)}{r^2} = \frac{q}{4\pi\varepsilon_0}\frac{(r^3 - a^3)}{(b^3 - a^3)}$$

(iii) $b < r < c \oint \vec{E} \cdot d\vec{A} = \dfrac{q}{\varepsilon_0} \Rightarrow E 4\pi r^2 = \dfrac{q}{\varepsilon_0} \Rightarrow E = \dfrac{q}{4\pi\varepsilon_0 r^2}$

(iv) $c < r < d \oint \vec{E} \cdot d\vec{A} = \dfrac{q}{\varepsilon_0} + \dfrac{1}{\varepsilon_0}\int \rho_{\text{outer}} dV \Rightarrow$

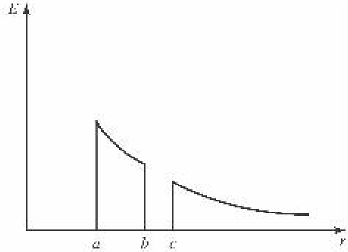
$$E 4\pi r^2 = \frac{q}{\varepsilon_0} + \frac{4\pi}{3\varepsilon_0}(r^3 - c^3)\rho_{\text{outer}}, \text{ so } E = \frac{q}{4\pi\varepsilon_0 r^2} - \frac{q(r^3 - c^3)}{4\pi\varepsilon_0 r^2 (d^3 - c^3)}$$

(v) $r > d \oint \vec{E} \cdot d\vec{A} = \dfrac{q}{\varepsilon_0} - \dfrac{q}{\varepsilon_0} = 0 \Rightarrow E = 0$

22.45: a) $a < r < b, E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2\lambda}{r}$, radially outward, as in **22.48 (b)**.

b) $r > c, E = \frac{1}{4\pi\varepsilon_0}\frac{2\lambda}{r}$, radially outward, since again the charge enclosed is the same as in part (a).

c)

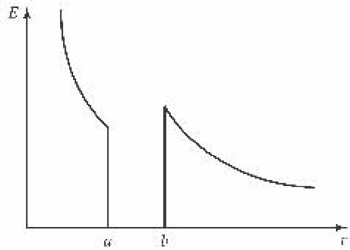


d) The inner and outer surfaces of the outer cylinder must have the same amount of charge on them: $\lambda l = -\lambda_{\text{inner}} l \Rightarrow \lambda_{\text{inner}} = -\lambda$, and $\lambda_{\text{outer}} = \lambda$.

22.46: a) (i) $r < a, E(2\pi rl) = \dfrac{q}{\varepsilon_0} = \dfrac{\alpha l}{\varepsilon_0} \Rightarrow E = \dfrac{\alpha}{2\pi\varepsilon_0 r}$.

(ii) $a < r < b$, there is no net charge enclosed, so the electric field is zero.

(iii) $r > b, E(2\pi rl) = \dfrac{q}{\varepsilon_0} = \dfrac{2\alpha l}{\varepsilon_0} \Rightarrow E = \dfrac{\alpha}{\pi\varepsilon_0 r}$.

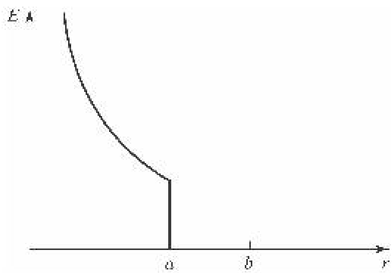


b) (i) Inner charge per unit length is $-\alpha$. (ii) Outer charge per length is $+2\alpha$.

22.47: a) (i) $r < a$, $E(2\pi rl) = \frac{q}{\varepsilon_0} = \frac{\alpha l}{\varepsilon_0} \Rightarrow E = \frac{\alpha}{2\pi\varepsilon_0 r}$, radially outward.

(ii) $a < r < b$, there is not net charge enclosed, so the electric field is zero.

(iii) $r > b$, there is no net charge enclosed, so the electric field is zero.



b) (i) Inner charge per unit length is $-\alpha$.

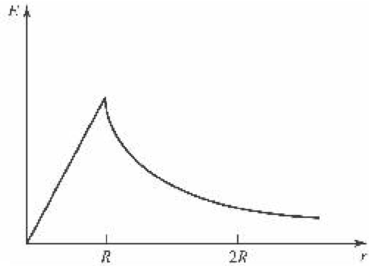
(ii) Outer charge per length is ZERO.

22.48: a) $r < R$, $E(2\pi rl) = \frac{q}{\varepsilon_0} = \frac{\rho\pi r^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho r}{2\varepsilon_0}$, radially outward.

b) $r > R$, and $\lambda = \rho\pi R^2$, $E(2\pi rl) = \frac{q}{\varepsilon_0} = \frac{\rho\pi R^2 l}{\varepsilon_0} \Rightarrow E = \frac{\rho R^2}{2\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2k\lambda}{r}$.

c) $r = R$. the electric field for BOTH regions is $E = \frac{\rho R}{2\varepsilon_0}$, so they are consistent.

d)



22.49: a) The conductor has the surface charge density on BOTH sides, so it has twice the enclosed charge and twice the electric field.

b) We have a conductor with surface charge density σ on both sides. Thus the electric field outside the plate is $\Phi = E(2A) = (2\sigma A)/\varepsilon_0 \Rightarrow E = \sigma/\varepsilon_0$. To find the field inside the conductor use a Gaussian surface that has one face inside the conductor, and one outside.

Then:

$$\Phi = E_{\text{out}}A + E_{\text{in}}A = (\sigma A)/\varepsilon_0 \text{ but } E_{\text{out}} = \sigma/\varepsilon_0 \Rightarrow E_{\text{in}}A = 0 \Rightarrow E_{\text{in}} = 0.$$

22.50: a) If the nucleus is a uniform positively charged sphere, it is only at its very center where forces on a charge would balance or cancel

b) $$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \Rightarrow E4\pi r^2 = \frac{e}{\varepsilon_0}\left(\frac{r^3}{R^3}\right) \Rightarrow E = \frac{er}{4\pi\varepsilon_0 R^3}$$

$$\Rightarrow F = qE = -\frac{1}{4\pi\varepsilon_0}\frac{e^2 r}{R^3}.$$

So from the simple harmonic motion equation:

$$F = -m\omega^2 r = -\frac{1}{4\pi\varepsilon_0}\frac{e^2 r}{R^3} \Rightarrow \omega = \sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mR^3}} \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mR^3}}.$$

c) If $f = 4.57 \times 10^{14}\ \text{Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{4\pi\varepsilon_0}\dfrac{e^2}{mR^3}}$

$$\Rightarrow R = \sqrt[3]{\frac{1}{4\pi\varepsilon_0}\frac{(1.60\times10^{-19}\ \text{C})^2}{4\pi^2(9.11\times10^{-31}\ \text{kg})(4.57\times10^{14}\ \text{Hz})^2}} = 3.13\times10^{-10}\ \text{m}.$$

$r_{\text{actual}}/r_{\text{Thompson}} \approx 1$

d) If $r > R$ then the electron would still oscillate but not undergo simple harmonic motion, because for $r > R$, $F \propto 1/r^2$, and is not linear.

22.51: The electrons are separated by a distance $2d$, and the amount of the positive nucleus's charge that is within radius d is all that exerts a force on the electron. So:
$F_e = \dfrac{ke^2}{(2d)^2} = F_{\text{nucleus}} = 2ke^2\frac{d}{R^3} \Rightarrow d^3 = R^3/8 \Rightarrow d = R/2.$

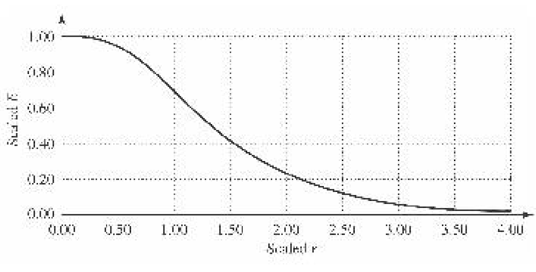
22.52: a) $Q(r) = Q - \int \rho dV = Q - \dfrac{Q}{\pi a_0^3}\iiint e^{-2r/a_0} r^2 dr\ \sin\theta d\theta\ d\phi = Q - \dfrac{4Q}{a_0^3}\int_0^r x^2 e^{-2x/a_0}\ dx$

$$\Rightarrow Q(r) = Q - \frac{4Qe^{-\alpha r}}{a_0{}^3\alpha^3}(2e^{\alpha r} - \alpha^2 r^2 - 2\alpha r - 2) = Qe^{-2r/a_0}[2(r/a_0)^2 + 2(r/a_0) + 1].$$

Note if $r \to \infty$, $Q(r) \to 0$.

b) The electric field is radially outward, and has magnitude:

$$\Rightarrow E = \frac{kQe^{-2r/a_0}}{r^2}(2(r/a_0)^2 + 2(r/a_0) + 1).$$



22.53: a) At $r = 2R$, $F = q_e E = \frac{1}{4\pi\varepsilon_0}\frac{q_e q_{Fe}}{4R^2} = \frac{1}{4\pi\varepsilon_0}\frac{(82)(1.6\times10^{-19}\ \text{C})^2}{4(7.1\times10^{-15}\ \text{m})^2} = 94\ \text{N}.$

So: $a = F/m = 94\ \text{N}/9.11\times10^{-31}\ \text{kg} = 1.0\times10^{32}\ \text{m/s}^2.$

b) At $r = R$, $a = 4a_{(a)} = 4.1\times10^{32}$ m/s^2.

c) At $r = R/2, Q = \frac{1}{8}(82e)$ ($\frac{1}{8}$ because the charge enclosed goes like r^3) so with the radius decreasing by 2, the acceleration from the change in radius goes up by $(2)^2 = 4$, but the charge decreased by 8, so $a = \frac{4}{8}a_{(b)} = 2.1\times10^{32}$ m/s^2.

d) At $r = 0, Q = 0$, so $F = 0$.

22.54: a) The electric field of the slab must be zero by symmetry. There is no preferred direction in the y-z plane, so the electric field can only point in the x-direction. But at the origin in the x-direction, neither the positive nor negative directions should be singled out as special, and so the field must be zero.

b) Use a Gaussian surface that has one face of area A on in the y-z plane at $x = 0$, and the other face at a general value x. Then:

$$x \le d : \Phi = EA = \frac{Q_{encl}}{\varepsilon_0} = \frac{\rho Ax}{\varepsilon_0} \Rightarrow E = \frac{\rho x}{\varepsilon_0},$$

with direction given by $\frac{x}{|x|}\hat{\boldsymbol{i}}$.

Note that E is zero at $x = 0$.

Now outside the slab, the enclosed charge is constant with x:

$$x \ge d : \Phi = EA = \frac{Q_{encl}}{\varepsilon_0} = \frac{\rho Ad}{\varepsilon_0} \Rightarrow E = \frac{\rho d}{\varepsilon_0},$$

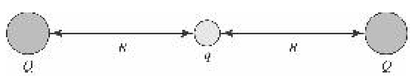
again with direction given by $\frac{x}{|x|}\hat{\boldsymbol{i}}$.

22.55: a) Again, E is zero at $x = 0$, by symmetry arguments.

b) $x \le d : \Phi = EA = \dfrac{Q_{encl}}{\varepsilon_0} = \dfrac{\rho_0 A}{\varepsilon_0 d^2}\displaystyle\int_0^x x'^2\,dx' = \dfrac{\rho_0 Ax^3}{3\varepsilon_0 d^2} \Rightarrow E = \dfrac{\rho_0 x^3}{3\varepsilon_0 d^2}$, in $\dfrac{x}{|x|}\hat{\boldsymbol{i}}$ direction.

$x \ge d : \Phi = EA = \dfrac{Q_{encl}}{\varepsilon_0} = \dfrac{\rho_0 A}{\varepsilon_0 d^2}\displaystyle\int_0^d x'^2\,dx' = \dfrac{\rho_0 Ad}{3\varepsilon_0} \Rightarrow E = \dfrac{\rho_0 d}{3\varepsilon_0}$, *in* $\dfrac{x}{|x|}\hat{\boldsymbol{i}}$ direction.

22.56: a) We could place two charges $+Q$ on either side of the charge $+q$:



b) In order for the charge to be stable, the electric field in a neighborhood around it must always point back to the equilibrium position.

c) If q is moved to infinity and we require there to be an electric field always pointing in to the region where q had been, we could draw a small Gaussian surface

there. We would find that we need a negative flux into the surface. That is, there has to be a negative charge in that region. However, there is none, and so we cannot get such a stable equilibrium.

d) For a negative charge to be in stable equilibrium, we need the electric field to always point away from the charge position. The argument in (c) carries through again, this time inferring that a positive charge must be in the space where the negative charge was if stable equilibrium is to be attained.

22.57: a) The total charge: $q = 4\pi \int_0^R \rho_0 (1 - r/R) r^2 dr = 4\pi[\int_0^R r^2 dr - \int_0^R r^3/R dr]$

$$\Rightarrow q = 4\pi\rho_0 \, [R^3/3 - R^3/4] = \frac{4\pi R^3 \rho_0}{12} = \frac{4\pi R^3}{12}\frac{3Q}{\pi R^3} = Q.$$

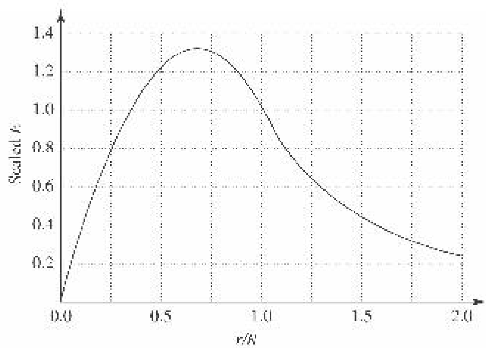
b) $r \geq R$, all the charge Q is enclosed, and: $\Phi = E(4\pi r^2) = Q/\varepsilon_0 \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, the same as a point charge.

c) $r \leq R$, then $Q(r) = q(r^3/R^3)$.

Also, $Q(r) = 4\pi \int \rho_0 (1 - r/R) r^2 dr = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$

$$\Rightarrow E(r) = \frac{12kQ}{r^2}\left(\frac{1}{3}\frac{r^3}{R^3} - \frac{1}{4}\frac{r^4}{R^4}\right) = \frac{kQ}{r^2}\left(\frac{4r^3}{R^3} - \frac{3r^4}{R^4}\right) = kQ\frac{r}{R^3}\left(4 - 3\frac{r}{R}\right)$$

d)



e) $\frac{\partial E}{\partial r} = 0 \; (r \leq R) \Rightarrow \frac{4kQ}{R^3} - \frac{6kQ}{R^4} r = 0 \Rightarrow r_{\max} = \frac{2}{3} R.$ So $E_{\max} = \frac{2}{3}\frac{kQ}{R^2}(4 - 2) = \frac{4kQ}{3R^2}$

22.58: a)

$$Q = 4\pi \int_0^\infty \rho(r) r^2 dr = 4\pi\rho_0 \int_0^R \left(1 - \frac{4r}{3R}\right) r^2 dr = 4\pi\rho_0 \left[\int_0^R r^2 dr - \frac{4}{3R}\int_0^R r^3 dr\right]$$

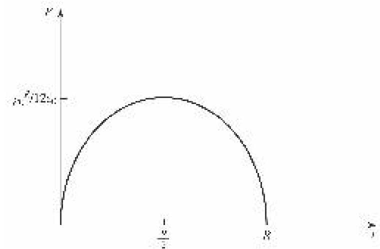
$$= 4\pi\rho_0\left[\frac{R^3}{3} - \frac{4}{3R}\cdot\frac{R^4}{4}\right] \Rightarrow Q = 0$$

b) $$r \geq R, \oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{encl}}}{\varepsilon_0} = 0 \Rightarrow E = 0$$

c) $$r \leq R, \oint \vec{E}\cdot d\vec{A} = \frac{4\pi}{\varepsilon_0}\int_0^r \rho(r')r'^2 dr' \Rightarrow E4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\left[\int_0^r r'^2 dr' - \frac{4}{3R}\int_0^r r'^3 dr'\right]$$

$$\Rightarrow E = \frac{\rho_0}{\varepsilon_0}\frac{1}{r^2}\left[\frac{r^3}{3} - \frac{r^4}{3R}\right] = \frac{\rho_0}{3\varepsilon_0} r\left[1 - \frac{r}{R}\right]$$

d)



e) $$\frac{\partial E}{\partial r} = 0 \Rightarrow \frac{\rho_0}{3\varepsilon_0} - \frac{2\rho_0 r_{\text{max}}}{3\varepsilon_0 R} = 0 \; r_{\text{max}} = \frac{R}{2}$$

$$E\left(r - \frac{R}{2}\right) = \frac{\rho_0}{3\varepsilon_0}\frac{R}{2}\left[1 - \frac{1}{2}\right] = \frac{\rho_0 R}{12\varepsilon_0}$$

22.59: a) $\Phi_g = \oint \vec{g}\cdot d\vec{A} = -Gm\oint \frac{r^2 \sin\theta \, dr \, d\theta \, d\phi}{r^2} = -4\pi Gm.$

b) For any closed surface, mass OUTSIDE the surface contributes zero to the flux passing through the surface. Thus the formula above holds for any situation where m is the mass enclosed by the Gaussian surface.

That is: $\Phi_g = \oint \vec{g}\cdot d\vec{A} = -4\pi GM_{\text{encl.}}$

22.60: a) $\Phi_g = g4\pi r^2 = -4\pi GM \Rightarrow g = -\frac{GM}{r^2}$, which is the same as for a point mass.

b) Inside a hollow shell, the $M_{\text{encl}} = 0$, so $g = 0$.

c) Inside a uniform spherical mass:

$$\Phi_g = g4\pi r^2 = -4\pi GM_{\text{encl}} = -4\pi G\left(M\frac{r^3}{R^3}\right) \Rightarrow g = -\frac{GMr}{R^3},$$

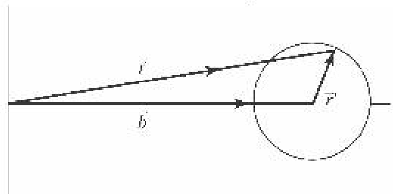
which is linear in r.

22.61: a) For a sphere NOT at the coordinate origin:

$$\vec{r}\,' = \vec{r} - \vec{b} \Rightarrow \Phi = 4\pi r'^2\, E = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0}\frac{4\pi r'^3}{3} \Rightarrow E = \frac{\rho r'}{3\varepsilon_0},$$

in the $\hat{r}\,'$-direction.

$$\Rightarrow \vec{E} = \frac{\rho(\vec{r} - \vec{b})}{3\varepsilon_0}.$$



b) The electric field inside a hole in a charged insulating sphere is:

$$\vec{E}_{\text{hole}} = \vec{E}_{\text{sphere}} - \vec{E}_{(a)} = \frac{\rho\vec{r}}{3\varepsilon_0} - \frac{\rho(\vec{r} - \vec{b})}{3\varepsilon_0} = \frac{\rho\vec{b}}{3\varepsilon_0}.$$

Note that $\vec{E}$ is uniform.

22.62: Using the technique of **22.61,** we first find the field of a cylinder off-axis, then the electric field in a hole in a cylinder is the difference between two electric fields—that of a solid cylinder on-axis, and one off-axis.

$$\vec{r}\,' = \vec{r} - \vec{b} \Rightarrow \Phi = 2\pi r' l\, E = \frac{Q_{\text{encl}}}{\varepsilon_0} = \frac{\rho}{\varepsilon_0} l\pi r'^2 \Rightarrow E = \frac{\rho r'}{2\varepsilon_0} \Rightarrow \vec{E} = \frac{\rho(\vec{r} - \vec{b})}{2\varepsilon_0}.$$

$$\vec{E}_{\text{hole}} = \vec{E}_{\text{cylinder}} - \vec{E}_{\text{above}} = \frac{\rho\,\vec{r}}{2\varepsilon_0} - \frac{\rho(\vec{r} - \vec{b})}{2\varepsilon_0} = \frac{\rho\,\vec{b}}{2\varepsilon_0}.$$ Note that $\vec{E}$ is uniform.

22.63: a) $x = 0$: no field contribution from the sphere centered at the origin, and the other sphere produces a point-like field:

$$\vec{E}(x = 0) = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{(2R)^2}\hat{\boldsymbol{i}} = -\frac{1}{4\pi\varepsilon_0}\frac{Q}{4R^2}\hat{\boldsymbol{i}}.$$

b) $x = R/2$: the sphere at the origin provides the field of a point charge of charge $q = Q/8$ since only one-eighth of the charge's volume is included. So:

$$\vec{E}\,(x = R/2) = \frac{1}{4\pi\varepsilon_0}\left(\frac{(Q/8)}{(R/2)^2} - \frac{Q}{(3R/2)^2}\right)\hat{\boldsymbol{i}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}(1/2 - 4/9)\hat{\boldsymbol{i}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{18R^2}\hat{\boldsymbol{i}}.$$

c) $x = R$: the two electric fields cancel, so $\vec{E} = 0$.

d) $x = 3R$: now both spheres contribute fields pointing to the right:

$$\vec{E}\ (x=3R)=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{(3R)^2}+\frac{Q}{R^2}\right)\hat{i}=\frac{1}{4\pi\varepsilon_0}\frac{10Q}{9R^2}\hat{i}.$$

22.64: (See Problem 22.63 with $Q\rightarrow -Q$ for terms associated with right sphere)

a) $\vec{E}\ (x=0)=+\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{4R^2}\hat{i}$

b) $\vec{E}\left(x=\dfrac{R}{2}\right)=\dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{(Q/8)}{(R/2)^2}+\dfrac{Q}{(3R/2)^2}\right]\hat{i}=\dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{Q}{2R^2}+\dfrac{4Q}{9R^2}\right]\hat{i}=\dfrac{1}{4\pi\varepsilon_0}\dfrac{17Q}{18R^2}\hat{i}$

c) $$\vec{E}\ (x=R)=\frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{R^2}+\frac{Q}{R^2}\right]\hat{i}=\frac{Q}{2\pi\varepsilon_0 R^2}\hat{i}$$

d) $\vec{E}\ (x=3R)=\dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{Q}{(3R)^2}-\dfrac{Q}{R^2}\right]\hat{i}=\dfrac{1}{4\pi\varepsilon_0}\left[\dfrac{Q}{9R^2}-\dfrac{Q}{R^2}\right]\hat{i}=\dfrac{-1}{4\pi\varepsilon_0}\dfrac{8Q}{9R^2}\hat{i}$

22.65: a) The charge enclosed:

$Q=Q_i+Q_0$, where $Q_i=\alpha\dfrac{4\pi(R/2)^3}{3}=\dfrac{\alpha\pi R^3}{6}$, and $Q_0=4\pi(2\alpha)\displaystyle\int_{R/2}^{R}(r^2-r^3/R)dr$

$$=8\alpha\pi\left(\frac{(R^3-R^3/8)}{3}-\frac{(R^4-R^4/16)}{4R}\right)=\frac{11\alpha\pi R^3}{24}$$

$$\Rightarrow Q=\frac{15\alpha\pi R^3}{24}\Rightarrow\alpha=\frac{8Q}{5\pi R^3}.$$

b) $r\le R/2:\Phi=E4\pi r^2=\dfrac{\alpha 4\pi r^3}{3\varepsilon_0}\Rightarrow E=\dfrac{\alpha r}{3\varepsilon_0}=\dfrac{8Qr}{15\pi\varepsilon_0 R^3}.$

$R/2\le r\le R:\Phi=E4\pi r^2=\dfrac{Q_i}{\varepsilon_0}+\dfrac{1}{\varepsilon_0}\left(8\alpha\pi\left(\dfrac{(r^3-R^3/8)}{3}-\dfrac{(r^4-R^4/16)}{4R}\right)\right)$

$\Rightarrow E=\dfrac{\alpha\pi R^3}{24\varepsilon_0(4\pi r^2)}(64(r/R)^3-48(r/R)^4-1)=\dfrac{kQ}{15r^2}(64(r/R)^3-48(r/R)^4-1).$

$r\ge R:E=\dfrac{Q}{4\pi\varepsilon_0 r^2}$, since all charge is enclosed.

c) $$\frac{Q_i}{Q}=\frac{(4Q/15)}{Q}=\frac{4}{15}=0.267.$$

d) $r\le R/2:F=-eE=-\frac{8eQ}{15\pi\varepsilon_0 R^3}r$, so the restoring force depends upon displacement to the first power, and we have simple harmonic motion.

e) $F=-kr,\ k=\dfrac{8eQ}{15\pi\varepsilon_0 R^3},\ \omega=\sqrt{\dfrac{k}{m_e}}=\sqrt{\dfrac{8eQ}{15\pi\varepsilon_0 R^3 m_e}},T=\dfrac{2\pi}{\omega}=2\pi\sqrt{\dfrac{15\pi\varepsilon_0 R^3 m_e}{8eQ}}.$

f) If the amplitude of oscillation is greater than $R/2$, the force is no longer linear in r, and is thus no longer simple harmonic.

22.66: a) Charge enclosed:

$$Q = Q_i + Q_0 \text{ where } Q_i = 4\pi \int_0^{R/2} \frac{3\alpha r^3}{2R} dr = \frac{6\pi\alpha}{R}\frac{1}{4}\frac{R^4}{16} = \frac{3}{32}\pi\alpha R^3.$$

$$\text{and } Q_0 = 4\pi\alpha \int_{R/2}^{R} (1-(r/R)^2)r^2\, dr = 4\pi\alpha R^3 \left(\frac{7}{24} - \frac{31}{160}\right) = \frac{47}{120}\pi\alpha R^3.$$

$$\text{Therefore, } Q = \left(\frac{3}{32} + \frac{47}{120}\right)\pi\alpha R^3 = \frac{233}{480}\pi\alpha R^3 \Rightarrow \alpha = \frac{480Q}{233\pi R^3}.$$

b) $$r \le R/2 : \Phi = E4\pi r^2 = \frac{4\pi}{\varepsilon_0}\int_0^r \frac{3\alpha r'^3}{2R} dr' = \frac{3\pi\alpha r^4}{2\varepsilon_0 R} \Rightarrow E = \frac{6\alpha r^2}{16\varepsilon_0 R} = \frac{180Qr^2}{233\pi\varepsilon_0 R^4}.$$

$$R/2 \le r \le R : \Phi = E4\pi r^2 = \frac{Q_i}{\varepsilon_0} + \frac{4\pi\alpha}{\varepsilon_0}\int_{R/2}^{r} (1-(r'/R)^2)r'^2\, dr'$$

$$= \frac{Q_i}{\varepsilon_0} + \frac{4\pi\alpha}{\varepsilon_0}\left(\frac{r^3}{3} - \frac{R^3}{24} - \frac{r^5}{5R^2} + \frac{R^3}{160}\right) = \frac{3}{128}\frac{4\pi\alpha R^3}{\varepsilon_0}$$

$$+ \frac{4\pi\alpha R^3}{\varepsilon_0}\left(\frac{1}{3}\left(\frac{r}{R}\right)^3 - \frac{1}{5}\left(\frac{r}{R}\right)^5 - \frac{17}{480}\right)$$

$$\Rightarrow E = \frac{480Q}{233\pi\varepsilon_0 r^2}\left(\frac{1}{3}\left(\frac{r}{R}\right)^3 - \frac{1}{5}\left(\frac{r}{R}\right)^5 - \frac{23}{1920}\right).$$

$r \ge R : E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$, since all charge is enclosed.

c) The fraction of Q between $R/2 \le r \le R$:

$$\frac{Q_o}{Q} = \frac{47}{120}\frac{480}{233} = 0.807.$$

d) $E(r = R/2) = \frac{180}{233}\frac{Q}{4\pi\varepsilon_0 R^2}$, using either of the electric field expressions above, evaluated at $r = R/2$.

e) The force an electron would feel never is proportional to $-r$ which is necessary for simple harmonic oscillations. It is oscillatory since the force is always attractive, but it has the wrong power of r to be *simple* harmonic.

23.1: $\Delta U = kq_1q_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right) = k(2.40\mu\text{C})(-4.30\mu\text{C})\left(\frac{1}{0.354\text{m}}-\frac{1}{0.150\text{m}}\right) = 0.357\text{J}$

$\Rightarrow W = -\Delta U = -0.357$ J.

23.2: $W = -1.9\times10^{-8}\text{J} = -\Delta U = U_i - U_f \Rightarrow U_f = 1.9\times10^{-8}\text{J} + 5.4\times10^{-8}\text{J} =$

$7.3\times10^{-8}\text{J}$

23.3: a)

$$E_i = K_i + U_i = \frac{1}{2}(0.0015\text{ kg})(22.0\text{ m/s})^2 + \frac{k(2.80\times10^{-6}\text{C})(7.50\times10^{-6}\text{C})}{0.800\text{ m}} = 0.608\text{ J}$$

$$E_i = E_f = \frac{1}{2}mv_f^2 + \frac{kq_1q_2}{r_f} \Rightarrow v_f = \sqrt{\frac{2(0.608\text{ J} - 0.491\text{ J})}{0.0015\text{ kg}}} = 12.5\text{ m/s}.$$

b) At the closest point, the velocity is zero:

$$\Rightarrow 0.608\text{ J} = \frac{kq_1q_2}{r} \Rightarrow r = \frac{k(2.80\times10^{-6}\text{C})(7.80\times10^{-6}\text{C})}{0.608\text{ J}} = 0.323\text{ m}.$$

23.4: $U = -0.400\text{ J} = \frac{kq_1q_2}{r} \Rightarrow r = \frac{-k(2.30\times10^{-6}\text{C})(7.20\times10^{-6}\text{C})}{-0.400\text{ J}} = 0.373\text{ m}.$

23.5: a) $U = \frac{kQq}{r} = \frac{k(4.60\times10^{-6}\text{C})(1.20\times10^{-6}\text{C})}{0.250\text{ m}} = 0.199\text{ J}.$

b) (i) $K_f = K_i + U_i - U_f$

$$= 0\text{ J} + k(4.60\times10^{-6}\text{C})(1.20\times10^{-6}\text{C})\left(\frac{1}{0.25\text{ m}} - \frac{1}{0.5\text{ m}}\right) = 0.0994\text{ J}$$

$$\Rightarrow K_f = 0.0994\text{ J} = \frac{1}{2}mv_f^2 \Rightarrow v_f = \sqrt{\frac{2(0.0994\text{ J})}{2.80\times10^{-4}\text{kg}}} = 26.6\text{ m/s}.$$

(ii) $K_f = 0.189\text{ J},\ v_f = 36.7\text{ m/s}.$

(iii) $K_f = 0.198\text{ J},\ v_f = 37.6\text{ m/s}.$

23.6: $U = \frac{kq^2}{0.500\text{m}} + \frac{2kq^2}{0.500\text{ m}} = 6kq^2 = 6k(1.2\times10^{-6}\text{ C})^2 = 0.078\text{ J}.$

23.7: a)

$$U = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_2}{r_{13}} + \frac{q_1q_2}{r_{23}}\right) = k\left(\frac{(4.00\text{ nC})(-3.00\text{ nC})}{(0.200\text{ m})} + \frac{(4.00\text{ nC})(2.00\text{ nC})}{(0.100\text{m})} + \frac{(-3.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})}\right)$$

$$= -3.60\times10^{-7}\text{ J}.$$

b) *If* $U = 0,\ 0 = k\left(\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{x} + \frac{q_2q_3}{r_{12}-x}\right)$. So solving for x we find:

$0 = -60 + \frac{8}{x} - \frac{6}{0.2-x} \Rightarrow 60x^2 - 26x + 1.6 = 0 \Rightarrow x = 0.074\text{ m}, 0.360\text{ m}$. Therefore $x = 0.074$ m since it is the only value between the two charges.

23.8: From Example 23.1, the initial energy E_i can be calculated:

$$E_i = K_i + U_i = \frac{1}{2}(9.11\times10^{-31}\text{kg})(3.00\times10^6\text{ m/s})^2 + \frac{k(-1.60\times10^{-19}\text{C})(3.20\times10^{-19}\text{C})}{10^{-10}\text{m}}$$

$$\Rightarrow E_i = -5.09\times10^{-19}\text{ J}.$$

When velocity equals zero, all energy is electric potential energy, so:

$$-5.09\times10^{-19}\text{ J} = -\frac{k2e^2}{r} \Rightarrow r = 9.06\times10^{-10}\text{ m}.$$

23.9: Since the work done is zero, the sum of the work to bring in the two equal charges q must equal the work done in bringing in charge Q.

$$W_{qq} = W_{qQ} \Rightarrow -\frac{kq^2}{d} = \frac{2kqQ}{d} \Rightarrow Q = -\frac{q}{2}.$$

23.10: The work is the potential energy of the combination.

$$U = U_{p\alpha} + U_{pe} + U_{e\alpha}$$

$$= \frac{ke(2e)}{5\sqrt{2}\times10^{-10}\text{m}} + \frac{ke(-e)}{5\times10^{-10}\text{m}} + \frac{k(-e)(2e)}{5\times10^{-10}\text{m}}$$

$$= \frac{ke^2}{5\times10^{-10}\text{m}}\left(\frac{2}{\sqrt{2}} - 1 - 2\right)$$

$$= \frac{(9.0\times10^9\text{ Nm}^2/\text{C}^2)\,(1.6\times10^{-19}\text{ C})^2}{5\times10^{-10}\text{m}}\left(\frac{2}{\sqrt{2}} - 3\right)$$

$$= -7.31\times10^{-19}\text{ J}$$

Since U is negative, we want do $+7.31\times10^{-19}$ J to separate the particles

23.11: $K_1+U_1=K_2+U_2$; $K_1=U_2=0$ so $K_2=U_1$

$$U_1=\frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r}+\frac{2}{r}+\frac{2}{r}\right)=\frac{1}{4\pi\varepsilon_0}\frac{5e^2}{r},\ \text{with } r=8.00\times10^{-10}\ \text{m}$$

$$U_1=1.44\times10^{-18}\ \text{J}=9.00\ \text{eV}$$

23.12: Get closest distance γ. Energy conservation: $\frac{1}{2}mv^2+\frac{1}{2}mv^2=\frac{ke^2}{\gamma}$

$$\gamma=\frac{ke^2}{mv^2}=\frac{(9\times10^9\ \text{Nm}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{(1.67\times10^{-27}\ \text{kg})(10^6\ \text{m/s})}=1.38\times10^{-13}\ \text{m}$$

Maximum force:

$$F=\frac{ke^2}{\gamma^2}$$

$$=\frac{(9\times10^9\ \text{Nm}^2/\text{C}^2)\ (1.6\times10^{-19}\ \text{C})^2}{(1.38\times10^{-13}\text{m})^2}$$

$$=0.012\ \text{N}$$

23.13: $K_A+U_A=K_B+U_B$

$U=qV$, so $K_A+qV_A=K_B+qV_B$

$$K_B=K_A+q(V_A-V_B)=0.00250\ \text{J}+(-5.00\times10^{-6}\ \text{C})\ (200\ \text{V}-800\ \text{V})=0.00550\ \text{J}$$

$$v_B=\sqrt{2K_B/m}=7.42\ \text{m/s}$$

It is faster at *B;* a negative charge gains speed when it moves to higher potential.

23.14: Taking the origin at the center of the square, the symmetry means that the potential is the same at the two corners not occupied by the $+5.00\ \mu\text{C}$ charges (The work done in moving to either corner from infinity is the same). But this also means that no net work is done is moving from one corner to the other.

23.15: $\vec{E}$ points from high potential to low potential, so $V_B>V_A$ and $V_C<V_A$.

The force on a positive test charge is east, so no work is done on it by the electric force when it moves due south (the force and displacement are perpendicular); $V_D=V_A$.

23.16: a) $W=-\Delta U=qEd=\Delta K=1.50\times10^{-6}$ J.

b) The initial point was at a higher potential than the latter since any positive charge, when free to move, will move from greater to lesser potential.

$\Delta V = \Delta U/q = (1.50\times 10^{-6}\ \text{J})/(4.20\ \text{nC}) = 357\ \text{V}.$

c) $qEd = 1.50\times 10^{-6}\ \text{J} \Rightarrow E = \dfrac{1.50\times 10^{-6}\ \text{J}}{(4.20\ \text{nC})(0.06\ \text{m})} = 5.95\times 10^{3}\ \text{N/C}.$

23.17: a) Work done is zero since the motion is along an equipotential, perpendicular to the electric field.

b) $W = qEd = (28.0\ \text{nC})\left(4.00\times 10^{4}\ \dfrac{\text{V}}{\text{m}}\right)(0.670\ \text{m}) = 7.5\times 10^{-4}\ \text{J}$

c) $W = qEd = (28.0\ \text{nC})\left(4.00\times 10^{4}\ \dfrac{\text{V}}{\text{m}}\right)(-2.60\cos 45°) = -2.06\times 10^{-3}\ \text{J}$

23.18: Initial energy equals final energy:

$$E_i = E_f \Rightarrow -\frac{keq_1}{r_{1i}} - \frac{keq_2}{r_{2i}} = -\frac{keq_1}{r_{1f}} - \frac{keq_2}{r_{2f}} + \frac{1}{2}m_e v_f^{\ 2}$$

$$E_i = k(-1.60\times 10^{-19}\ \text{C})\left(\frac{(3.00\times 10^{-9}\ \text{C})}{0.25\ \text{m}} + \frac{(2.00\times 10^{-9}\ \text{C})}{0.25\ \text{m}}\right) = -2.88\times 10^{-17}\ \text{J}$$

$$E_f = k(-1.60\times 10^{-19}\ \text{C})\left(\frac{(3.00\times 10^{-9}\ \text{C})}{0.10\ \text{m}} + \frac{(2.00\times 10^{-9}\ \text{C}}{0.40\ \text{m}}\right) + \frac{1}{2}m_e v_f^2$$

$$= -5.04\times 10^{-17}\ \text{J} + \frac{1}{2}m_e v_f^{\ 2}$$

$$\Rightarrow v_f = \sqrt{\frac{2}{9.11\times 10^{-31}\ \text{kg}}(5.04\times 10^{-17}\ \text{J} - 2.88\times 10^{-17}\ \text{J})}$$

$$= 6.89\times 10^{6}\ \text{m/s}.$$

23.19: a) $V = \dfrac{kq}{r} \Rightarrow r = \dfrac{kq}{V} = \dfrac{k(2.50\times 10^{-11}\ \text{C})}{90.0\ \text{V}} = 2.5\times 10^{-3}\ \text{m}.$

b) $V = \dfrac{kq}{r} \Rightarrow r = \dfrac{kq}{V} = \dfrac{k(2.50\times 10^{-11}\ \text{C})}{30.0\ \text{V}} = 7.5\times 10^{-3}\ \text{m}.$

23.20: a) $V = \dfrac{kq}{r} \Rightarrow q = \dfrac{rV}{k} = \dfrac{(0.250\ \text{m})(48.0\ \text{V})}{k} = 1.33\times 10^{-9}\ \text{C}.$

b) $V = \dfrac{k(1.33 \times 10^{-9}\ \text{C})}{(0.750\ \text{m})} = 16\ \text{V}$

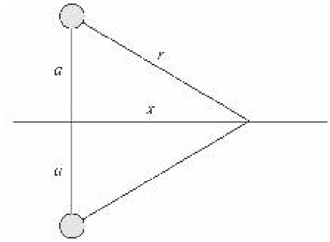
23.21: a) At A : $V_A = k\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2}\right) = k\left(\dfrac{2.40 \times 10^{-9}\ \text{C}}{0.05\ \text{m}} + \dfrac{-6.50 \times 10^{-9}\ \text{C}}{0.05\ \text{m}}\right) = -738\ \text{V}.$

b) At B : $V_B = k\left(\dfrac{q_1}{r_1} + \dfrac{q_2}{r_2}\right) = k\left(\dfrac{2.40 \times 10^{-9}\ \text{C}}{0.08\ \text{m}} + \dfrac{-6.50 \times 10^{-9}\ \text{C}}{0.06\ \text{m}}\right) = -705\ \text{V}.$

c) $W = q\Delta V = (2.50 \times 10^{-9}\ \text{C})(-33\ \text{V}) = -8.25 \times 10^{-8}\ \text{J}.$

The negative sign indicates that the work is done *on* the charge. So the work done by the field is 8.25×10^{-8} J.

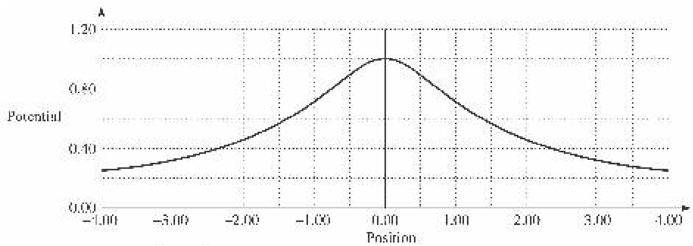
23.22: a)



b) $V = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{a}.$

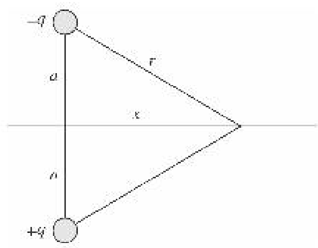
c) Looking at the diagram in (a): $V(x) = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r} = 2\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{\sqrt{a^2 + x^2}}$

d)



e) When $x >> a, V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{2q}{x}$, just like a point charge of charge $+2q$.

23.23: a)



b) $V_x = \dfrac{kq}{r} + \dfrac{k(-q)}{r} = 0.$

c) The potential along the x-axis is always zero, so a graph would be flat.

d) If the two charges are interchanged, then the results of (b) and (c) still hold. The potential is zero

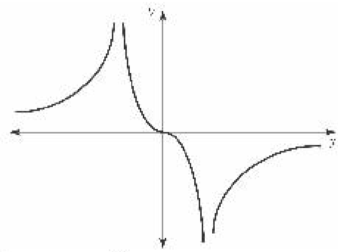
23.24: a) $|y| < a : V = \dfrac{kq}{(a+y)} - \dfrac{kq}{(a-y)} = \dfrac{2kqy}{y^2 - a^2}.$

$$y > a : V = \frac{kq}{(a+y)} - \frac{kq}{y-a} = \frac{-2kqa}{y^2 - a^2}.$$

$$y < -a : V = \frac{-kq}{(a+y)} - \frac{kq}{(-y+a)} = \frac{2kqa}{y^2 - a^2}.$$

Note: This can also be written as $V = k\left(\frac{-q}{|y-a|} + \frac{q}{|y+a|}\right)$

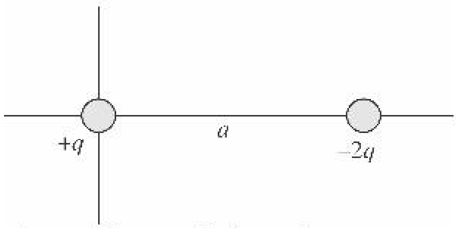
b)



c) $y >> a : V = \frac{kq}{(a+y)} - \frac{kq}{(y-a)} = \frac{-2kqa}{y^2}.$

d) If the charges are interchanged, then the potential is of the opposite sign.

23.25: a)



b)

$$x > a : V = \frac{kq}{x} - \frac{2kq}{x-a} = \frac{-kq(x+a)}{x(x-a)}.$$

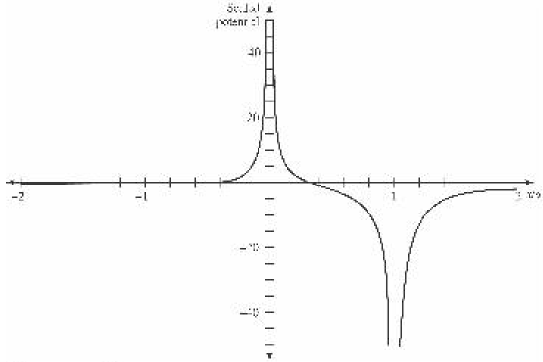
$$0 < x < a : V = \frac{kq}{x} - \frac{2kq}{a-x} = \frac{kq(3x-a)}{x(x-a)}.$$

$$x < 0 : V = \frac{-kq}{x} + \frac{2kq}{x-a} = \frac{kq(x+a)}{x(x-a)}.$$

Note: This can be also be written as $V = k(\frac{q}{|x|} - \frac{2q}{|x-a|})$

c) The potential is zero at $x = -a$ and $a/3$.

d)

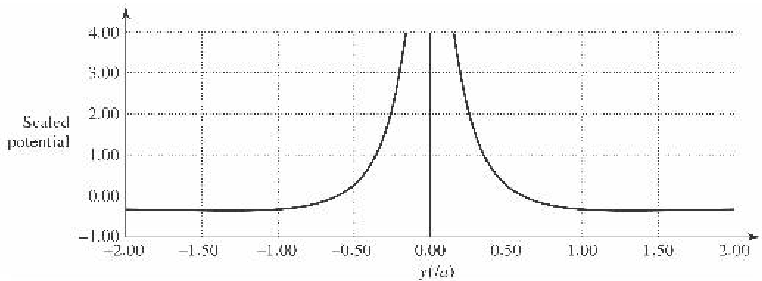


e) For $x >> a : V \approx \dfrac{-kqx}{x^2} = \dfrac{-kq}{x}$, which is the same as the potential of a point charge $-q$. (Note: The two charges must be added with the correct sign.)

23.26: a) $V = \dfrac{kq}{|y|} - \dfrac{2kq}{r} = kq\left(\dfrac{1}{|y|} - \dfrac{2}{\sqrt{a^2+y^2}}\right)$.

b) $V = 0$, when $y^2 = \dfrac{a^2+y^2}{4} \Rightarrow 3y^2 = a^2 \Rightarrow y = \pm\dfrac{a}{\sqrt{3}}$.

c)



d) $y >> a : V \approx kq\left(\dfrac{1}{y} - \dfrac{2}{y}\right) = -\dfrac{kq}{y}$, which is the potential of a point charge $-q$.

23.27: $W = -\Delta U = -Vq = (295\text{ V})\ (1.60\times10^{-19}\text{ C}) = 4.72\times10^{-17}$ J. But also:

$$W = \Delta K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2(4.72\times10^{-17}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = 1.01\times10^7\text{ m/s}.$$

23.28: a) $E = \dfrac{V}{d} \Rightarrow d = \dfrac{V}{E} = \dfrac{4.98\text{ V}}{12.0\text{ N/C}} = 0.415\text{ m}.$

b) $V = \dfrac{kq}{d} \Rightarrow q = \dfrac{Vd}{k} = \dfrac{(4.98\ \text{V})\ (0.415\ \text{m})}{k} = 2.30 \times 10^{-10}\ \text{C}.$

c) The electric field is directed away from q since it is a positive charge.

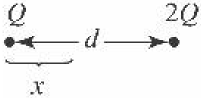
23.29: a) Point b has a higher potential since it is "upstream" from where the positive charge moves.

$$V_a - V_b = E(b-a) = -|E|(b-a) \Rightarrow V_b - V_a = |E|(b-a) > 0$$

b) $E = \dfrac{V}{d} = \dfrac{240\,V}{0.3\,\text{m}} = 800\ \text{N/C}.$

c) $W = -\Delta U = -q\Delta V = -(-0.20 \times 10^{-6}\ \text{C})(-240\ \text{V}) = -4.8 \times 10^{-5}\ \text{J}.$

23.30:(a) $V = V_Q + V_{2Q} > 0$, so V is zero nowhere except for infinitely far from the charges.

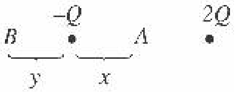


The fields can cancel only between the charges

$$E_Q = E_{2Q} \rightarrow \frac{kQ}{x^2} = \frac{k(2Q)}{(d-x)^2} \rightarrow (d-x)^2 = 2x^2$$

$x = \frac{d}{1+\sqrt{2}}$. The other root, $x = \frac{d}{1-\sqrt{2}}$, does not lie between the charges.

(b)



V can be zero in 2 places, A and B.

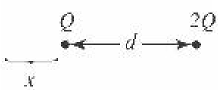
$$at\ A: \frac{k(-Q)}{x} + \frac{k(2Q)}{d-x} = 0 \rightarrow x = d/3$$

$$at\ B: \frac{k(-Q)}{y} + \frac{k(2Q)}{d+y} = 0 \rightarrow y = d$$

$E_Q = E_{2Q}$ to the left of $-Q$.

$$\frac{kQ}{x^2} = \frac{k(2Q)}{(d+x)^2} \rightarrow x = \frac{d}{\sqrt{2}-1}$$

(c)



Note that E and V are not zero at the same places.

23.31: a) $K_1 + qV_1 = K_2 + qV_2$

$$q(V_1 - V_2) = K_2 - K_1; \qquad q = -1.602 \times 10^{-19}\ \text{C}$$

$$K_1 = \tfrac{1}{2} m_e v_1^2 = 4.099 \times 10^{-18}\ \text{J}; \qquad K_2 = \tfrac{1}{2} m_e v_2^2 = 2.915 \times 10^{-17}\ \text{J}$$

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = -156\ \text{V}$$

The electron gains kinetic energy when it moves to higher potential.

b) Now $K_1 = 2.915 \times 10^{-17}$ J, $K_2 = 0$

$$V_1 - V_2 = \frac{K_2 - K_1}{q} = +182\ \text{V}$$

The electron loses kinetic energy when it moves to lower potential

23.32: a) $V = \dfrac{kq}{r} = \dfrac{k(3.50 \times 10^{-9}\ \text{C})}{0.48\ \text{m}} = 65.6\ \text{V}.$

b) $V = \dfrac{k(3.50 \times 10^{-9}\ \text{C})}{0.240\ \text{m}} = 131.3\ \text{V}$

c) Since the sphere is metal, its interior is an equipotential, and so the potential inside is 131.3 V.

23.33: a) The electron will exhibit simple harmonic motion for $x << a$, but will otherwise oscillate between ± 30.0 cm.

b) From Example 23.11,

$$V = \frac{kQ}{\sqrt{x^2 + a^2}} \Rightarrow \Delta V = kQ\left(\frac{1}{a} - \frac{1}{\sqrt{x^2 + a^2}}\right)$$

$$\Rightarrow \Delta V = k(24.0 \times 10^{-9}\ \text{C})\left(\frac{1}{0.150\ \text{m}} - \frac{1}{\sqrt{(0.300\ \text{m})^2 + (0.150\ \text{m})^2}}\right)$$

$$= 796\ \text{V}$$

But $W = -q\Delta V = \dfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2(1.60 \times 10^{-19}\ \text{C})\,(796\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}} = 1.67 \times 10^7\ \text{m/s}.$

23.34: Energy is conserved:

$$\frac{1}{2}mv^2 = q\Delta V \Rightarrow \Delta V = \frac{(1.67 \times 10^{-27}\ \text{kg})\,(1500\ \text{m/s})^2}{2(1.60 \times 10^{-19}\ \text{C})} = 0.0117\ \text{V}.$$

But:

$$\Delta V = \frac{\lambda}{2\pi\varepsilon_0}\ln(r_0/r) \Rightarrow r_0 = r\exp\left(\frac{2\pi\varepsilon_0\Delta V}{\lambda}\right) \Rightarrow r = r_0\exp\left(-\frac{2\pi\varepsilon_0\Delta V}{\lambda}\right)$$

$$\Rightarrow r = (0.180\text{ m})\exp\left(-\frac{2\pi\varepsilon_0(0.0117\text{ V})}{5.00\times10^{-12}\text{ C/m}}\right) = 0.158\text{ m.}$$

23.35: a) $E = \frac{V}{d} = \frac{360\,V}{0.0450\text{ m}} = 8000\text{ N/C.}$

b) $F = Eq = (8000\text{ N/C})\,(2.40\times10^{-9}\text{ C}) = 1.92\times10^{-5}\text{ N.}$

c) $W = Fd = (1.92\times10^{-5}\text{ N})\,(0.0450\text{ m}) = 8.64\times10^{-7}\text{ J.}$

d) $\Delta U = \Delta Vq = (-360\text{ V})\,(2.40\times10^{-9}\text{ C}) = -8.64\times10^{-7}\text{ J.}$

23.36: a) $V = Ed = (480\text{ N/C})\,(3.8\times10^{-2}\text{ m}) = 18.2\text{ V.}$

b) The higher potential is at the positive sheet.

c) $E = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0(480\text{ N/C}) = 4.25\times10^{-9}\text{ C/m}^2.$

23.37: a) $E = \frac{V}{d} \Rightarrow d = \frac{V}{E} = \frac{4750\text{ V}}{3.00\times10^6\text{ V/m}} = 1.58\times10^{-3}\text{ m.}$

b) $E = \frac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0(3.00\times10^6\text{ V/m}) = 2.66\times10^{-5}\text{ C/m}^2.$

23.38: a) $E = \frac{\sigma}{\varepsilon_0} = \frac{47.0\times10^{-9}\text{ C/m}^2}{\varepsilon_0} = 5311\text{ N/C.}$

b) $V = Ed = (5311\text{ N/C})\,(0.0220\text{ m}) = 117\text{ V.}$

c) The electric field stays the same if the separation of the plates doubles, while the potential between the plates doubles.

23.39: a) The electric field outside the shell is the same as for a point charge at the center of the shell, so the potential outside the shell is the same as for a point charge:

$$V = \frac{q}{4\pi\varepsilon_0 r}\text{ for } r > R.$$

The electric field is zero inside the shell, so no work is done on a test charge as it moves inside the shell and all points inside the shell are at the same potential as the surface of the shell: $V = \frac{q}{4\pi\varepsilon_0 R}$ for $r \le R$.

b) $V = \dfrac{kq}{R}$ so $q = \dfrac{RV}{k} = \dfrac{(0.15\text{ m})(-1200\text{ V})}{k} = -20\text{ nC}$

c) No, the amount of charge on the sphere is very small.

23.40: For points outside this spherical charge distribution the field is the same as if all the charge were concentrated at the center.

Therefore

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

and

$$q = 4\pi\varepsilon_0 E r^2 = \frac{(3800\text{ N/C})(0.200\text{ m})^2}{9\times 10^9\text{ N.m}^2/C^2} = 1.69\times 10^{-8}\text{C}$$

Since the field is directed inward, the charge must be negative. The potential of a point charge, taking ∞ as zero, is

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{(9\times 10^9\text{ N.m}^2/\text{C}^2)(-1.69\times 10^{-8}\text{C})}{0.200\text{ m}} = -760\text{ V}$$

at the surface of the sphere. Since the charge all resides on the surface of a conductor, the field inside the sphere due to this symmetrical distribution is zero. No work is therefore done in moving a test charge from just inside the surface to the center, and the potential at the center must also be -760 V.

23.41: a) $E = -\nabla V$.

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(Axy - Bx^2 + Cy) = -Ay + 2Bx.$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(Axy - Bx^2 + Cy) = -Ax - C.$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}(Axy - Bx^2 + Cy) = 0.$$

b) $-Ay + 2Bx = 0 \Rightarrow y = \dfrac{2B}{A}x,\ -Ax - C = 0 \Rightarrow x = -\dfrac{C}{A}$ so $y = \dfrac{2B}{A}.\left(\dfrac{-C}{A}\right) =$
$\dfrac{-2BC}{A^2}$, $E = 0$ at $\left(\dfrac{-C}{A}, \dfrac{-2BC}{A^2}, z\right)$

23.42: a) $E = -\nabla V$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{kQ}{\sqrt{x^2+y^2+z^2}}\right) = \frac{kQx}{(x^2+y^2+z^2)^{3/2}} = \frac{kQx}{r^3}.$$

Similarly, $E_y = \dfrac{kQy}{r^3}$ and $E_z = \dfrac{kQz}{r^3}$.

b) So from (a), $E = \dfrac{kQ}{r^2}\left(\dfrac{x\hat{\boldsymbol{i}}}{r} + \dfrac{y\hat{\boldsymbol{j}}}{r} + \dfrac{z\hat{\boldsymbol{k}}}{r}\right) = \dfrac{kQ}{r^2}\hat{\boldsymbol{r}}$, which agrees with Equation (21.7).

23.43: a) There is no dependence of the potential on x or y, and so it has no components in those directions. However, there is z dependence:

$$E = -\nabla V \Rightarrow E_z = -\frac{\partial V}{\partial z} = -C \Rightarrow E = -C\hat{\boldsymbol{k}},\ \ \mathit{for}\ \ 0 < z < d.$$

and $\vec{\boldsymbol{E}} = 0$, for $z > d$, since the potential is constant there.

(b) Infinite parallel plates of opposite charge could create this electric field, where the surface charge is $\sigma = \pm C\varepsilon_0$.

23.44: a)

(i) $r < r_a : V = \dfrac{kq}{r_a} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$.

(ii) $r_a < r < r_b : V = \dfrac{kq}{r} - \dfrac{kq}{r_b} = kq\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right)$.

(iii) $r > r_b : V = 0$, since outside a sphere the potential is the same as for point charge. Therefore we have the identical potential to two oppositely charged point charges at the same location. These potentials cancel.

b) $V_a = \dfrac{1}{4\pi\varepsilon_0}\left(\dfrac{q}{r_a} - \dfrac{q}{r_b}\right)$ and $V_b = 0 \Rightarrow V_{ab} = \dfrac{1}{4\pi\varepsilon_0}q\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)$

c) $r_a < r < r_b : E = -\dfrac{\partial V}{\partial r} = -\dfrac{q}{4\pi\varepsilon_0}\dfrac{\partial}{\partial r}\left(\dfrac{1}{r} - \dfrac{1}{r_b}\right) = +\dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2} = \dfrac{V_{ab}}{\left(\dfrac{1}{r_a} - \dfrac{1}{r_b}\right)}\dfrac{1}{r^2}$.

d) From Equation (24.23): $E = 0$, since V is zero outside the spheres.

e) If the outer charge is different, then outside the outer sphere the potential is no longer zero but is $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r} - \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{(q-Q)}{r}$. All potentials inside the outer shell are just shifted by an amount $V = -\dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r_b}$. Therefore relative potentials within the

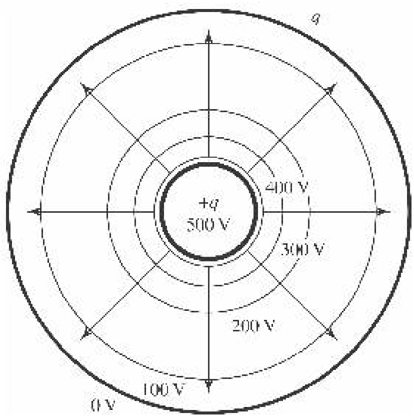
shells are not affected. Thus (b) and (c) do not change. However, now that the potential does vary outside the spheres, there is an electric field there:

$$E = -\frac{\partial V}{\partial r} = -\frac{\partial}{\partial r}\left(\frac{kq}{r} + \frac{-kQ}{r}\right) = \frac{kq}{r^2}\left(1 - \frac{Q}{q}\right).$$

23.45: a) $$V_{ab} = kq\left(\frac{1}{r_a} - \frac{1}{r_b}\right) = 500 \text{ V}$$

$$\Rightarrow q = \frac{500 \text{ V}}{k\left(\frac{1}{0.012 \text{ m}} - \frac{1}{0.096 \text{ m}}\right)} = 7.62 \times 10^{-10} \text{ C}.$$

b)



c) The equipotentials are closest when the electric field is largest.

23.46: a) $$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(\frac{kQ}{2a}\ln\left(\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right)\right)$$

$$\Rightarrow E_x = -\frac{kQ}{2a}\left[\frac{\partial}{\partial x}\ln(\sqrt{a^2 + x^2} + a) - \frac{\partial}{\partial x}\ln(\sqrt{a^2 + x^2} - a)\right]$$

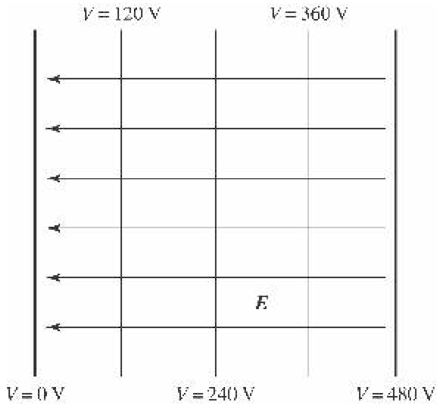
$$= -\frac{kQ}{2a}\left[\frac{x(a^2 + x^2)^{-1/2}}{\sqrt{a^2 + x^2} + a} - \frac{x(a^2 + x^2)^{-1/2}}{\sqrt{a^2 + x^2} - a}\right] = \frac{kQ}{x\sqrt{a^2 + x^2}}$$

$$\Rightarrow E_x = \frac{(2a\lambda)}{4\pi\varepsilon_0 x a\sqrt{1 + x^2/a^2}} = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{x\sqrt{1 + x^2/a^2}}.$$

b) The potential was evaluated at y and z equal to zero, and thus shows no dependence on them. However, the electric field depends upon the derivative of the

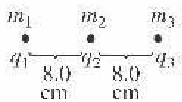
potential and the potential could still have a functional dependence on the variables y and z, and hence E_y and E_z may be non-zero.

23.47:



a) Equipotentials and electric field lines of two large parallel plates are shown above.
b) The electric field lines and the equipotential lines are mutually perpendicular.

23.48:



(a) $\Sigma F = m_1 a = F_{12} + F_{13}$

$$\frac{kq_1q_2}{r_{12}^2} + \frac{kq_1q_3}{r_{13}^2} = m_1 a$$

$$q_1 = q_2 = q_3 = q$$

$$ma = kq^2\left(\frac{1}{r_{12}^2} + \frac{1}{r_{13}^2}\right)$$

$$(0.02\ \text{kg})a = (9\times10^9\ \text{Nm}^2/\text{C}^2)\,(2.0\times10^{-6}\,\text{C})^2\left[\frac{1}{(0.08\ \text{m})^2} + \frac{1}{(0.16\ \text{m})^2}\right]$$

$$a = 352\ \text{m/s}^2$$

(b) Maximum speed occurs at "infinity". The center charge does not move since the forces on it balance. Energy conservation gives $U_i = K_f$.

$$\frac{kq_1q_2}{r_{12}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_3}{r_{23}} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_3v_3^2.$$

$v_1 = v_3$, $m_1 = m_3$, and $q_1 = q_2 = q_3 = q$

$$v_1 = \sqrt{\frac{kq^2}{m_1}\left(\frac{1}{r_{12}} + \frac{1}{r_{13}} + \frac{1}{r_{23}}\right)}$$

$$= \sqrt{\frac{(9\times10^9\ \text{Nm}^2/\text{C}^2)\,(2\times10^{-6}\,\text{C})^2}{0.020\ \text{kg}}\left(\frac{1}{0.08\ \text{m}} + \frac{1}{0.16\ \text{m}} + \frac{1}{0.08\ \text{m}}\right)} = 7.5\ \text{m/s}$$

23.49: a) $W_E = \Delta K - W_F = 4.35\times10^{-5}\ \text{J} - 6.50\times10^{-5}\ \text{J} = -2.15\times10^{-5}\ \text{J}.$

b) $W_E = -q\Delta V \Rightarrow \Delta V = \dfrac{-W_E}{q} = \dfrac{2.15\times10^{-5}\ \text{J}}{7.60\times10^{-9}\ \text{C}} = +2829\ \text{V}$. So the initial point is –2829 V with respect to the final point.

c) $E = \dfrac{V}{d} = \dfrac{2829\ \text{V}}{0.08\ \text{m}} = 3.54\times10^4\ \dfrac{\text{V}}{\text{m}}.$

23.50: a) $\dfrac{mv^2}{r} = \dfrac{ke^2}{r^2} \Rightarrow v = \sqrt{\dfrac{ke^2}{mr}}.$

b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}\dfrac{ke^2}{r} = -\dfrac{1}{2}U.$

c) $E = K + U = \dfrac{1}{2}U = -\dfrac{1}{2}\dfrac{ke^2}{r} = -\dfrac{1}{2}\dfrac{k(1.60\times10^{-19}\ \text{C})^2}{5.29\times10^{-11}\ \text{m}} = -2.17\times10^{-18}\ \text{J} = -13.6\ \text{eV}.$

23.51: a) $V = Cx^{4/3} \Rightarrow C = (240\ \text{V})\,(0.0130\ \text{m})^{-4/3} = 7.85\times10^4\ \text{V/m}^{4/3}$.

b) $E = -\dfrac{\partial V}{\partial x} = -\dfrac{4}{3}Cx^{1/3} = -\dfrac{4}{3}(7.85\times10^4\ \text{V/m}^{4/3})x^{1/3} = \left(-1.05\times10^5\ \dfrac{\text{V}}{\text{m}^{4/3}}x^{1/3}\right)\text{V/m},$ toward cathode.

c) $F = -eE = ((1.05\times10^5)\,(0.00650)^{1/3}\ \text{V/m})\,(1.60\times10^{-19}\ \text{C}) = 3.14\times10^{-15}\ \text{N},$ toward anode.

23.52: From Problem 22.51, the electric field of a sphere with radius R and q distributed uniformly over its volume is $E = \dfrac{qr}{4\pi\varepsilon_0 R^3}$ for $r \le R$ and $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ for $r \ge R$

$V_a - V_b = \int_a^b E dr$. Take b at infinity and $V_\infty = 0$. Let point a be a distance $r < R$ from the center of the sphere.

$$V_r = \int_r^R \frac{qr}{4\pi\varepsilon_0 R^3} dr + \int_R^\infty \frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{q}{8\pi\varepsilon_0 R}\left(3 - \frac{r^2}{R^2}\right)$$

Set $q = +2e$ to get V_r for the sphere. The work done by the attractive force of the sphere when one electron is removed from $r = d$ to ∞ is

$$W_{\text{sphere}} = -eV_r = -\frac{2e^2}{8\pi\varepsilon_0 R}\left(3 - \frac{d^2}{R^2}\right)$$

The total work done by the attractive force of the sphere when both electrons are removed is twice this, $2W_{\text{sphere}}$. The work done by the repulsive force of the two electrons is $W_{ee} = \dfrac{e^2}{4\pi\varepsilon_0 (2d)}$ The total work done by the electrical forces is $2W_{\text{sphere}} + W_{ee}$. The energy required to remove the two electrons is the negative of this,

$$\frac{e^2}{2\pi\varepsilon_0 R}\left(3 - \frac{R}{4d} - \frac{d^2}{R^2}\right)$$

We can check this result in the special case of $d = R$, when the electrons initially sit on the surface of the sphere. The potential due to the sphere is the same as for a point charge $+2e$ at the center of the sphere.

$$W_{a\to b} = U_a - U_b$$

$$U_b = 0.\ U_a = 2\left(\frac{-2e^2}{4\pi\varepsilon_0 R}\right) + \frac{e^2}{4\pi\varepsilon_0 (2R)} = \frac{e^2}{4\pi\varepsilon_0 R}\left(-2 + \frac{1}{4}\right) = \frac{-7e^2}{8\pi\varepsilon_0 R}$$

The work done by the electric forces when the electrons are removed is $-7e^2/8\pi\varepsilon_0 R$ and the energy required to remove them is $7e^2/8\pi\varepsilon_0 R$. Setting $d = R$ in our general expression yields this same result.

23.53: a)

$$U = kq^2\left(-\frac{3}{d} + \frac{3}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right) + kq^2\left(-\frac{2}{d} + \frac{3}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right) + kq^2\left(-\frac{2}{d} + \frac{2}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right)$$

$$+ kq^2\left(-\frac{1}{d} + \frac{2}{\sqrt{2}d} - \frac{1}{\sqrt{3}d}\right) + kq^2\left(-\frac{2}{d} + \frac{1}{\sqrt{2}d}\right) + kq^2\left(-\frac{1}{d} + \frac{1}{\sqrt{2}d}\right) + kq^2\left(-\frac{1}{d}\right)$$

$$\Rightarrow U = kq^2\left(-\frac{12}{d} + \frac{12}{\sqrt{2}d} - \frac{4}{\sqrt{3}d}\right) = -\frac{12kq^2}{d}\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{3\sqrt{3}}\right) = -1.46\, q^2/\pi\varepsilon_0 d$$

b) The fact that the electric potential energy is less than zero means that it is energetically favourable for the crystal ions to be together.

23.54: a) $U = 2kq^2\left(-\dfrac{1}{d}+\dfrac{1}{2d}-\dfrac{1}{3d}+\cdots\right) = -\dfrac{2kq^2}{d}\sum_{i=1}^{\infty}\dfrac{(-1)^{i-1}}{i}.$

b) $U = -\dfrac{2kq^2}{d}\ln(2)$

c) The potential energy is the same for the negative ions—the equations are identical if we examine (a).

d) If $d = 2.82\times 10^{-10}$ m, then $U = -\dfrac{2k(1.60\times 10^{-19}\text{ C})^2\ln(2)}{2.82\times 10^{-10}\text{ m}} = -1.13\times 10^{-18}$ J.

e) The real energy $(-0.80\times 10^{-18}\text{ J})$ is about 70% of that calculated above.

23.55: a) $U_e = \dfrac{-2ke^2}{r} = \dfrac{-2k(1.60\times 10^{-19}\text{ C})^2}{0.535\times 10^{-10}\text{ m}} = -8.61\times 10^{-18}$ J.

b) If all the kinetic energy goes into potential energy:

$$U_t = U_e + K = -8.61\times 10^{-18}\text{ J} + 1.02\times 10^{-18}\text{ J} = \frac{2ke^2}{\sqrt{d^2+x^2}} = -7.59\times 10^{-18}\text{ J}$$

$$\Rightarrow x^2 = \frac{4k^2e^4}{U_t^2} - d^2 = 8.24\times 10^{-22}\text{m}^2\ (d = 5.35\times 10^{-11}\text{ m})$$

(Note that we must be careful to keep all digits along the way.) $\Rightarrow x = 2.87\times 10^{-11}$ m.

23.56: $F_e = mg\tan\theta = (1.50\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan(30°) = 0.0085\text{ N}$. (Balance forces in x and y directions.) But also:

$$F_e = Eq = \frac{Vq}{d} \Rightarrow V = \frac{Fd}{q} = \frac{(0.0085\text{ N})(0.0500\text{ m})}{8.90\times 10^{-6}\text{ C}} = 47.8\text{ V}.$$

23.57: a) (i) $V = \dfrac{\lambda}{2\pi\varepsilon_0}(\ln(b/a) - \ln(b/b)) = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(b/a).$

(ii) $V = \dfrac{\lambda}{2\pi\varepsilon_0}(\ln(b/r) - \ln(b/b)) = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(b/r).$

(iii) $V = 0.$

b) $V_{ab} = V(a) - V(b) = \dfrac{\lambda}{2\pi\varepsilon_0}\ln(b/a).$

c) Between the cylinders:

$$V = \frac{\lambda}{2\pi\varepsilon_0}\ln(b/r) = \frac{V_{ab}}{\ln(b/a)}\ln(b/r)$$

$$\therefore E = -\frac{\partial V}{\partial r} = -\frac{V_{ab}}{\ln(b/a)}\frac{\partial}{\partial r}(\ln(b/r)) = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r}.$$

d) The potential difference between the two cylinders is identical to that in part (b) even if the outer cylinder has no charge.

23.58: Using the results of Problem 23.57, we can calculate the potential difference:

$$E = \frac{V_{ab}}{\ln(b/a)}\frac{1}{r} \Rightarrow V_{ab} = E\ln(b/a)r$$

$$\Rightarrow V_{ab} = (2.00\times 10^4\ \text{N/C})\,(\ln(0.018\ \text{m}/145\times 10^{-6}\ \text{m}))\ 0.012\ \text{m} = 1157\ \text{V}.$$

23.59: a) $F = Eq = (1.10\times 10^3\,\text{V/m})\,(1.60\times 10^{-19}\ \text{C}) = 1.76\times 10^{-16}\ \text{N}$, downward.

b) $a = F/m_e = (1.76\times 10^{-16}\ \text{N})/(9.11\times 10^{-31}\ \text{kg}) = 1.93\times 10^{14}\ \text{m/s}^2$, downward.

c) $t = \dfrac{0.060\ \text{m}}{6.50\times 10^6\ \text{m/s}} = 9.23\times 10^{-9}\ \text{s},\ y - y_0 = \dfrac{1}{2}at^2 = \dfrac{1}{2}(1.93\times 10^{14}\ \text{m/s}^2)\,(9.23\times 10^{-9}\text{s})^2$

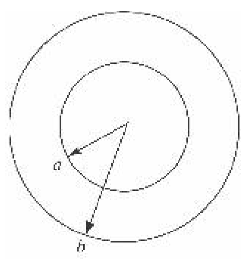
$= 8.22\times 10^{-3}$ m.

d) Angle $\theta = \arctan(v_y/v_x) = \arctan(at/v_x) = \arctan(1.78/6.50) = 15.3°$.

e) The distance below center of the screen is:

$$D = d_y + v_y t = 8.22\times 10^{-3}\ \text{m} + (1.78\times 10^6\ \text{m/s})\frac{0.120\ \text{m}}{6.50\times 10^6\ \text{m/s}} = 0.0411\ \text{m}.$$

23.60:



(a) Use ΔV_{ab} to get $\lambda : \Delta V = \int_a^b E \cdot dl = \int_a^b \frac{\lambda}{2\pi\varepsilon_0} dr = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{b}{a}$

$$\lambda = \frac{2\pi\varepsilon_0 \Delta V}{\ln b/a}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2\pi\varepsilon_0 \Delta V / \ln b/a}{2\pi\varepsilon_0 r} = \frac{\Delta V}{r \ln b/a}$$

at outer surface of the wire, $r = a = \frac{0.127\text{ mm}}{2}$

$$E = \frac{850\text{ V}}{(\frac{0.000127\text{ m}}{2}) \ln \left[\frac{1.00\text{ cm}}{(\frac{0.0127\text{ cm}}{2})}\right]} = 2.65 \times 10^6\text{ V/m}$$

(b) at the inner surface of the cylinder, $r = 1.00\text{ cm}$, which gives

$$E = 1.68 \times 10^4\text{ V/m}$$

23.61: a) From Problem 23.57,

$$E = \frac{V_{ab}}{\ln(b/a)} \frac{1}{r} = \frac{50{,}000\text{ V}}{\ln(0.140/9.00 \times 10^{-5})} \frac{1}{0.070\text{ m}}$$

$$\Rightarrow E = 9.72 \times 10^4\text{ V/m.}$$

b) $F = Eq = 10\,mg \Rightarrow q = \dfrac{10\,(3.00 \times 10^{-8}\text{ kg})(9.80\text{ m/s}^2)}{9.72 \times 10^4\text{ V/m}} = 3.02 \times 10^{-11}\text{ C.}$

23.62: Recall from Example 23.12 for a line of charge of length a:

$$V = \frac{kQ}{a} \ln \left[\frac{\sqrt{a^2/4 + x^2} + a/2}{\sqrt{a^2/4 + x^2} - a/2}\right]$$

a) For a square with two sets of oppositely charged sides, the potentials cancel and $V = 0$.

b) If all sides have the same charge we have:

$$V = \frac{4kQ}{a} \ln \left[\frac{\sqrt{a^2/4 + x^2} + a/2}{\sqrt{a^2/4 + x^2} - a/2}\right], \text{ but here } x = a/2, \text{ so:}$$

$$\Rightarrow V = \frac{4kQ}{a} \ln \left[\frac{\sqrt{a^2 + 4x^2} + a}{\sqrt{a^2 + 4x^2} - a}\right] = \frac{4kQ}{a} \ln \left[\frac{(\sqrt{2} + 1)}{(\sqrt{2} - 1)}\right].$$

23.63: a)

$$dV = \frac{kQ}{\sqrt{x^2 + r^2}}\left[\frac{2\pi r\,dr}{\pi R^2}\right] = \frac{2kQ}{R^2}\frac{r\,dr}{\sqrt{x^2 + r^2}}$$

$$V = \int_0^R dV = \frac{2kQ}{R^2}\int_0^R \frac{r\,dr}{\sqrt{x^2 + r^2}} = \frac{2kQ}{R^2}(z^{1/2})\Big|_{z=x^2}^{z=x^2+R^2} = \frac{2kQ}{R^2}\left[\sqrt{x^2 + R^2} - x\right] =$$

$$\frac{\sigma}{2\varepsilon_0}\left[\sqrt{x^2 + R^2} - x\right]$$

b) $$E_x = -\frac{\partial V}{\partial x} = -\frac{2kQ}{R^2}\left[\frac{x}{\sqrt{x^2 + R^2}} - 1\right] = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{1}{\sqrt{1 + R^2/x^2}}\right].$$

23.64: a) From Example 23.12:

$$V(x) = \frac{kQ}{2a}\ln\left[\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right] = \frac{kQ}{2a}\ln\left[\frac{\sqrt{1 + a^2/x^2} + a/x}{\sqrt{1 + a^2/x^2} - a/x}\right]$$

If $a << x$, $\sqrt{1 + a^2/x^2} \pm a/x \approx 1 + \frac{1}{2}\left(\frac{a}{x}\right)^2 \pm \frac{a}{x} \approx 1 + \frac{a}{x}$, and $\ln(1+\alpha) \approx \alpha + \frac{1}{2}\alpha^2 + \cdots$

$$\Rightarrow V(x) \approx \frac{kQ}{2a}\left[\left(\frac{a}{x} + \frac{1}{2}\left(\frac{a}{x}\right)^2 + \cdots\right) - \left(-\frac{a}{x} + \frac{1}{2}\left(\frac{a}{x}\right)^2 + \cdots\right)\right] = \frac{kQ}{2a}\left[\frac{2a}{x}\right] = \frac{kQ}{x}.$$

That is, the finite rod acts like a point charge when you are a long way from it.

b) From Example 23.12:

$$V(x) = \frac{kQ}{2a}\ln\left[\frac{\sqrt{a^2 + x^2} + a}{\sqrt{a^2 + x^2} - a}\right] = \frac{kQ}{2a}\ln\left[\frac{\sqrt{1 + x^2/a^2} + 1}{\sqrt{1 + x^2/a^2} - 1}\right].$$

If $x << a$, $\sqrt{1 + x^2/a^2} \pm 1 \approx 1 \pm 1 + \frac{1}{2}\left(\frac{x}{a}\right)^2$, and $\ln(1+\alpha) \approx \alpha + \frac{1}{2}\alpha^2 + \cdots$

$$\Rightarrow V(x) \approx \frac{kQ}{2a}\left[\ln\left(\frac{(2 + x^2/2a^2)}{(x^2/2a^2)}\right)\right] = \frac{kQ}{2a}\left[\ln\left(\frac{4a^2}{x^2} + 1\right)\right] \approx \frac{kQ}{a}\ln(2a/x) =$$

$$\frac{Q}{4\pi\varepsilon_0 a}\ln(2a/x) = \frac{\lambda}{2\pi\varepsilon_0}\ln(2a/x).$$

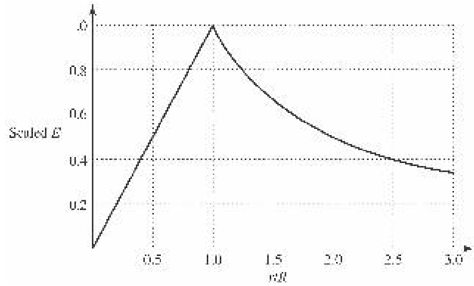
Thus $\lambda \equiv \frac{Q}{2a}$, and $R = 2a$, which is the only natural length in the problem.

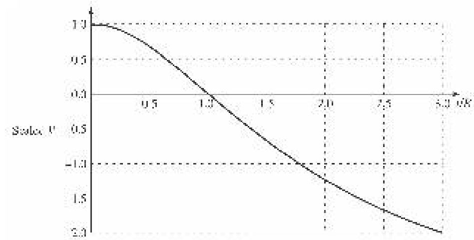
23.65: a) Recall: $r \le R : E = \frac{\rho r}{2\varepsilon_0} \Rightarrow V = -\int_R^r \vec{E}\cdot d\vec{r} = -\frac{\rho}{2\varepsilon_0}\int_R^r r\,dr = -\frac{\rho}{4\varepsilon_0}(r^2 - R^2)$

So with $\lambda = \pi R^2 \rho$, $V = -k\lambda(r^2/R^2 - 1)$.

For $r \geq R : E = \dfrac{\rho R^2}{2\varepsilon_0 r} \Rightarrow V = -\int_R^r \vec{E}\cdot d\,\vec{r} = -\dfrac{\rho R^2}{2\varepsilon_0}\int_R^r \dfrac{dr}{r} = -\dfrac{\lambda}{2\pi\varepsilon_0}\ln\left[\frac{r}{R}\right] = -2k\lambda\ln\left[\frac{r}{R}\right].$

b)





23.66: a) $V(0.03, 0) = \dfrac{k(5.00\times10^{-9}\ \text{C})}{0.0300\ \text{m}} + \dfrac{k(-2.00\times10^{-9}\ \text{C})}{0.01\ \text{m}} = -300\ \text{V}.$

$$V(0.03, 0.05) = \frac{k(5.00\times10^{-9}\ \text{C})}{\sqrt{(0.0300^2 + 0.0500^2)}\ \text{m}} + \frac{k(-2.00\times10^{-9}\ \text{C})}{\sqrt{0.0100^2 + 0.0500^2}} = 419\ \text{V}.$$

b) $W = -q\Delta V = -(+6.00\times10^{-9}\ \text{C})(718\ \text{V}) = -4.31\times10^{-6}\ \text{J}.$

Note that the work done by the field is negative, since the charge is moved AGAINST the electric field.

23.67: From Example 21.10, we have: $E_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$

$$\Rightarrow V = -\frac{Q}{4\pi\varepsilon_0}\int_\infty^x \frac{x'}{(x'^2+a^2)^{3/2}}dx' = \frac{Q}{4\pi\varepsilon_0}u^{-1/2}\Big|_{u=\infty}^{u=x^2+a^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+a^2}} = \text{Equation}$$

(23.16).

23.68:

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\,dl}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\pi a}\frac{dl}{a} = \frac{1}{4\pi\varepsilon_0}\frac{Q\,d\theta}{\pi a} \Rightarrow V = \frac{1}{4\pi\varepsilon_0}\int_0^\pi \frac{Q\,d\theta}{\pi a} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a}.$$

23.69: a) S_1 and $S_3: V_{13} = -\int_0^{0.3} (-5x\hat{\boldsymbol{i}} + 3z\hat{\boldsymbol{k}}) \cdot \hat{\boldsymbol{j}}\, dy = 0$; S_1 and S_3 are at equal potentials.

b) S_2 and $S_4: V_{24} = -\int_0^{0.3} (-5x\hat{\boldsymbol{i}} + 3z\hat{\boldsymbol{k}}) \cdot \hat{\boldsymbol{k}} dz = -3\int_0^{0.3} z\, dz = \frac{-3}{2} z^2 |_0^{0.3} = \frac{-3}{2}(0.3)^2 =$

-0.135 V. S_4 is higher.

c) S_5 and $S_6: V_{56} = -\int_0^{0.3} (-5x\hat{\boldsymbol{i}} + 3z\hat{\boldsymbol{k}}) \cdot \hat{\boldsymbol{i}} dx = 5\int_0^{0.3} x\, dx = \frac{5}{2} x^2 |_0^{0.3} = \frac{5}{2}(0.3)^2 = 0.225$ V.

S_5 is higher.

23.70: From Example 22.9, we have:

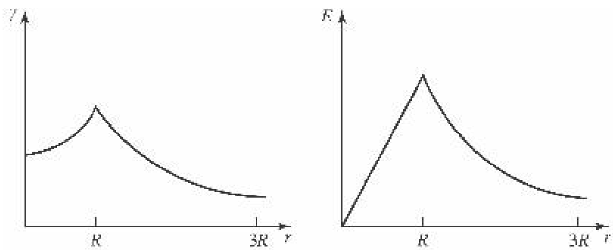
$$r > R: E = \frac{kQ}{r^2} \Rightarrow V = -kQ\int_\infty^r \frac{dr'}{r'^2} = \frac{kQ}{r}$$

$$r < R: E = \frac{kQr}{R^3} \Rightarrow V = -\int_\infty^R \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{r}}' - \int_R^r \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{r}}' = \frac{kQ}{R} - \frac{kQ}{R^3}\int_R^r r'\, dr'$$

$$\Rightarrow V = \frac{kQ}{R} - \frac{kQ}{R^3}\frac{1}{2}r'^2\Big|_R^r = \frac{kQ}{R} + \frac{kQ}{2R} - \frac{kQr^2}{2R^3}$$

$$\therefore V = \frac{kQ}{2R}\left[3 - \frac{r^2}{R^2}\right]$$

b)



23.71: a) Problem 23.70 shows that

$$V_r = \frac{Q}{8\pi\varepsilon_0 R}(3 - r^2/R^2) \text{ for } r \le R \text{ and } V_r = \frac{Q}{4\pi\varepsilon_0 r} \text{ for } r \ge R$$

$$V_0 = \frac{3Q}{8\pi\varepsilon_0 R}, V_R = \frac{Q}{4\pi\varepsilon_0 R}, \text{and } V_0 - V_R = \frac{Q}{8\pi\varepsilon_0 R}$$

b) If $Q > 0, V$ is higher at the center. If $Q < 0, V$ is higher at the surface.

23.72: (a) Points $a, b,$ and c are all at the same potential because $E = 0$ inside the spherical shell of charge on the outer surface. So $\Delta V_{ab} = \Delta V_{bc} = \Delta V_{ac} = 0.$

$$\Delta V_{c\infty} = \frac{kq}{R} = \frac{(9\times10^{9}\ \text{Nm}^2/\text{C}^2)\,(150\times10^{-6}\ \text{C})}{0.60\ \text{m}}$$

$$= 2.25\times10^{6}\ \text{V}.$$

(b) They are all at the same potential

(c) Only $\Delta V_{c\infty}$ would change; it would be -2.25×10^{6} V.

23.73: a) The electrical potential energy for a spherical shell with uniform surface charge density and a point charge q outside the shell is the same as if the shell is replaced by a point charge at its center. Since $F_r = -dU/dr$, this means the force the shell exerts on the point charge is the same as if the shell were replaced by a point charge at its center. But by Newton's 3rd law, the force q exerts on the shell is the same as if the shell were a point charge. But q can be replaced by a spherical shell with uniform surface charge and the force is the same, so the force between the shells is the same as if they were both replaced by point charges at their centers. And since the force is the same as for point charges, the electrical potential energy for the pair of spheres is the same as for a pair of point charges.

b) The potential for solid insulating spheres with uniform charge density is the same outside of the sphere as for a spherical shell, so the same result holds.

c) The result doesn't hold for conducting spheres or shells because when two charged conductors are brought close together, the forces between them causes the charges to redistribute and the charges are no longer distributed uniformly over the surfaces.

23.74: Maximum speed occurs at "infinity" Energy conservation gives

$$\frac{kq_1q_2}{r} = \frac{1}{2}m_{50}v_{50}^2 + \frac{1}{2}m_{150}v_{150}^2$$

Momentum conservation: $m_{50}v_{50} = m_{150}v_{150}$ and $v_{50} = 3v_{150}$

Solve for v_{50} and v_{150}, where $r = 0.50$ m

$$v_{50} = 12.7\ \text{m/s},\ v_{150} = 4.24\ \text{m/s}$$

Maximum acceleration occurs just after spheres are released. $\sum F = ma$ gives

$$\frac{kq_1q_2}{r^2} = m_{150}a_{150}$$

$$\frac{(9\times10^{9}\ \text{Nm}^2/\text{C}^2)\,(10^{-5}\ \text{C})\,(3\times10^{-5}\ \text{C})}{(0.50\ \text{m})^2} = (0.15\ \text{kg})a_{150}$$

$$a_{150} = 72.0\ \text{m/s}^2$$

$$a_{50} = 3a_{150} = 216\ \text{m/s}^2$$

23.75: Using the electric field from Problem 22.37, the potential difference between the conducting sphere and insulating shell is:

$$V=-\int_{2R}^{R}\vec{E}\cdot d\vec{r}=-\int_{2R}^{R}\frac{kQ}{r^2}dr=kQ\left[\frac{1}{R}-\frac{1}{2R}\right]\Rightarrow V=\frac{kQ}{2R}.$$

23.76: a) At $r=c: V=-\int_{\infty}^{c}\frac{kq}{r^2}dr=\frac{kq}{c}.$

b) At $r=b: V=-\int_{\infty}^{c}\vec{E}\cdot d\vec{r}-\int_{c}^{b}\vec{E}\cdot d\vec{r}=\frac{kq}{c}-0=\frac{kq}{c}.$

c) At $r=a: V=-\int_{\infty}^{c}\vec{E}\cdot d\vec{r}-\int_{c}^{b}\vec{E}\cdot d\vec{r}-\int_{b}^{a}\vec{E}\cdot d\vec{r}=\frac{kq}{c}-kq\int_{b}^{a}\frac{dr}{r^2}=kq\left[\frac{1}{c}-\frac{1}{b}+\frac{1}{a}\right].$

d) At $r=0: V=kq\left[\frac{1}{c}-\frac{1}{b}+\frac{1}{a}\right]$ since it is inside a metal sphere, and thus at the same potential as its surface.

23.77: Using the electric field from Problem 22.54, the potential difference between the two faces of the uniformly charged slab is:

$$V=-\int_{-d}^{d}\vec{E}\cdot d\vec{r}=-\int_{-d}^{d}\frac{\rho x}{2\varepsilon_0}dx=\frac{\rho}{2\varepsilon_0}\left(\frac{x^2}{2}\right)\Big|_{-d}^{d}\Rightarrow V=0.$$

23.78: a) $V=\frac{kQ}{r}=\frac{k(-1.20\times10^{-12}\text{ C})}{6.50\times10^{-4}\text{ m}}=-16.6\text{ V}.$

b) The volume doubles, so the radius increases by the cube root of two: $R_{\text{new}}=\sqrt[3]{2}R=8.19\times10^{-4}\text{ m}$ and the new charge is $Q_{\text{new}}=2Q=-2.40\times10^{-12}\text{ C}$. So the new potential is:

$$V_{\text{new}}=\frac{kQ_{\text{new}}}{R_{\text{new}}}=\frac{k(-2.40\times10^{-12}\text{ C})}{8.19\times10^{-4}\text{ m}}=-26.4\text{ V}.$$

23.79: a)

$$dV_p=\frac{kdq}{z+x}=\frac{kQ}{a}\frac{dz}{z+x}\Rightarrow V=\frac{kQ}{a}\int_0^a\frac{dz}{z+x}=\frac{kQ}{a}\ln\left(\frac{x+a}{x}\right)=\frac{kQ}{a}\ln\left(1+\frac{a}{x}\right).$$

b)

$$dV_R=\frac{kQ}{a}\frac{dz}{r}=\frac{kQ}{a}\frac{dz}{\sqrt{z^2+y^2}}\Rightarrow V_R=\frac{kQ}{a}\int_0^a\frac{dz}{\sqrt{z^2+y^2}}=\frac{kQ}{a}\ln\left(\frac{\sqrt{a^2+y^2}+a}{y}\right).$$

c)

$x >> a : V_p \approx \frac{kQ}{a}\frac{a}{x} = \frac{kQ}{x}$, Since $\ln(1+a) \approx a$.

$y >> a : V_R \approx \frac{kQ}{a}\frac{a}{y} = \frac{kQ}{y}$, Since $\ln\left(\frac{\sqrt{a^2+y^2}+a}{y}\right) \approx \ln\left(\frac{y+a}{y}\right) = \ln\left(1+\frac{a}{y}\right) \approx \frac{a}{y}$.

23.80: Set the alpha particle's kinetic energy equal to its potential energy:

$$K = U \Rightarrow 11.0 \text{ MeV} = \frac{k(2e)(82e)}{r} \Rightarrow r = \frac{k(164)(1.60\times10^{-19}\text{ C})^2}{(11.0\times10^6 \text{ eV})(1.60\times10^{-19}\text{ J/eV})}$$
$$= 2.15\times10^{-14}\text{ m}.$$

23.81: a) $V = \frac{kQ_B}{R_B} = \frac{kQ_A}{R_A} = \frac{kQ_B}{R_A/3} \Rightarrow Q_A = 3Q_B \Rightarrow \frac{Q_B}{Q_A} = \frac{1}{3}$.

b) $E_B = -\left.\frac{\partial V}{\partial r}\right|_{r=R_B} = \frac{kQ_B}{R_B^{\ 2}} = \frac{k(Q_A/3)}{(R_A/3)^2} = \frac{3kQ_A}{R_A^{\ 2}} = 3E_A \Rightarrow \frac{E_B}{E_A} = 3.$

23.82: a) From Problem 22.57 we have the electric field:

$$r \geq R : E = \frac{kQ}{r^2} \Rightarrow V = -\int_\infty^r \frac{kQ}{r'^2}\,dr' = \frac{kQ}{r},$$

which is the potential of a point charge.

b) $$r \leq R : E = \frac{kQ}{r^2}\left[4\frac{r^3}{R^3} - 3\frac{r^4}{R^4}\right] \Rightarrow V = -\int_\infty^R E\,dr' - \int_R^r E\,dr'$$

$$\Rightarrow V = \frac{kQ}{R}\left[1 - 2\frac{r^2}{R^2} + 2\frac{R^2}{R^2} + \frac{r^3}{R^3} - \frac{R^3}{R^3}\right] = \frac{kQ}{R}\left[\frac{r^3}{R^3} - 2\frac{r^2}{R^2} + 2\right].$$

23.83: a) $E = \frac{kQ_1}{R_1^{\ 2}}$, $V = \frac{kQ_1}{R_1}$, so $V = RE$

b) After electrostatic equilibrium is reached, with charge Q_1' now on the original sphere we have:

$Q_1 = Q_1' + Q_2$ and $V_1 = V_2 \Rightarrow \frac{Q_1'}{R_1} = \frac{Q_2}{R_2} \Rightarrow Q_1' = Q_2\frac{R_1}{R_2}$

$Q_1 = Q_2\frac{R_1}{R_2} + Q_2 \Rightarrow Q_2 = \frac{Q_1}{(1+\frac{R_1}{R_2})} = \frac{R_2 Q_1}{(R_2+R_1)}$ and $Q_1' = \frac{(R_1/R_2)Q_1}{\left(1+\frac{R_1}{R_2}\right)} = \frac{R_1 Q_1}{(R_2+R_1)}$

c) The new potential is the same at each sphere's surface:

$$V_1 = \frac{kQ_1'}{R_1} = \frac{kQ_1}{R_2\left(1+\frac{R_1}{R_2}\right)} = \frac{kQ_1}{(R_2+R_1)} = V_2$$

d) The new electric field is not the same at each sphere's surface:

$$E_1 = \frac{kQ_1'}{R_1^{\ 2}} = \frac{kQ_1}{R_1 R_2(1+\frac{R_1}{R_2})} = \frac{kQ_1}{R_1(R_2+R_1)}$$

$$E_2 = \frac{kQ_2}{R_2^{\ 2}} = \frac{kQ_1}{R_2^{\ 2}(1+\frac{R_1}{R_2})} = \frac{kQ_1}{R_2(R_2+R_1)}$$

23.84: a) We have $V(x, y, z) = A(x^2 - 3y^2 + z^2)$. So:

$$\boldsymbol{E} = -\frac{\partial V}{\partial x}\hat{\boldsymbol{i}} - \frac{\partial V}{\partial y}\hat{\boldsymbol{j}} - \frac{\partial V}{\partial z}\hat{\boldsymbol{k}} = -2Ax\hat{\boldsymbol{i}} + 6Ay\hat{\boldsymbol{j}} - 2Az\hat{\boldsymbol{k}}$$

b) A charge is moved in along the z-axis. So the work done is given by:

$$W = q\int_{z_0}^{0} \vec{\boldsymbol{E}}\cdot\hat{\boldsymbol{k}}\, dz = q\int_{z_0}^{0}(-2Az)\, dz = +(Aq)z_0^{\ 2} \Rightarrow A = \frac{W}{qz_0^{\ 2}}$$

$$A = \frac{6.00\times10^{-5}\ \text{J}}{(1.5\times10^{-6}\ \text{C})(0.250\ \text{m})^2} = 640\ \text{V/m}^2.$$

c) $\boldsymbol{E}(0, 0, 0.250) = -2(640\ \text{V/m}^2)(0.250\ \text{m})\hat{\boldsymbol{k}} = -320\ \text{V/m}\hat{\boldsymbol{k}}.$

d) In every plane parallel to the x-z plane, y is constant, so:

$$V(x, y, z) = Ax^2 + Az^2 - C \Rightarrow x^2 + z^2 = \frac{V+C}{A} \equiv R^2,$$

which is the equation for a circle since R is constant as long as we have constant potential on those planes.

e) $V = 1280\ \text{V}$, and $y = 2\text{m}$: $x^2 + z^2 = \dfrac{1280\ \text{V} + 3(640\ \text{V/m}^2)(2.00\ \text{m})^2}{640\ \text{V/m}^2} = 14\ \text{m}^2.$

Thus the radius of the circle is 3.74 m.

23.85: a) $E_i = E_f \Rightarrow 2\left[\dfrac{1}{2}m_p v^2\right] = \dfrac{ke^2}{2r_p} \Rightarrow v = \sqrt{\dfrac{k(1.60\times10^{-19}\ \text{C})^2}{2(1.2\times10^{-15}\ \text{m})(1.67\times10^{-27}\ \text{kg})}}$

$\Rightarrow v = 7.58\times10^6\ \text{m/s}.$

b) For a helium-helium collision, the charges and masses change from (a):

$$v = \sqrt{\frac{k(2(1.60\times10^{-19}\ \text{C}))^2}{(3.5\times10^{-15}\ \text{m})(2.99)(1.67\times10^{-27}\ \text{kg})}} = 7.26\times10^6\ \text{m/s}.$$

c)

$$K=\frac{3kT}{2}=\frac{mv^2}{2}\Rightarrow T_{\mathrm{p}}=\frac{m_{\mathrm{p}}v^2}{3k}=\frac{(1.67\times10^{-27}\ \mathrm{kg})(7.58\times10^6\ \mathrm{m/s})^2}{3(1.38\times10^{-23}\ \mathrm{J/K})}=2.3\times10^9\ \mathrm{K}$$

$$\Rightarrow T_{\mathrm{He}}=\frac{m_{\mathrm{He}}v^2}{3k}=\frac{(2.99)(1.67\times10^{-27}\ \mathrm{kg})(7.26\times10^6\ \mathrm{m/s})^2}{3(1.38\times10^{-23}\ \mathrm{J/K})}=6.4\times10^9\,\mathrm{K}.$$

d) These calculations were based on the particles' average speed. The distribution of speeds ensures that there are always a certain percentage with a speed greater than the average speed, and these particles can undergo the necessary reactions in the sun's core.

23.86: a) The two daughter nuclei have half the volume of the original uranium nucleus, so their radii are smaller by a factor of the cube root of 2:

$$r=\frac{7.4\times10^{-15}\ \mathrm{m}}{\sqrt[3]{2}}=5.9\times10^{-15}\ \mathrm{m}.$$

b) $U=\dfrac{k(46e)^2}{2r}=\dfrac{k(46)^2(1.60\times10^{-19}\ \mathrm{C})^2}{1.17\times10^{-14}\ \mathrm{m}}=4.14\times10^{-11}\ \mathrm{J}$

Each daughter has half of the potential energy turn into its kinetic energy when far from each other, so:

$$K=U/2=(4.15\times10^{-11}\ \mathrm{J})/2=2.07\times10^{-11}\ \mathrm{J}.$$

c) If we have 10.0 kg of uranium, then the number of nuclei is:

$$n=\frac{10.0\ \mathrm{kg}}{236\ \mathrm{u}\ (1.66\times10^{-27}\ \mathrm{kg/u})}=2.55\times10^{25}\ \text{nuclei}.$$

And each releases energy $\mathrm{U}: E=nU=(2.55\times10^{25})(4.15\times10^{-11}\ \mathrm{J})=1.06\times10^{15}\ \mathrm{J}=$ 253 kilotons of TNT.

d) We could call an atomic bomb an "electric" bomb since the electric potential energy provides the kinetic energy of the particles.

23.87: Angular momentum and energy must be conserved, so:

$$mv_1b=mv_2r_2\ \textit{and}\ E_1=E_2\Rightarrow E_1=\frac{1}{2}mv_2{}^2+\frac{kq_1q_2}{r_2}\ \text{and}\ E_1=11\ \mathrm{MeV}=1.76\times10^{-12}\ \mathrm{J}.$$

Substituting in for v_2 we find:

$$E_1=E_1\frac{b^2}{r_2{}^2}+\frac{kq_1q_2}{r_2}\Rightarrow(E_1)r_2{}^2-(kq_1q_2)r_2-E_1b^2=0,\ \text{and note}\ q_1=2e\ \text{and}\ q_2=82e.$$

$(i)\ b=10^{-12}\ \mathrm{m}\Rightarrow r_2=1.01\times10^{-12}\ \mathrm{m}$

$(\mathrm{ii})\ b=10^{-13}\ \mathrm{m}\Rightarrow r_2=1.11\times10^{-13}\ \mathrm{m}.$

$(\mathrm{iii})\ b=10^{-14}\ \mathrm{m}\Rightarrow r_2=2.54\times10^{-14}\ \mathrm{m}.$

23.88: a) $r \le a: V = \dfrac{\rho_0 a^2}{18\varepsilon_0}\left[1 - 3\dfrac{r^2}{a^2} + 2\dfrac{r^3}{a^3}\right]$ and $E = -\dfrac{\partial V}{\partial r}$

$$\Rightarrow E = -\frac{\rho_0 a^2}{18\varepsilon_0}\left[-6\frac{r}{a^2} + 6\frac{r^2}{a^3}\right] = \frac{\rho_0 a}{3\varepsilon_0}\left[\frac{r}{a} - \frac{r^2}{a^2}\right].$$

$r \ge a: V = 0$ and $E = -\dfrac{\partial V}{\partial r} = 0.$

b) $r \le a: E_r 4\pi r^2 = \dfrac{Q_r}{\varepsilon_0} = \dfrac{\rho_0 a}{3\varepsilon_0}\left[\dfrac{r}{a} - \dfrac{r^2}{a^2}\right]4\pi r^2$

$$E_{r+dr}4\pi(r^2 + 2rdr) = \frac{Q_{r+dr}}{\varepsilon_0} = \frac{\rho_0 a}{3\varepsilon_0}\left[\frac{r+dr}{a} - \frac{(r^2 + 2r\,dr)}{a^2}\right]4\pi(r^2 + 2r\,dr)$$

$$\Rightarrow \frac{Q_{r+dr} - Q_r}{\varepsilon_0} = \frac{\rho(r)4\pi r^2\,dr}{\varepsilon_0} \approx \frac{\rho_0 a 4\pi r^2 dr}{3\varepsilon_0}\left[-\frac{2r}{a^2} + \frac{2}{a} - \frac{2r}{a^2} + \frac{1}{a}\right]$$

$$\Rightarrow \rho(r) = \frac{\rho_0}{3}\left[3 - \frac{4r}{a}\right] = \rho_0\left[1 - \frac{4r}{3a}\right].$$

c) $r \ge a: \rho(r) = 0,$ so the total charge enclosed will be given by:

$$Q = 4\pi\int_0^a \rho(r)r^2 dr = 4\pi\rho_0\int_0^a\left[r^2 - \frac{4r^3}{3a}\right]dr = 4\pi\rho_0\left[\frac{1}{3}r^3 - \frac{r^4}{3a}\right]_0^a = 0.$$

Therefore, by Gauss's Law, the electric field must equal zero for any position $r \ge a$.

23.89: a) $F_g = mg = \dfrac{4\pi r^3}{3}\rho g = qV_{ab}/d = qE = F_e \Rightarrow q = \dfrac{4\pi}{3}\dfrac{\rho r^3 gd}{V_{ab}}.$

b) $F_g = mg = \dfrac{4\pi r^3}{3}\rho g = 6\pi\eta r v_t = F_v \Rightarrow r = \sqrt{\dfrac{9\eta v_t}{2\rho g}}$

$$\Rightarrow q = \frac{4\pi}{3}\frac{\rho g d}{V_{ab}}\left[\sqrt{\frac{9\eta v_t}{2\rho g}}\right]^3 = 18\pi\frac{d}{V_{ab}}\sqrt{\frac{\eta^3 v_t^3}{2\rho g}}.$$

c) $q = 18\pi\dfrac{10^{-3}\text{ m}}{9.16\text{ V}}\sqrt{\dfrac{(1.81\times10^{-5}\text{ Ns/m}^2)^3(10^{-3}\text{ m/39.3 s})^3}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 4.80\times10^{-19}\text{ C} = 3e.$

$$r = \sqrt{\frac{9(1.81\times10^{-5}\text{ Ns/m}^2)(10^{-3}\text{ m/39.3 s})}{2(824\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = 5.07\times10^{-7}\text{ m}$$

23.90: For an infinitesimal slice of a finite cylinder, we have the potential:

$$dV = \frac{k\,dQ}{\sqrt{(x-z)^2+R^2}} = \frac{kQ}{L}\frac{dz}{\sqrt{(x-z)^2+R^2}}$$

$$\Rightarrow V = \frac{kQ}{L}\int_{-L/2}^{L/2}\frac{dz}{\sqrt{(x-z)^2+R^2}} = \frac{kQ}{L}\int_{-L/2-x}^{L/2-x}\frac{du}{\sqrt{u^2+R^2}} \text{ where } u = x - z.$$

$$\Rightarrow V = \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2-x)^2+R^2}+(L/2-x)}{\sqrt{(L/2+x)^2+R^2}-L/2-x}\right] \text{ on the cylinder's axis.}$$

b) For $L << R$:

$$V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{(L/2-x)^2+R^2}+L/2-x}{\sqrt{(L/2+x)^2+R^2}-L/2-x}\right] \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{x^2-xL+R^2}+L/2-x}{\sqrt{x^2+xL+R^2}-L/2-x}\right]$$

$$\Rightarrow V \approx \frac{kQ}{L}\ln\left[\frac{\sqrt{1-xL/(R^2+x^2)}+(L/2-x)/\sqrt{R^2+x^2}}{\sqrt{1+xL/(R^2+x^2)}+(-L/2-x)/\sqrt{R^2+x^2}}\right]$$

$$\Rightarrow V \approx \frac{kQ}{L}\ln\left[\frac{1-xL/2(R^2+x^2)+(L/2-x)/\sqrt{R^2+x^2}}{1+xL/2(R^2+x^2)+(-L/2-x)/\sqrt{R^2+x^2}}\right]$$

$$\Rightarrow V \approx \frac{kQ}{L}\ln\left[\frac{1+L/2\sqrt{R^2+x^2}}{1-L/2\sqrt{R^2+x^2}}\right] = \frac{kQ}{L}\left(\ln\left[1+\frac{L}{2\sqrt{R^2+x^2}}\right]-\ln\left[1-\frac{L}{2\sqrt{R^2+x^2}}\right]\right)$$

$$\Rightarrow V \approx \frac{kQ}{L}\frac{2L}{2\sqrt{x^2+R^2}} = \frac{kQ}{\sqrt{x^2+R^2}}, \text{ which is the same as for a ring.}$$

c) $$E = -\frac{\partial V}{\partial x} = \frac{2kQ\left(\sqrt{(L-2x)^2+4R^2}-\sqrt{(L+2x)^2+4R^2}\right)}{\sqrt{(L-2x)^2+4R^2}\cdot\sqrt{(L+2x)^2+4R^2}}.$$

23.91: a)

$$v_{cm} = \frac{m_1v_1+m_2v_2}{m_1+m_2} = \frac{(6\times10^{-5}\text{ kg})(400\text{m/s})+(3\times10^{-5}\text{ kg})(1300\text{ m/s})}{6.0\times10^{-5}\text{ kg}+3.0\times10^{-5}\text{ kg}} = 700\text{ m/s}$$

b) $E_{rel} = \frac{1}{2}m_1v_1{}^2+\frac{1}{2}m_2v_2{}^2+\frac{kq_1q_2}{r}-\frac{1}{2}(m_1+m_2)v_{cm}{}^2.$

After expanding the center of mass velocity and collecting like terms:

$$\Rightarrow E_{rel} = \frac{1}{2}\frac{m_1m_2}{m_1+m_2}[v_1{}^2+v_2{}^2-2v_1v_2]+\frac{kq_1q_2}{r} = \frac{1}{2}\mu(v_1-v_2)^2+\frac{kq_1q_2}{r}.$$

c) $E_{rel} = \frac{1}{2}(2.0\times10^{-5}\text{ kg})(900\text{ m/s})^2+\frac{k(2.0\times10^{-6}\text{ C})(-5.0\times10^{-6}\text{ C})}{0.0090\text{ m}} = -1.9\text{ J}.$

d) Since the energy is less than zero, the system is "bound."

e) The maximum separation is when the velocity is zero:

$$-1.9\ \text{J} = \frac{kq_1q_2}{r} \Rightarrow r = \frac{k(2.0\times10^{-6}\ \text{C})(-5.0\times10^{-6}\ \text{C})}{-1.9\ \text{J}} = 0.047\ \text{m}.$$

f) Now using $v_1 = 400\ \text{m/s}$ and $v_2 = 1800\ \text{m/s}$, we find : $E_{rel} = +9.6$ J. so the particles do escape, and the final relative velocity is :

$$|v_1 - v_2| = \sqrt{\frac{2E_{rel}}{\mu}} = \sqrt{\frac{2(9.6\ \text{J})}{2.0\times10^{-5}\ \text{kg}}} = 980\ \text{m/s}.$$

24.1: $Q = CV = (25.0\ \text{V})(7.28\ \mu\text{F}) = 1.82\times10^{-4}\ \text{C}.$

24.2: a) $C = \varepsilon_0 \dfrac{A}{d} = \varepsilon_0 \dfrac{0.00122\ \text{m}^2}{0.00328\ \text{m}} = 3.29\ \text{pF}.$

b) $V = \dfrac{Q}{C} = \dfrac{4.35\times10^{-8}\ \text{C}}{3.29\times10^{-12}\ \text{F}} = 13.2\ \text{kV}.$

c) $E = \dfrac{V}{d} = \dfrac{13.2\times10^{3}\ \text{V}}{0.00328\ \text{m}} = 4.02\times10^{6}\ \text{V/m}.$

24.3: a) $V = \dfrac{Q}{C} = \dfrac{0.148\times10^{-6}\ \text{C}}{2.45\times10^{-10}\ \text{F}} = 604\ \text{V}.$

b) $A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(2.45\times10^{-10}\ \text{F})(0.328\times10^{-3}\ \text{m})}{\varepsilon_0} = 0.0091\ \text{m}^2.$

c) $E = \dfrac{V}{d} = \dfrac{604\ \text{V}}{0.328\times10^{-3}\ \text{m}} = 1.84\times10^{6}\ \text{V/m}.$

d) $E = \dfrac{\sigma}{\varepsilon_0} \Rightarrow \sigma = \varepsilon_0 E = \varepsilon_0(1.84\times10^{6}\ \text{V/m}) = 1.63\times10^{-5}\ \text{C/m}^2.$

24.4: $\Delta V = Ed = \dfrac{\sigma}{\varepsilon_0} d$

$$= \frac{(5.60\times10^{-12}\ \text{C/m}^2)(0.00180\ \text{m})}{8.85\times10^{-12}\ \text{C}^2/\text{Nm}^2}$$

$= 1.14\ \text{mV}$

24.5: a) $Q = CV = 120\ \mu C$

b) $C = \varepsilon_0 A/d$

$d \to d/2$ means $C \to C/2$ and $Q \to Q/2 = 60\ \mu\text{C}$

c) $r \to 2r$ means $A \to 4A, C \to 4C$, and $Q \to 4Q = 480\ \mu\text{C}$

24.6: (a) 12.0 V since the plates remain charged.

(b) (i) $V = \frac{Q}{C}$

Q does not change since the plates are disconnected from the battery.

$$C = \frac{\varepsilon \cdot \text{A}}{d}$$

If d is doubled, $C \to \frac{1}{2}C$, so $V \to 2V = 24.0\,\text{V}$

(ii) $A = \pi r^2$, so if $r \to 2r$, then $A \to 4A$, and $C \to 4C$ which means that

$$V \to \frac{1}{4}V = 3.00\ \text{V}$$

24.7: Estimate $r = 1.0$ cm

$$C = \frac{\varepsilon_0 A}{d} \text{ so } d = \frac{\varepsilon_0 \pi r^2}{C} = \frac{\varepsilon_0 \pi (0.010\text{ m})^2}{1.00 \times 10^{-12}\text{ F}} = 2.8\text{ mm}$$

The separation between the pennies is nearly a factor of 10 smaller than the diameter of a penny, so it is a reasonable approximation to treat them as infinite sheets.

24.8: (a) $\Delta V = Ed$

$100\text{ V} = (10^4\text{ N/C})d$

$$d = 10^{-2}\text{ m} = 1.00\text{cm}$$

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 \pi R^2}{d}$$

$$R = \sqrt{\frac{Cd}{\pi\varepsilon_0}} = \sqrt{\frac{4Cd}{4\pi\varepsilon_0}}$$

$$R = \sqrt{4(5.00 \times 10^{-12}\text{F})(10^{-2}\text{m})(9 \times 10^9\ \frac{\text{Nm}^2}{\text{C}^2})}$$

$$R = 4.24 \times 10^{-2}\text{m} = 4.24\text{ cm}$$

(b) $Q = CV = (5\text{pF})(100\text{ V}) = 500\text{ pC}$

24.9: a) $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)}$

$$C = \frac{(0.180\text{ m})2\pi\varepsilon_0}{\ln(5.00/0.50)} = 4.35 \times 10^{-12}\text{ F}$$

b) $V = Q/C = (10.0 \times 10^{-12}\text{ C})/(4.35 \times 10^{-12}\text{ F}) = 2.30\text{ V}$

24.10: a) $\dfrac{C}{L} = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)} \Rightarrow \ln(r_b/r_a) = \dfrac{2\pi\varepsilon_0}{C/L} = \dfrac{2\pi\varepsilon_0}{31.5 \times 10^{-12}\text{ F/m}} = 1.77 \Rightarrow \dfrac{r_b}{r_a} = 5.84.$

b) $\dfrac{Q}{L} = V\dfrac{C}{L} = (2.60\text{V})(31.5 \times 10^{-12}\text{ F/m}) = 8.19 \times 10^{-11}\text{ C/m}.$

24.11: a) $C/L = \dfrac{2\pi\varepsilon_0}{\ln(r_b/r_a)} = \dfrac{2\pi\varepsilon_0}{\ln(3.5\text{ mm}/1.5\text{ mm})} = 6.56 \times 10^{-11}\text{ F/m}.$

b) The charge on each conductor is equal but opposite. Since the inner conductor is at a higher potential it is positively charged, and the magnitude is:

$$Q = CV = \frac{2\pi\varepsilon_0 LV}{\ln(r_b/r_a)} = \frac{2\pi\varepsilon_0(2.8\text{ m})(0.35\text{ V})}{\ln(3.5\text{ mm}/1.5\text{ mm})} = 6.43 \times 10^{-11}\text{C}.$$

24.12: a) For two concentric spherical shells, the capacitance is:

$$C = \frac{1}{k}\left(\frac{r_a r_b}{r_b - r_a}\right) \Rightarrow kCr_b - kCr_a = r_a r_b \Rightarrow r_b = \frac{kCr_a}{kC - r_a}$$

$$\Rightarrow r_b = \frac{k(116 \times 10^{-12}\ \text{F})(0.150\ \text{m})}{k(116 \times 10^{-12}\ \text{F}) - 0.150\ \text{m}} = 0.175\ \text{m}.$$

b) $V = 220\ \text{V}$, and $Q = CV = (116 \times 10^{-12}\ \text{F})(220\ \text{V}) = 2.55 \times 10^{-8}\ \text{C}.$

24.13: a) $C = \frac{1}{k}\left(\frac{r_b r_a}{r_b - r_a}\right) = \frac{1}{k}\left(\frac{(0.148\ \text{m})(0.125\ \text{m})}{0.148\ \text{m} - 0.125\ \text{m}}\right) = 8.94 \times 10^{-11}\ \text{F}.$

b) The electric field at a distance of 12.6 cm:

$$E = \frac{kQ}{r^2} = \frac{kCV}{r^2} = \frac{k(8.94 \times 10^{-11}\ \text{F})(120\ \text{V})}{(0.126\ \text{m})^2} = 6082\ \text{N/C}.$$

c) The electric field at a distance of 14.7 cm:

$$E = \frac{kQ}{r^2} = \frac{kCV}{r^2} = \frac{k(8.94 \times 10^{-11}\ \text{F})(120\ \text{V})}{(0.147\ \text{m})^2} = 4468\ \text{N/C}.$$

d) For a spherical capacitor, the electric field is not constant between the surfaces.

24.14: a) $\frac{1}{C_{eq}} = \frac{1}{C_1 + C_2} + \frac{1}{C_3} = \frac{1}{((3.0 + 5.0) \times 10^{-6}\ \text{F})} + \frac{1}{(6.0 \times 10^{-6}\ \text{F})}$

$$\Rightarrow C_{eq} = 3.42 \times 10^{-6}\ \text{F}.$$

The magnitude of the charge for capacitors in series is equal, while the charge is distributed for capacitors in parallel. Therefore,

$$Q_3 = Q_1 + Q_2 = VC_{eq} = (24.0\ \text{V})(3.42 \times 10^{-6}\ \text{F}) = 8.21 \times 10^{-5}\ \text{C}.$$

Since C_1 and C_2 are at the same potential, $\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q_2 = \frac{C_2}{C_1}Q_1 = \frac{5}{3}Q,$

$$Q_3 = \tfrac{8}{3}Q_1 = 8.21 \times 10^{-5}\ \text{C} \Rightarrow Q_1 = 3.08 \times 10^{-5}\ \text{C, and } Q_2 = 5.13 \times 10^{-5}\ \text{C}.$$

b) $V_2 = V_1 = Q_1/C_1 = (3.08 \times 10^{-5}\ \text{C})/(3.00 \times 10^{-6}\ \text{F}) = 10.3\ \text{V}$. And $V_3 =$ $24.0\ \text{V} - 10.3\ \text{V} = 13.7\ \text{V}.$

c) The potential difference between a and d: $V_{ad} = V_1 = V_2 = 10.3\ \text{V}.$

24.15: a) $\frac{1}{C_{eq}} = \frac{1}{(\frac{1}{C_1} + \frac{1}{C_2}) + C_3} + \frac{1}{C_4} = \frac{1}{(2.00\ \mu\text{F} + 4.0\ \mu\text{F})} + \frac{1}{(4.0\ \mu\text{F})}$

$$\Rightarrow C_{eq} = 2.40\ \mu\text{F}.$$

Then, $Q_{12}+Q_3=Q_4=Q_{\text{total}}=C_{\text{eq}}V=(2.40\times10^{-6}\text{ F})(28.0\text{ V})=6.72\times10^{-5}\text{ C}$ and $2Q_{12}=Q_3\Rightarrow Q_{12}=\dfrac{Q_{\text{total}}}{3}=\dfrac{6.72\times10^{-5}\text{ C}}{3}=2.24\times10^{-5}\text{ C}$, and $Q_3=4.48\times10^{-5}\text{ C}$. But also, $Q_1=Q_2=Q_{12}=2.24\times10^{-5}\text{ C}$.

b) $V_1=Q_1/C_1=(2.24\times10^{-5}\text{ C})/(4.00\times10^{-6}\text{ F})=5.60\text{ V}=V_2$

$V_3=Q_3/C_3=(4.48\times10^{-5}\text{C})/(4.00\times10^{-6}\text{ F})=11.2\text{ V}.$

$V_4=Q_4/C_4=(6.72\times10^{-5}\text{ C})/(4.00\times10^{-6}\text{ F})=16.8\text{ V}.$

c) $V_{ad}=V_{ab}-V_4=28.0\text{ V}-16.8\text{ V}=11.2\text{ V}.$

24.16: a)

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{(3.0\times10^{-6}\text{ F})}+\frac{1}{(5.0\times10^{-6}\text{ F})}$$
$$=5.33\times10^{5}\text{ F}^{-1}\Rightarrow C_{\text{eq}}=1.88\times10^{-6}\text{ F}$$
$$\Rightarrow Q=VC_{\text{eq}}=(52.0\text{ V})(1.88\times10^{-6}\text{ F})=9.75\times10^{-5}\text{ C}$$

b) $V_1=Q/C_1=9.75\times10^{-5}\text{ C}/3.0\times10^{-6}\text{ F}=32.5\text{ V}.$

$V_2=Q/C_2=9.75\times10^{-5}\text{ C}/5.0\times10^{-6}\text{ F}=19.5\text{ V}.$

24.17: a) $Q_1=VC_1=(52.0\text{ V})(3.0\times10^{-6}\text{ F})=1.56\times10^{-4}\text{ C}.$

$Q_2=VC_2=(52.0\text{ V})(5.0\times10^{-6}\text{ F})=2.6\times10^{-4}\text{ C}.$

b) For parallel capacitors, the voltage over each is the same, and equals the voltage source: 52.0 V.

24.18: $C_{\text{eq}}=\left(\frac{1}{C_1}+\frac{1}{C_2}\right)^{-1}=\left(\frac{d_1}{\varepsilon_0 A}+\frac{d_2}{\varepsilon_0 A}\right)^{-1}=\frac{\varepsilon_0 A}{d_1+d_2}$. So the combined capacitance for two capacitors in series is the same as that for a capacitor of area A and separation (d_1+d_2).

24.19: $C_{\text{eq}}=C_1+C_2=\frac{\varepsilon_0 A_1}{d}+\frac{\varepsilon_0 A_2}{d}=\frac{\varepsilon_0(A_1+A_2)}{d}$. So the combined capacitance for two capacitors in parallel is that of a single capacitor of their combined area (A_1+A_2) and common plate separation d.

24.20: a) and b) The equivalent resistance of the combination is 6.0 μF, therefore the total charge on the network is: $Q=C_{\text{eq}}V_{\text{eq}}(6.0\ \mu\text{F})(36\text{ V})=2.16\times10^{-4}\text{C}$. This is also the charge on the 9.0 μF capacitor because it is connected in series with the point b. So:

$$V_9 = \frac{Q_9}{C_9} = \frac{2.16\times10^{-4}\ \text{C}}{9.0\times10^{-6}\ \text{F}} = 24\ \text{V}.$$

Then $V_3 = V_{11} = V_{12} + V_6 = V - V_9 = 36\ \text{V} - 24\ \text{V} = 12\ \text{V}.$

$$\Rightarrow Q_3 = C_3V_3 = (3.0\ \mu\text{F})(12\ \text{V}) = 3.6\times10^{-5}\ \text{C}.$$

$$\Rightarrow Q_{11} = C_{11}V_{11} = (11\ \mu\text{F})(12\ \text{V}) = 1.32\times10^{-4}\ \text{C}.$$

$$\Rightarrow Q_6 = Q_{12} = Q - Q_3 - Q_{11}$$

$$= 2.16\times10^{-4}\ \text{C} - 3.6\times10^{-5}\ \text{C} - 1.32\times10^{-4}\ \text{C}.$$

$$= 4.8\times10^{-5}\ \text{C}.$$

So now the final voltages can be calculated:

$$V_6 = \frac{Q_6}{C_6} = \frac{4.8\times10^{-5}\ \text{C}}{6.0\times10^{-6}\ \text{F}} = 8\ \text{V}.$$

$$V_{12} = \frac{Q_{12}}{C_{12}} = \frac{4.8\times10^{-5}\ \text{C}}{12\times10^{-6}\ \text{F}} = 4\ \text{V}.$$

c) Since the $3\ \mu\text{F}, 11\ \mu\text{F}$ and $6\ \mu\text{F}$ capacitors are connected in parallel and are in series with the $9\ \mu\text{F}$ capacitor, their charges must add up to that of the $9\ \mu\text{F}$ capacitor. Similarly, the charge on the $3\ \mu\text{F}, 11\ \mu\text{F}$ and $12\ \mu\text{F}$ capacitors must add up to the same as that of the $9\ \mu\text{F}$ capacitor, which is the same as the whole network. In short, charge is conserved for the whole system. It gets redistributed for capacitors in parallel and it is equal for capacitors in series.

24.21: Capacitances in parallel simply add, so:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{8.0\ \mu\text{F}} = \left(\frac{1}{(11+4.0+x)\ \mu\text{F}} + \frac{1}{9.0\ \mu\text{F}}\right) \Rightarrow (15+x)\ \mu\text{F} = 72\ \mu\text{F} \Rightarrow x = 57\ \mu\text{F}.$$

24.22: a) C_1 and C_2 are in parallel and so have the same potential across them:

$$V = \frac{Q_2}{C_2} = \frac{40.0\times10^{-6}\text{C}}{3.00\times10^{-6}\ \text{F}} = 13.33\ \text{V}$$

Thus $Q_1 = VC_1 = (13.33\ \text{V})(3.00\times10^{-6}\ \text{F}) = 80.0\times10^{-6}\ \text{C}$. Since Q_3 is in series with the parallel combination of C_1 and C_2, its charge must be equal to their combined charge: $40.0\times10^{-6}\text{C} + 80.0\times10^{-6}\ \text{C} = 120.0\times10^{-6}\text{C}$ b) The total capacitance is found from:

$$\frac{1}{C_{\text{tot}}} = \frac{1}{C_{\|}} + \frac{1}{C_3} = \frac{1}{9.00\times10^{-6}\ \text{F}} + \frac{1}{5.00\times10^{-6}\ \text{F}}$$

$$C_{\text{tot}} = 3.21\ \mu\text{F}$$

and

$$V_{ab} = \frac{Q_{\text{tot}}}{C_{\text{tot}}} = \frac{120.0\times10^{-6}\ \text{C}}{3.21\times10^{-6}\ \text{F}} = 37.4\ \text{V}$$

24.23: $V_1 = Q_1/C_1 = (150\ \mu\text{C})/(3.00\ \mu\text{F}) = 50\ \text{V}$

C_1 and C_2 are in parallel, so $V_2 = 50\ \text{V}$

$V_3 = 120\ \text{V} - V_1 = 70\ \text{V}$

24.24: a) $V = Q/C = (2.55\ \mu\text{C})/(920 \times 10^{-12}\ \text{F}) = 2772\ \text{V}.$

b) Since the charge is kept constant while the separation doubles, that means that the capacitance halves and the voltage doubles to 5544 V.

c) $U = \frac{1}{2}CV^2 = \frac{1}{2}(920 \times 10^{-12}\ \text{F})(2772\ \text{V})^2 = 3.53 \times 10^{-3}\ \text{J}.$ Now if the separation is doubled, the capacitance halves, and the energy stored doubles. So the amount of work done to move the plates equals the difference in energy stored in the capacitor, which is $3.53 \times 10^{-3}\ \text{J}.$

24.25: $E = V/d = (400\ \text{V})/(0.005\ \text{m}) = 8.00 \times 10^4\ \text{V/m}.$

And $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 (8.00 \times 10^4\ \text{V/m})^2 = 0.0283\ \text{J/m}^3.$

24.26: a) $C = Q/V = (0.0180\ \mu\text{C})/(200\ \text{V}) = 9.00 \times 10^{-11}\ \text{F}.$

b) $C = \dfrac{\varepsilon_0 A}{d} \Rightarrow A = \dfrac{Cd}{\varepsilon_0} = \dfrac{(9.00 \times 10^{-11}\ \text{F})(0.0015\ \text{m})}{\varepsilon_0} = 0.0152\ \text{m}^2.$

c) $E_{\text{max}} = V_{\text{max}}/d \Rightarrow V_{\text{max}} = E_{\text{max}} d = (3.00 \times 10^6\ \text{V/m})(0.0015\ \text{m}) = 4500\ \text{V}.$

d) $U = \dfrac{Q^2}{2C} = \dfrac{(1.80 \times 10^{-8}\ \text{C})^2}{2(9.00 \times 10^{-11}\ \text{F})} = 1.80 \times 10^{-6}\ \text{J}.$

24.27: $U = \frac{1}{2}CV^2 = \frac{1}{2}(4.50 \times 10^{-4}\ \text{F})(295\ \text{V})^2 = 19.6\ \text{J}.$

24.28: a) $Q = CV_0$.

b) They must have equal potential difference, and their combined charge must add up to the original charge. Therefore:

$$V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} \text{ and also } Q_1 + Q_2 = Q = CV_0$$

$$C_1 = C \text{ and } C_2 = \frac{C}{2} \text{ so } \frac{Q_1}{C} = \frac{Q_2}{(C/2)} \Rightarrow Q_2 = \frac{Q_1}{2}$$

$$\Rightarrow Q = \frac{3}{2}Q_1 \Rightarrow Q_1 = \frac{2}{3}Q \text{ so } V = \frac{Q_1}{C} = \frac{2}{3}\frac{Q}{C} = \frac{2}{3}V_0$$

c) $U = \frac{1}{2}\left(\frac{Q_1^2}{C_1} + \frac{Q_2^{\ 2}}{C_2}\right) = \frac{1}{2}\left[\frac{(\frac{2}{3}Q)^2}{C} + \frac{2(\frac{1}{3}Q)^2}{C}\right] = \frac{1}{3}\frac{Q^2}{C} = \frac{1}{3}CV_0^{\ 2}$

d) The original U was $U = \frac{1}{2}CV_0^{\ 2} \Rightarrow \Delta U = \frac{-1}{6}CV_0^{\ 2}$.

e) Thermal energy of capacitor, wires, etc., and electromagnetic radiation.

24.29: a) $U_0 = \dfrac{Q^2}{2C} = \dfrac{xQ^2}{2\varepsilon_0 A}$.

b) Increase the separation by $dx \Rightarrow U = \frac{(x+dx)Q^2}{2\varepsilon_0 A} = U_0(1 + dx/x)$. The change is then $\frac{Q^2}{2\varepsilon_0 A}dx$.

c) The work done in increasing the separation is given by:

$$dW = U - U_0 = \frac{dxQ^2}{2\varepsilon_0 A} = Fdx \Rightarrow F = \frac{Q^2}{2\varepsilon_0 A}.$$

d) The reason for the difference is that E is the field due to both plates. The force is QE if E is the field due to one plate is Q is the charge on the other plate.

24.30: a) If the separation distance is halved while the charge is kept fixed, then the capacitance increases and the stored energy, which was 8.38 J, decreases since $U = Q^2/2C$. Therefore the new energy is 4.19 J.

b) If the voltage is kept fixed while the separation is decreased by one half, then the doubling of the capacitance leads to a doubling of the stored energy to 16.76 J, using $U = CV^2/2$, when V is held constant throughout.

24.31: a) $U = Q^2/2C$

$Q = \sqrt{2UC} = \sqrt{2(25.0\ \text{J})(5.00 \times 10^{-9}\ \text{F})} = 5.00 \times 10^{-4}\ \text{C}$

The number of electrons N that must be removed from one plate and added to the other is $N = Q/e = (5.00 \times 10^{-4}\ \text{C})/(1.602 \times 10^{-19}\ \text{C}) = 3.12 \times 10^{15}$ electrons.

b) To double U while keeping Q constant, decrease C by a factor of 2.

$C = \varepsilon_0 A/d$; halve the plate area or double the plate separation.

24.32: $C = \dfrac{Q}{V} = \dfrac{8.20 \times 10^{-12}\ \text{C}}{2.40\ \text{V}} = 3.417 \times 10^{-12}$ farad

Since $C = K\varepsilon_0\, A/d$ for a parallel plate capacitor

$$d = \frac{K\varepsilon_0 A}{C} = \frac{(1.00)(8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2)(2.60 \times 10^{-3}\,\text{m}^2)}{3.417 \times 10^{-12}\ \text{farad}}$$

$= 6.734 \times 10^{-3}\,\text{m}$

The energy density is thus

$$u = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}(3.42\times10^{-12}\text{ farad})(2.40\text{ V})^2}{(2.60\times10^{-3}\text{m}^2)(6.734\times10^{-3}\text{m})} = 5.63\times10^{-7}\ \frac{\text{J}}{\text{m}^3}$$

24.33: a) $U = \frac{1}{2}QV \Rightarrow Q = \frac{2U}{V} = \frac{2(3.20\times10^{-9}\text{ J})}{4.00\text{ V}} = 1.60\times10^{-9}\text{ C}.$

b) $\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(r_a/r_b)} \Rightarrow \frac{r_a}{r_b} = \exp(2\pi\varepsilon_0 L/C) = \exp(2\pi\varepsilon_0 LV/Q)$

$\Rightarrow \frac{r_a}{r_b} = \exp(2\pi\varepsilon_0(15.0\text{ m})(4.00\text{ V})/(1.60\times10^{-9}\text{ C})) = 8.05.$

24.34: a) For a spherical capacitor:

$C = \frac{1}{k}\frac{r_a r_b}{r_b - r_a} = \frac{1}{k}\frac{(0.100\text{ m})(0.115\text{ m})}{(0.115\text{ m} - 0.100\text{ m})} = 8.53\times10^{-11}\text{ F}$

$\Rightarrow V = Q/C = (3.30\times10^{-9}\text{ C})/(8.53\times10^{-11}\text{ F}) = 38.7\text{ V}.$

b) $U = \frac{1}{2}CV^2 = \frac{(8.53\times10^{-11}\text{ F})(38.7\text{ V})^2}{2} = 6.38\times10^{-8}\text{ J}.$

24.35: a) $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}\left(\frac{kq}{r^2}\right)^2 = \frac{\varepsilon_0}{2}\left(\frac{kVC}{r^2}\right)^2 = \frac{\varepsilon_0 k^2}{2}\frac{(120\text{ V})^2(8.94\times10^{-11}\text{ F})^2}{(0.126\text{ m})^4}$

$\Rightarrow u = 1.64\times10^{-4}\text{ J/m}^3.$

b) The same calculation for $r = 14.7\text{ cm} \Rightarrow u = 8.83\times10^{-5}\text{ J/m}^3.$

c) No, the electric energy density is NOT constant within the spheres.

24.36: a) $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\right)^2 = \frac{1}{32\pi^2\varepsilon_0}\frac{(8.00\times10^{-9}\text{ C})^2}{(0.120\text{ m})^4} = 1.11\times10^{-4}\text{ J/m}^3.$

b) If the charge was –8.00 nC, the electric field energy would remain the same since U only depends on the square of E.

24.37: Let the applied voltage be V. Let each capacitor have capacitance C. $U = \frac{1}{2}CV^2$ for a single capacitor with voltage V.

a) **series**

Voltage across each capacitor is $V/2$. The total energy stored is

$$U_s = 2\left(\frac{1}{2}C[V/2]^2\right) = \frac{1}{4}CV^2$$

parallel

Voltage across each capacitor is V. The total energy stored is

$U_{\mathrm{p}} = 2\left(\frac{1}{2}CV^2\right) = CV^2$

$U_{\mathrm{p}} = 4U_{\mathrm{s}}$ b) $Q = CV$ for a single capacitor with voltage V.

$Q_{\mathrm{s}} = 2(C[V/2]) = CV; \quad Q_{\mathrm{p}} = 2(CV) = 2CV; \quad Q_{\mathrm{p}} = 2Q_{\mathrm{s}}$

c) $E = V/d$ for a capacitor with voltage V

$E_{\mathrm{s}} = V/2d; \quad E_{\mathrm{p}} = V/d; \quad E_{\mathrm{p}} = 2E_{\mathrm{s}}$

24.38: a) $C = K\varepsilon_0 A/d$ gives us the area of the plates:

$$A = \frac{Cd}{K\varepsilon_0} \frac{(5.00\times10^{-12}\ \text{farad})(1.50\times10^{-3}\ \text{m})}{(1.00)(8.85\times10^{-12}\text{C}^2/\text{N}\cdot\text{m}^2)} = 8.475\times10^{-4}\ \text{m}^2$$

We also have $C = K\varepsilon_0 A/d = Q/V$, so $Q = K\varepsilon_0 A(V/d)$. V/d is the electric field between the plates, which is not to exceed $3.00\times10^4\ \text{N/C}$. Thus

$$Q = (1.00)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(8.475\times10^{-4}\ \text{m}^2)(3.00\times10^4\ \text{N/C})$$
$$= 2.25\times10^{-10}\ \text{C}$$

b) Again, $Q = K\varepsilon_0 A(V/d) = 2.70\varepsilon_0 A(V/d)$. If we continue to think of V/d as the electric field, only K has changed from part (a); thus Q in this case is $(2.70)(2.25\times10^{-10}\,\text{C}) = 6.08\times10^{-10}\,\text{C}$.

24.39: a) $\sigma_i = \varepsilon_0\ ((3.20 - 2.50)\times 10^5\ \text{V/m}) = 6.20\times 10^{-7}\ \text{C/m}^2$. The field induced in the dielectric creates the bound charges on its surface.

b) $K = \dfrac{E_0}{E} = \dfrac{3.20\times 10^5\ \text{V/m}}{2.50\times 10^5\ \text{V/m}} = 1.28.$

24.40: a) $E_0 = KE = (3.60)(1.20\times 10^6\ \text{V/m}) = 4.32\times 10^6\ \text{V/m} \Rightarrow \sigma = \varepsilon_0 E_0 =$ $3.82\times 10^{-5}\ \text{C/m}^2$.

b) $\sigma_i = \sigma\left(1 - \dfrac{1}{K}\right) = (3.82\times 10^{-5}\ \text{C/m}^2)(1 - 1/3.60) = 2.76\times 10^{-5}\ \text{C/m}^2.$

c) $U = \frac{1}{2}CV^2 = uAd = \frac{1}{2}K\varepsilon_0 E^2 Ad$

$\Rightarrow U = \frac{1}{2}(3.60)\varepsilon_0(1.20\times 10^6\ \text{V/m})^2(0.0018\ \text{m})(2.5\times 10^{-4}\ \text{m}^2) = 1.03\times 10^{-5}\ \text{J}.$

24.41: $C = \dfrac{K\varepsilon_0 A}{d} = \dfrac{K\varepsilon_0 AE}{V} \Rightarrow A = \dfrac{CV}{K\varepsilon_0 E} = \dfrac{(1.25\times 10^{-9}\ \text{F})(5500\ \text{V})}{(3.60)\varepsilon_0(1.60\times 10^7\ \text{V/m})} = 0.0135\ \text{m}^2.$

24.42: Placing a dielectric between the plates just results in the replacement of ε for ε_0 in the derivation of Equation (24.20). One can follow exactly the procedure as shown for Equation (24.11).

24.43: a) $\varepsilon = K\varepsilon_0 = (2.6)\varepsilon_0 = 2.3\times 10^{-11}\ \text{C}^2/\text{Nm}^2.$

b) $V_{\max} = E_{\max} d = (2.0\times 10^7\ \text{V/m})(2.0\times 10^{-3}\ \text{m}) = 4.0\times 10^4\ \text{V}.$

c) $E = \dfrac{\sigma}{K\varepsilon_0} \Rightarrow \sigma = \varepsilon E = (2.3\times 10^{-11}\ \text{C}^2/\text{Nm}^2)(2.0\times 10^7\ \text{V/m}) = 0.46\times 10^{-3}\ \text{C/m}^2.$

And $\sigma_i = \sigma\left(1 - \dfrac{1}{K}\right) = (0.46\times 10^{-3}\ \text{C/m}^2)(1 - 1/2.6) = 2.8\times 10^{-4}\ \text{C/m}^2.$

24.44: a) $\Delta Q = Q - Q_0 = (K-1)Q_0 = (K-1)C_0V_0 = (2.1)(2.5\times 10^{-7}\ \text{F})(12\ \text{V}) =$ $6.3\times 10^{-6}\ \text{C}.$

b) $Q_i = Q\left(1 - \frac{1}{K}\right) = (9.3\times 10^{-6}\ \text{C})(1 - 1/3.1) = 6.3\times 10^{-6}\ \text{C}.$

c) The addition of the mylar doesn't affect the electric field since the induced charge cancels the additional charge drawn to the plates.

24.45: a) $U_0 = \frac{1}{2}C_0V^2 \Rightarrow V = \sqrt{\frac{2U_0}{C_0}} = \sqrt{\frac{2(1.85\times10^{-5}\ \text{J})}{(3.60\times10^{-7}\ \text{F})}} = 10.1\ \text{V}.$

b) $U = \frac{1}{2}KC_0V^2 \Rightarrow K = \frac{U}{C_0V^2} = \frac{2(2.32\times10^{-5} + 1.85\times10^{-5}\ \text{J})}{(3.60\times10^{-7}\ \text{F})(10.1\ \text{V})^2} = 2.27.$

24.46: a) The capacitance changes by a factor of K when the dielectric is inserted. Since V is unchanged (The battery is still connected),

$$\frac{C_{\text{after}}}{C_{\text{before}}} = \frac{Q_{\text{after}}}{Q_{\text{before}}} = \frac{45.0\ \text{pC}}{25.0\ \text{pC}} = K = 1.80$$

b) The area of the plates is $\pi r^2 = \pi(0.0300\ \text{m})^2 = 2.827\times10^{-3}\text{m}^2$, and the separation between them is thus

$$d = \frac{K\varepsilon_0 A}{C} = \frac{(1.00)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)}{12.5\times10^{-12}\ \text{farad}}$$

$$= 2.002\times10^{-3}\ \text{m}$$

Before the dielectric is inserted,

$$C = \frac{K\varepsilon_0 A}{d} = \frac{Q}{V}$$

$$V = \frac{Qd}{K\varepsilon_0 A} = \frac{(25.0\times10^{-12}\ \text{C})(2.00\times10^{-3}\ \text{m})}{(1.00)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.827\times10^{-3}\ \text{m}^2)}$$

$$= 2.000\ \text{V}$$

The battery remains connected, so the potential difference is unchanged after the dielectric is inserted.

c) Before the dielectric is inserted,

$$E = \frac{Q}{\varepsilon_0 KA} = \frac{25.0\times10^{-12}\ \text{C}}{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.00)(2.827\times10^{-3}\ \text{m}^2)}$$

$$= 999\ \text{N/C}$$

Again, since the voltage is unchanged after the dielectric is inserted, the electric field is also unchanged.

24.47: a) before: $V_0 = Q_0/C_0 = (9.00\times10^{-6}\ \text{C})/(3.00\times10^{-6}\ \text{F}) = 3.00\ \text{V}$

after: $C = KC_0 = 15.0\ \text{F}; Q = Q_0$

$V = Q/C = 0.600\ \text{V}$; V decreases by a factor of K

b) $E = V/d$, the same at all points between the plates (as long as far from the edges of the plates)

before: $E = (3.00\ \text{V})/(2.00\times10^{-3}\ \text{m}) = 1500\ \text{V/m}$

after: $E = (0.600\ \text{V})/(2.00\times10^{-3}\ \text{m}) = 300\ \text{V/m}$

24.48: a) $\oint K\vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = \dfrac{Q_{free}}{\varepsilon_0} \Rightarrow KE4\pi d^2 = \dfrac{q}{\varepsilon_0} \Rightarrow E = \dfrac{q}{4\pi\varepsilon d^2}.$

b) $\oint \vec{E}\cdot d\vec{A} = \dfrac{q_{total}}{\varepsilon_0} = \dfrac{q_f + q_b}{\varepsilon_0} \Rightarrow E4\pi d^2 = \dfrac{q + q_b}{\varepsilon_0} \Rightarrow E = \dfrac{q + q_b}{4\pi\varepsilon_0 d^2}$

$\Rightarrow q_{total} = q + q_b = q/K.$

c) The total bound change is $q_b = q\left(\frac{1}{K} - 1\right)$

24.49: a) Equation (25.22): $\oint K\vec{E}\cdot d\vec{A} = \frac{Q_{free}}{\varepsilon_0} \Rightarrow KEA = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{K\varepsilon_0 A} = \frac{Q}{\varepsilon A}.$

b) $V = Ed = \dfrac{Qd}{K\varepsilon_0 A} = \dfrac{Qd}{\varepsilon A}.$

c) $C = \dfrac{Q}{V} = \dfrac{\varepsilon A}{d} = K\dfrac{\varepsilon_0 A}{d} = KC_0.$

24.50: a) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{\varepsilon_0 (0.16\,\text{m})^2}{4.7\times10^{-3}\ \text{m}} = 4.8\times10^{-11}\ \text{F}.$

b) $Q = CV = (4.8\times10^{-11}\text{F})\,(12\ \text{V}) = 0.58\times10^{-9}\ \text{C}.$

c) $E = V/d = (12\ \text{V})/(4.7\times10^{-3}\text{m}) = 2553\ \text{V/m}.$

d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(4.8\times10^{-11}\text{F})(12\ \text{V})^2 = 3.46\times10^{-9}\ \text{J}.$

e) If the battery is disconnected, so the charge remains constant, and the plates are pulled further apart to 0.0094 m, then the calculations above can be carried out just as before, and we find:

a) $C = 2.41\times10^{-11}\ \text{F}$ b) $Q = 0.58\times10^{-9}\ \text{C}.$

c) $E = 2553\ \text{V/m}$ d) $U = \dfrac{Q^2}{2C} = \dfrac{(0.58\times10^{-9}\ \text{C})^2}{2(2.41\times10^{-11}\ \text{F})} = 6.91\times10^{-9}\ \text{J}.$

24.51: If the plates are pulled out as in Problem 24.50 the battery is connected, ensuring that the voltage remains constant. This time we find:

a) $C = 2.4\times10^{-11}\ \text{F}$ b) $Q = 2.9\times10^{-10}\ \text{C}$ c) $E = \dfrac{V}{d} = \dfrac{12\ \text{V}}{0.0094} = 1.3\times10^3\ \dfrac{\text{V}}{\text{m}}$

d) $U = \dfrac{CV^2}{2} = \dfrac{(2.4\times10^{-11}\ \text{F})\,(12\ \text{V})^2}{2} = 1.73\times10^{-9}\ \text{J}.$

24.52: a) System acts like two capacitors in series so $C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$

$$C_1 = C_2 = \frac{\varepsilon_0 L^2}{d} \text{ so } C_{eq} = \frac{\varepsilon_0 L^2}{2d}. \quad U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{\left(\frac{\varepsilon_0 L^2}{2d}\right)} = \frac{Q^2 d}{\varepsilon_0 L^2}.$$

b) After rearranging, the E fields should be calculated. Use superposition recalling $E = \frac{Q}{2\varepsilon_0 A}$ for a single plate (not $\frac{Q}{\varepsilon_0 A}$ since charge Q is only on one face).

$$\text{between 1 and 3: } E = \left(\frac{Q}{2\varepsilon_0 L^2}\right)_1 - \left(\frac{Q}{2\varepsilon_0 L^2}\right)_3 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_2 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_4 = \frac{Q}{\varepsilon_0 L^2}$$

$$\text{between 3 and 2: } E = \left(\frac{Q}{2\varepsilon_0 L^2}\right)_1 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_3 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_2 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_4 = \frac{2Q}{\varepsilon_0 L^2}$$

$$\text{between 2 and 4: } E = \left(\frac{Q}{2\varepsilon_0 L^2}\right)_1 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_3 - \left(\frac{Q}{2\varepsilon_0 L^2}\right)_2 + \left(\frac{Q}{2\varepsilon_0 L^2}\right)_4 = \frac{Q}{\varepsilon_0 L^2}$$

$$U_{new} = \left(\frac{1}{2}\varepsilon_0 E^2\right) L^2 d = \frac{1}{2}\varepsilon_0\left(\frac{Q^2}{\varepsilon_0{}^2 L^4} + \frac{4Q^2}{\varepsilon_0{}^2 L^4} + \frac{Q^2}{\varepsilon_0{}^2 L^4}\right) L^2 d = \frac{3Q^2 d}{\varepsilon_0 L^2}$$

$$\Delta U = U_{new} - U = \frac{3Q^2 d}{\varepsilon_0 L^2} - \frac{Q^2 d}{\varepsilon_0 L^2} = \frac{2Q^2 d}{\varepsilon_0 L^2}$$

This is the work required to rearrange the plates.

24.53: a) The power output is 600 W, and 95% of the original energy is converted.
$\Rightarrow E = Pt = (2.70 \times 10^5 \text{W})(1.48 \times 10^{-3}\text{ s}) = 400\text{ J} \therefore E_0 = \frac{400\text{ J}}{0.95} = 421\text{ J}.$

b) $U = \frac{1}{2}CV^2 \Rightarrow C = \frac{2U}{V^2} = \frac{2(421\text{ J})}{(125\text{ V})^2} = 0.054\text{ F}$

24.54: $C_0 = \frac{A\varepsilon_0}{d} = \frac{(4.20 \times 10^{-5}\text{ m}^2)\varepsilon_0}{7.00 \times 10^{-4}\text{ m}} = 5.31 \times 10^{-13}\text{ F}$

$$\Rightarrow C = C_0 + 0.25\text{ pF} = 7.81 \times 10^{-13}\text{ F}.$$

But $C = \frac{A\varepsilon_0}{d'} \Rightarrow d' = \frac{A\varepsilon_0}{C} = \frac{(4.20 \times 10^{-5}\text{ m}^2)\varepsilon_0}{7.81 \times 10^{-13}\text{ F}} = 4.76 \times 10^{-4}\text{m}.$

Therefore the key must be depressed by a distance of:

$$7.00 \times 10^{-4}\text{ m} - 4.76 \times 10^{-4}\text{ m} = 0.224\text{ mm}.$$

24.55: a) $d << r_a : C = \frac{2\pi\varepsilon_0 L}{\ln(r_b/r_a)} = \frac{2\pi\varepsilon_0 L}{\ln((d + r_a)/r_a)} = \frac{2\pi\varepsilon_0 L}{\ln(1 + d/r_a)} \approx \frac{2\pi r_a L\varepsilon_0}{d} = \frac{\varepsilon_0 A}{d}.$

b) At the scale of part (a) the cylinders appear to be flat, and so the capacitance should appear like that of flat plates.

24.56: Originally: $Q_1 = C_1V_1 = (9.0\ \mu\text{F})\,(28\ \text{V}) = 2.52\times10^{-4}\ \text{C}$; $Q_2 = C_2V_2 = (4.0\ \mu\text{F})\times$ $(28\ \text{V}) = 1.12\times10^{-4}\ \text{C}$, and $C_{eq} = C_1 + C_2 = 13.0\ \mu\text{F}$. So the original energy stored is $U = \frac{1}{2}C_{eq}V^2 = \frac{1}{2}(13.0\times10^{-6}\ \text{F})\,(28\ \text{V})^2 = 5.10\times10^{-3}\ \text{J}$. Disconnect and flip the capacitors, so now the total charge is $Q = Q_2 - Q_1 = 1.4\times10^{-4}\ \text{C}$, and the equivalent capacitance is still the same, $C_{eq} = 13.0\ \mu\text{F}$. So the new energy stored is :

$$U = \frac{Q^2}{2C_{eq}} = \frac{(1.4\times10^{-4}\ \text{C})^2}{2(13.0\times10^{-6}\ \text{F})} = 7.54\times10^{-4}\ \text{J}$$

$$\Rightarrow \Delta U = 7.45\times10^{-4}\ \text{J} - 5.10\times10^{-3}\ \text{J} = -\,4.35\times10^{-3}\ \text{J}.$$

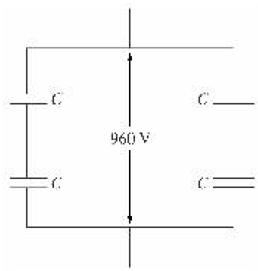
24.57: a) $C_{eq} = 4.00\ \mu F + 6.00\mu\text{F} = 10.00\ \mu\text{F}$, and $Q_{total} = C_{eq}\ V = (10.00\ \mu\text{F})\ (660\ \text{V}) = 6.6 \times 10^{-3}\text{C}$. The voltage over each is 660 V since they are in parallel. So:

$$Q_1 = C_1 V_1 = (4.00\ \mu\text{F})\ (660\ \text{V}) = 2.64 \times 10^{-3}\ \text{C}.$$
$$Q_2 = C_2 V_2 = (6.00\ \mu\text{F})\ (660\ \text{V}) = 3.96 \times 10^{-3}\ \text{C}.$$

b) $Q_{total} = 3.96 \times 10^{-3}\ \text{C} - 2.64 \times 10^{-3}\ \text{C} = 1.32 \times 10^{-3}\ \text{C}$, and still $C_{eq} = 10.00\ \mu\text{F}$, so the voltage is $V = Q/C = (1.32 \times 10^{-3}\ \text{C})/(10.00\ \mu\text{F}) = 132\ \text{V}$, and the new charges:

$$Q_1 = C_1 V_1 = (4.00\ \mu\text{F})(132\ \text{V}) = 5.28 \times 10^{-4}\ \text{C}.$$
$$Q_2 = C_2 V_2 = (6.00\ \mu\text{F})(132\ \text{V}) = 7.92 \times 10^{-4}\ \text{C}.$$

24.58: a)



$C_{eq} = \frac{C}{2} + \frac{C}{2} = C$. So the total capacitance is the same as each individual capacitor, and the voltage is spilt over each so that $V = 480\ \text{V}$. Another solution is two capacitors in parallel that are in series with two others in parallel.

b) If one capacitor is a moderately good conductor, then it can be treated as a "short" and thus removed from the circuit, and one capacitor will have greater than 600 V over it.

24.59: a) $\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2 + \left(\frac{1}{C_3} + \frac{1}{C_4}\right)^{-1}} + \dfrac{1}{C_5} \Rightarrow C_1 = C_5 = 2C_2$ and

$$C_2 = C_3 = C_4 \text{ so } \frac{1}{C_{eq}} = \frac{2}{C_1} + \frac{2}{3C_2} = \frac{5}{3}C_2 \Rightarrow C_{eq} = \frac{3}{5}C_2 = 2.52\ \mu\text{F}.$$

b) $Q = CV = (2.52\ \mu\text{F})(220\ \text{V}) = 5.54 \times 10^{-4}\ \text{C} = Q_1 = Q_5$

$\Rightarrow V_1 = V_5 = (5.54 \times 10^{-4}\ \text{C}) / (8.4 \times 10^{-6}\ \text{F}) = 66\ \text{V}.$

So $V_2 = 220 - 2(66) = 88\ \text{V} \Rightarrow Q_2 = (88\ \text{V})(4.2\ \mu\text{F}) = 3.70 \times 10^{-4}\ \text{C}$. Also $V_3 = V_4 = \frac{1}{2}(88\ \text{V}) = 44\ \text{V} \Rightarrow Q_3 = Q_4 = (44\ \text{V})(4.2\ \mu\text{F}) = 1.85 \times 10^{-4}\,\text{C}$.

24.60: a) With the switch open: $C_{eq} = \left(\left(\frac{1}{3\,\mu F} + \frac{1}{6\,\mu F}\right)^{-1} + \left(\frac{1}{3\,\mu F} + \frac{1}{6\,\mu F}\right)^{-1}\right) = 4.00\ \mu\text{F}$

$\Rightarrow Q_{total} = C_{eq}V = (4.00\ \mu\text{F})\,(210\ \text{V}) = 8.4 \times 10^{-4}\ \text{C}$. By symmetry, each capacitor carries 4.20×10^{-4} C. The voltages are then just calculated via $V{=}Q/C$. So: $V_{ad} = Q/C_3 = 140\ \text{V}$, and $V_{ac} = Q/C_6 = 70\ \text{V} \Rightarrow V_{cd} = V_{ad} - V_{ac} = 70\ \text{V}$.

b) When the switch is closed, the points c and d must be at the same potential, so the equivalent capacitance is:

$$C_{eq} = \left(\frac{1}{(3+6)\ \mu\text{F}} + \frac{1}{(3+6)\ \mu\text{F}}\right)^{-1} = 4.5\ \mu\text{F}.$$

$\Rightarrow Q_{total} = C_{eq}V = (4.50\ \mu\text{F})\,(210\ \text{V}) = 9.5 \times 10^{-4}\ \text{C}$, and each capacitor has the same potential difference of 105 V (again, by symmetry)

c) The only way for the sum of the positive charge on one plate of C_2 and the negative charge on one plate of C_1 to change is for charge to flow through the switch. That is, the quantity of charge that flows through the switch is equal to the charge in $Q_2 - Q_1 = 0$. With the switch open, $Q_1 = Q_2$ and $Q_2 - Q_1 = 0$. After the switch is closed, $Q_2 - Q_1 = 315\ \mu\text{C}$; $315\ \mu\text{C}$ of charge flowed through the switch.

24.61: a) $C_{eq} = \left(\frac{1}{8.4\ \mu\text{F}} + \frac{1}{8.4\ \mu\text{F}} + \frac{1}{4.2\ \mu\text{F}}\right)^{-1} = 2.1\,\mu\text{F}$

$$\Rightarrow Q = C_{eq}V = (2.1\ \mu\ \text{F})\,(36\ \text{V}) = 7.50 \times 10^{-5}\ \text{C}.$$

b) $U = \frac{1}{2}CV^2 = \frac{1}{2}(2.1\ \mu\text{F})\,(36\ \text{V})^2 = 1.36 \times 10^{-3}\ \text{J}$.

c) If the capacitors are all in parallel, then:

$C_{eq} = (8.4\ \mu\text{F} + 8.4\ \mu\text{F} + 4.2\ \mu\text{F}) = 21\ \mu\text{F}$ and $Q = 3(7.56 \times 10^{-5}\text{C}) = 2.27 \times 10^{-4}\ \text{C}$,

and $V = Q/C = (2.27 \times 10^{-4}\ \text{C})/(21\ \mu\text{F}) = 10.8\ \text{V}$.

d) $U = \frac{1}{2}CV^2 = \frac{1}{2}(21\ \mu\text{F})\ (10.8\ \text{V})^2 = 1.22 \times 10^{-3}\ \text{J}$.

24.62: a) $C_{eq} = \left(\frac{1}{4.0\ \mu\text{F}} + \frac{1}{6.0\ \mu\text{F}}\right)^{-1} = 2.4 \times 10^{-6}\ \text{F}$

$$\Rightarrow Q = C_{eq}V = (2.4 \times 10^{-6}\ \text{F})\ (600\ \text{V}) = 1.58 \times 10^{-3}\text{C}$$

and $V_2 = Q/C_2 = (1.58 \times 10^{-3}\ \text{C})/(4.0\ \mu\text{F}) = 395\ \text{V} \Rightarrow V_3 = 660\ \text{V} - 395\ \text{V} = 265\ \text{V}$.

b) Disconnecting them from the voltage source and reconnecting them to themselves we must have equal potential difference, and the sum of their charges must be the sum of the original charges:

$$Q_1 = C_1V \text{ and } Q_2 = C_2V \Rightarrow 2Q = Q_1 + Q_2 = (C_1 + C_2)V$$

$$\Rightarrow V = \frac{2Q}{C_1 + C_2} = \frac{2(1.58 \times 10^{-3}\ \text{C})}{10.0 \times 10^{-6}\ \text{F}} = 316\ \text{V}.$$

$$\Rightarrow Q_1 = (4.00 \times 10^{-6}\ \text{F})(316\ \text{V}) = 1.26 \times 10^{-3}\ \text{C}.$$

$$\Rightarrow Q_2 = (6.00 \times 10^{-6}\ \text{F})(316\ \text{V}) = 1.90 \times 10^{-3}\ \text{C}.$$

24.63: a) Reducing the furthest right leg yields $C = \left(\frac{1}{6.9\ \mu\text{F}} + \frac{1}{6.9\ \mu\text{F}} + \frac{1}{6.9\ \mu\text{F}}\right)^{-1} =$ $2.3\ \mu\text{F} = C_1/3.$ It combines in parallel with a $C_2 \Rightarrow C = 4.6\ \mu\text{F} + 2.3\ \mu\text{F} = 6.9\ \mu\text{F} = C_1$. So the next reduction is the same as the first: $C = 2.3\ \mu\text{F} = C_1/3.$ And the next is the same as the second, leaving $3C_1$'s in series so $C_{\text{eq}} = 2.3\ \mu\text{F} = C_1/3.$

b) For the three capacitors nearest points a and b:

$$Q_{C_1} = C_{\text{eq}}V = (2.3 \times 10^{-6}\ \text{F})(420\ \text{V}) = 9.7 \times 10^{-4}\ \text{C}$$

and $Q_{C_2} = C_2V_2 = (4.6 \times 10^{-6}\ \text{F})(420\ \text{V})/3 = 6.44 \times 10^{-4}\ \text{C}.$

c) $V_{cd} = \frac{1}{3}\left(\frac{420}{3}\ \text{V}\right) = 46.7\ \text{V}$, since by symmetry the total voltage drop over the equivalent capacitance of the part of the circuit from the junctions between a, c and d, b is $\frac{420}{3}$ V, and the equivalent capacitance is that of three equal capacitors C_1 in series. V_{cd} is the voltage over just one of those capacitors, i.e., 1/3of $\frac{420}{3}$ V.

24.64: (a) $C_{\text{equiv}} = C_1 + C_2 + C_3 = 60\ \mu\text{F}$

$$Q = CV = (60\ \mu\text{F})\ (120\ \text{V}) = 7200\ \mu\text{C}$$

(b) $\dfrac{1}{C_{\text{equiv}}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}$

$$C_{\text{equiv}} = 5.45\ \mu\text{F}$$

$$Q = CV = (5.45\ \mu\text{F})(120\ \text{V}) = 654\ \mu\text{C}$$

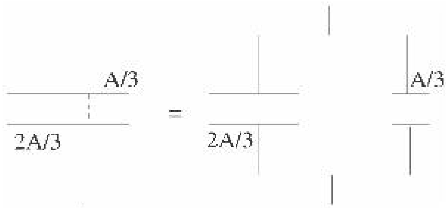
24.65: a) Q is constant.

with the dielectric: $V = Q/C = Q/(KC_0)$

without the dielectric: $V_0 = Q/C_0$

$V_0/V = K$, so $K = (45.0\ \text{V})/(11.5\ \text{V}) = 3.91$

b)



Let $C_0 = \varepsilon_0 A/d$ be the capacitance with only air between the plates. With the dielectric filling one-third of the space between the plates, the capacitor is equivalent to C_1 and C_2 in parallel, where C_1 has $A_1 = A/3$ and C_2 has $A_2 = 2A/3$

$$C_1 = K\,C_0/3,\ C_2 = 2C_0/3;\ C_{eq} = C_1 + C_2 = (C_0/3)(K+2)$$

$$V = \frac{Q}{C_{eq}} = \frac{Q}{C_0}\left(\frac{3}{K+2}\right) = V_0\left(\frac{3}{K+2}\right) = (45.0\ \text{V})\left(\frac{3}{5.91}\right) = 22.8\ \text{V}$$

24.66: a) This situation is analagous to having two capacitors C_1 in series, each with separation $\frac{1}{2}(d-a)$. Therefore $C = \left(\frac{1}{C_1} + \frac{1}{C_1}\right)^{-1} = \frac{1}{2}C_1 = \frac{1}{2}\frac{\varepsilon_0 A}{(d-a)/2} = \frac{\varepsilon_0 A}{d-a}$.

b) $C = \dfrac{\varepsilon_0 A}{d-a} = \dfrac{\varepsilon_0 A}{d}\dfrac{d}{d-a} = C_0\dfrac{d}{d-a}$.

c) As $a \to 0,\ C \to C_0$. And as $a \to d,\ C \to \infty$.

24.67: a) One can think of "infinity" as a giant conductor with $V = 0$.

b) $C = \frac{Q}{V} = \frac{Q}{(Q/4\pi\varepsilon_0 R)} = 4\pi\varepsilon_0 R,$ where we've chosen $V = 0$ at infinity.

c) $C_{earth} = 4\pi\varepsilon_0 R_{earth} = 4\pi\varepsilon_0(6.4\times10^6\ \text{m}) = 7.1\times10^{-4}\ \text{F}$. Larger than, but comparable to the capacitance of a typical capacitor in a circuit.

24.68: a) $r < R : u = \dfrac{1}{2}\varepsilon_0 E^2 = 0.$

b) $r > R : u = \frac{1}{2}\varepsilon_0 E^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{Q}{4\pi\varepsilon_0 r^2}\right)^2 = \dfrac{Q^2}{32\pi^2\varepsilon_0 r^4}.$

c) $U = \int u\,dV = 4\pi\int_R^\infty r^2 u\,dr = \dfrac{Q^2}{8\pi\varepsilon_0}\int_R^\infty \dfrac{dr}{r^2} = \dfrac{Q^2}{8\pi\varepsilon_0 R}.$

d) This energy is equal to $\frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R}$ which is just the energy required to assemble all the charge into a spherical distribution. (Note, being aware of double counting gives the factor of 1/2 in front of the familiar potential energy formula for a charge Q a distance R from another charge Q.)

e) From Equation (24.9): $U = \frac{Q^2}{2C} = \frac{Q^2}{8\pi\varepsilon_0 R}$ from part (c) $\Rightarrow C = 4\pi\varepsilon_0 R,$ as in Problem (24.67).

24.69: a) $r < R : u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{kQr}{R^3}\right)^2 = \frac{kQ^2 r^2}{8\pi R^6}.$

b) $r > R : u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{kQ}{r^2}\right)^2 = \frac{kQ^2}{8\pi r^4}.$

c) $r < R : U = \int u dV = 4\pi \int_0^R r^2 u dr = \frac{kQ^2}{2R^6}\int_0^R r^4 dr = \frac{kQ^2}{10R}.$

$r > R : U = \int u dV = 4\pi \int_R^\infty r^2 u dr = \frac{kQ^2}{2}\int_R^\infty \frac{dr}{r^2} = \frac{kQ^2}{2R} \Rightarrow U = \frac{3kQ^2}{5R}.$

24.70: a) $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{\lambda}{2\pi\varepsilon_0 r}\right)^2 = \frac{\lambda^2}{8\pi^2 \varepsilon_0 r^2}.$

b) $U = \int u dV = 2\pi L \int u r dr = \frac{L\lambda^2}{4\pi\varepsilon_0}\int_{r_a}^{r_b} \frac{dr}{r} \Rightarrow \frac{U}{L} = \frac{\lambda^2}{4\pi\varepsilon_0}\ln(r_b / r_a).$

c) Using Equation (24.9):

$$U = \frac{Q^2}{2C} = \frac{Q^2}{4\pi\varepsilon_0 L}\ln(r_b / r_a) = \frac{\lambda^2 L}{4\pi\varepsilon_0}\ln(r_b / r_a) = U \text{ of part (b).}$$

24.71: $C_{eq} = \left(\left(\frac{\varepsilon_1 A}{d/2}\right)^{-1} + \left(\frac{\varepsilon_2 A}{d/2}\right)^{-1}\right)^{-1} = \left(\left(\frac{d}{2\varepsilon_1 A}\right) + \left(\frac{d}{2\varepsilon_2 A}\right)\right)^{-1} = \left(\frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)\right)^{-1}$

$$\Rightarrow C_{eq} = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right).$$

24.72: This situation is analagous to having two capacitors in parallel, each with an area $\frac{A}{2}$. So:

$$C_{eq} = C_1 + C_2 = \frac{\varepsilon_1 A/2}{d} + \frac{\varepsilon_2 A/2}{d} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2).$$

24.73: a) $E = \frac{\sigma}{K\varepsilon_0} = \frac{0.50 \times 10^{-3}\ \text{C/m}^2}{(5.4)\varepsilon_0} = 1.0 \times 10^7\ \text{V/m}.$

b) $V = Ed = (1.0 \times 10^7\ \text{V/m})(5.0 \times 10^{-9}\ \text{m}) = 0.052\ \text{V}.$ The outside is at the higher potential.

c) volume $= 10^{-16}\ \text{m}^3 \Rightarrow R \approx 2.88 \times 10^{-6}\ \text{m}$

$\Rightarrow$ shell volume $= 4\pi R^2 d = 4\pi(2.88\times10^{-6}\ \text{m})^2(5.0\times10^{-9}\,\text{m}) = 5.2\times10^{-19}\ \text{m}^3$
$\Rightarrow U = uV = (\frac{1}{2}K\varepsilon_0 E^2)\text{V} = \frac{1}{2}(5.4)\varepsilon_0(1.0\times10^7\ \text{V/m})^2(5.2\times10^{-19}\ \text{m}^3) = 1.36\times10^{-15}\ \text{J}.$

24.74: a) $Q = CV = \dfrac{K\varepsilon_0 A}{d}V = \dfrac{(2.50)\varepsilon_0(0.200\ \text{m}^2)(3000\ \text{V})}{1.00\times10^{-2}\ \text{m}} = 1.33\times10^{-6}\ \text{C}.$

b) $Q_i = Q(1-1/K) = (1.33\times10^{-6}\ \text{C})\,(1-1/2.50) = 7.98\times10^{-7}\ \text{C}.$

c) $E = \dfrac{\sigma}{\varepsilon} = \dfrac{Q}{K\varepsilon_0 A} = \dfrac{1.33\times10^{-6}\ \text{C}}{(2.50)\,\varepsilon_0\ (0.200\ \text{m}^2)} = 3.01\times10^5\ \text{V/m}.$

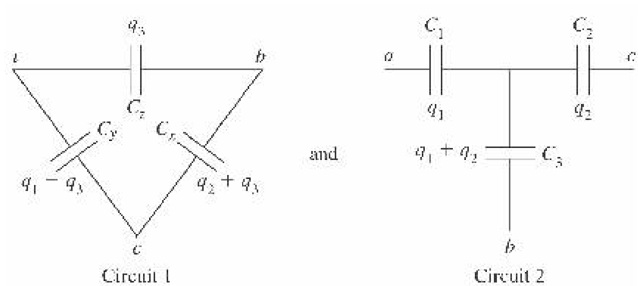
d) $U = \dfrac{1}{2}QV = \dfrac{1}{2}(1.33\times10^{-6}\ \text{C})\,(3000\ \text{V}) = 2.00\times10^{-3}\ \text{J}.$

e) $u = \dfrac{U}{Ad} = \dfrac{2.00\times10^{-3}\ \text{J}}{(0.200\ \text{m}^2)\,(0.0100\ \text{m})} = 1.00\ \text{J/m}^3 \Rightarrow$ or

$u = \frac{1}{2}K\varepsilon_0 E^2 = \frac{1}{2}(2.50)\varepsilon_0(3.01\times10^5\ \text{V/m})^2 = 1.00\ \text{J/m}^3.$

f) In this case, one does work by pushing the slab into the capacitor since the constant potential requires more charges to be brought onto the plates. When the charge is kept constant, the field pulls the dielectric into the gap, with the field (or charges) doing the work.

24.75: a) We are to show the transformation from one circuit to the other:



From Circuit 1: $V_{ac} = \dfrac{q_1 - q_3}{C_y}$ and $V_{bc} = \dfrac{q_2 + q_3}{C_x}$, where q_3 is derived from V_{ab} :

$$V_{ab} = \frac{q_3}{C_z} = \frac{q_1 - q_3}{C_y} - \frac{q_2 - q_3}{C_x} \Rightarrow q_3 = \frac{C_x C_y C_z}{C_x + C_y + C_z}\left(\frac{q_1}{C_y} - \frac{q_2}{C_x}\right) \equiv K\left(\frac{q_1}{C_y} - \frac{q_2}{C_x}\right)$$

From Circuit 2: $V_{ac} = \dfrac{q_1}{C_1} + \dfrac{q_1 + q_2}{C_3} = q_1\left(\dfrac{1}{C_1} + \dfrac{1}{C_3}\right) + q_2\dfrac{1}{C_3}$ and

$$V_{bc} = \frac{q_2}{C_2} + \frac{q_1 + q_2}{C_3} = q_1\frac{1}{C_3} + q_2\left(\frac{1}{C_2} + \frac{1}{C_3}\right).$$

Setting the coefficients of the charges equal to each other in matching potential equations from the two circuits results in three independent equations relating the two sets of capacitances. The set of equations are:

$$\frac{1}{C_1}=\frac{1}{C_y}\left(1-\frac{1}{KC_y}-\frac{1}{KC_x}\right),\ \frac{1}{C_2}=\frac{1}{C_x}\left(1-\frac{1}{KC_y}-\frac{1}{KC_x}\right)\text{and }\frac{1}{C_3}=\frac{1}{KC_yC_x}.$$

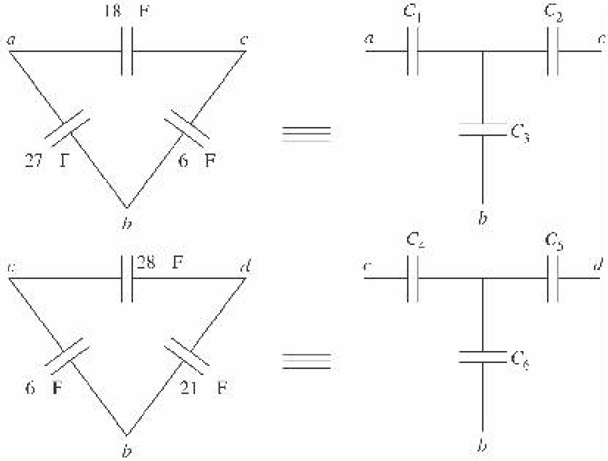
From these, subbing in the expression for K, we get:

$$C_1=(C_xC_y+C_yC_z+C_zC_x)/C_x.$$

$$C_2=(C_xC_y+C_yC_z+C_zC_x)/C_y.$$

$$C_3=(C_xC_y+C_yC_z+C_zC_x)/C_z.$$

b) Using the transformation of part (a) we have:



Where $C_1 = 126\ \mu\text{F}, C_2 = 28\ \mu\text{F}, C_3 = 42\ \mu\text{F}, C_4 = 42\ \mu\text{F}, C_5 = 147\ \mu\text{F},$ and $C_6 = 32\ \mu\text{F}$. Now the total equivalent capacitance is:

$$C_{\text{eq}} = \left(\frac{1}{72\ \mu\text{F}} + \frac{1}{126\ \mu\text{F}} + \frac{1}{34.8\ \mu\text{F}} + \frac{1}{147\ \mu\text{F}} + \frac{1}{72\ \mu\text{F}}\right)^{-1} = 14.0\ \mu\text{F},$$

where the $34.8\ \mu\text{F}$ comes from:

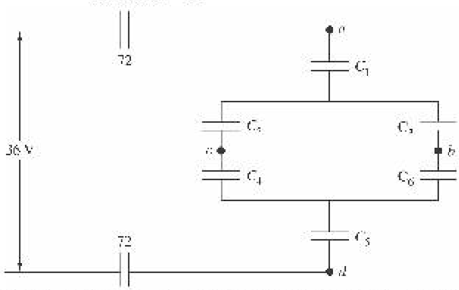
$$34.8\ \mu\text{F} = \left(\left(\frac{1}{42\ \mu\text{F}} + \frac{1}{32\ \mu\text{F}}\right)^{-1} + \left(\frac{1}{28\ \mu\text{F}} + \frac{1}{42\ \mu\text{F}}\right)^{-1}\right).$$

c) The circuit diagram can be re-drawn as shown on the next page. The overall charge is given by:

$$Q = C_{\text{eq}}V = (14.0\ \mu\text{F})\ (36\ \text{V}) \Rightarrow Q = 5.04 \times 10^{-4}\ \text{C}.$$

And this is also the charge over the $72\ \mu\text{F}$ capacitors.

$$\Rightarrow V_{72} = \frac{5.04 \times 10^{-4}\ \text{C}}{71 \times 10^{-6}\ \text{F}} = 7.0\ \text{V}.$$



Next we will find the voltage over the numbered capacitors, and their associated voltages. Then those voltages will be changed back into voltage of the original capacitors, and then their charges.

$$Q_{C_1} = Q_{C_5} = Q_{72} = 5.04 \times 10^{-4}\ \text{C}$$

$$\Rightarrow V_{C_5} = \frac{5.04\times10^{-4}\ \text{C}}{147\times10^{-6}\ \text{F}} = 3.43\ \text{V and } V_{C_1} = \frac{5.04\times10^{-4}\ \text{C}}{126\times10^{-6}\ \text{F}} = 4.00\ \text{V}.$$

$$\Rightarrow V_{C_2C_4} = V_{C_3C_6} = (36.0 - 7.00 - 7.00 - 4.00 - 3.43)\ \text{V} = 14.6\ \text{V}.$$

But $C_{eq}(C_2C_4) = \left(\frac{1}{C_2}+\frac{1}{C_4}\right)^{-1} = 16.8\ \mu\text{F}$ and $C_{eq}(C_3C_6) = \left(\frac{1}{C_3}+\frac{1}{C_6}\right)^{-1} = 18.2\ \mu\text{F}$, so:

$$Q_{C_2} = Q_{C_4} = V_{C_2C_4}C_{eq(C_2C_4)} = 2.45\times10^{-4}\ \text{C},$$

$$Q_{C_3} = Q_{C_6} = V_{C_3C_6}C_{eq(C_3C_6)} = 2.64\times10^{-4}\ \text{C}.$$

$$\Rightarrow V_{C_2} = \frac{Q_{C_2}}{C_2} = 8.8\ \text{V},\ V_{C_3} = \frac{Q_{C_3}}{C_3} = 6.3\ \text{V}, V_{C_4} = \frac{Q_{C_4}}{C_4} = 5.8\ \text{V},\ V_{C_6} = \frac{Q_{C_6}}{C_6} = 8.3\ \text{V}.$$

$$\Rightarrow V_{ac} = V_{C_1} + V_{C_2} = V_{18} = 13\ \text{V} \Rightarrow Q_{18} = C_{18}V_{18} = 2.3\times10^{-4}\ \text{C}.$$

$$V_{ab} = V_{C_1} + V_{C_3} = V_{27} = 10\ \text{V} \Rightarrow Q_{27} = C_{27}V_{27} = 2.8\times10^{-4}\ \text{C}.$$

$$V_{cd} = V_{C_4} + V_{C_5} = V_{28} = 9\ \text{V} \Rightarrow Q_{28} = C_{28}V_{28} = 2.6\times10^{-4}\ \text{C}.$$

$$V_{bd} = V_{C_5} + V_{C_6} = V_{21} = 12\ \text{V} \Rightarrow Q_{21} = C_{21}V_{21} = 2.5\times10^{-4}\ \text{C}.$$

$$V_{bc} = V_{C_3} - V_{C_2} = V_6 = 2.5\ \text{V} \Rightarrow Q_6 = C_6V_6 = 1.5\times10^{-5}\ \text{C}.$$

24.76: a) The force between the two parallel plates is:

$$F = qE = \frac{q\sigma}{2\varepsilon_0} = \frac{q^2}{2\varepsilon_0 A} = \frac{(CV)^2}{2\varepsilon_0 A} = \frac{\varepsilon_0{}^2A^2}{z^2}\frac{V^2}{2\varepsilon_0 A} = \frac{\varepsilon_0 AV^2}{2z^2}.$$

b) When $V = 0$, the separation is just z_0. So:

$$F_{4\ springs} = 4k(z_0 - z) = \frac{\varepsilon_0 AV^2}{2z^2} \Rightarrow 2z^3 - 2z^2z_0 + \frac{\varepsilon_0 AV^2}{4k} = 0.$$

c) For $A = 0.300\ \text{m}^2$, $z_0 = 1.2\times10^{-3}$ m, $k = 25$ N/m, and $V = 120$ V,

$2z^3 - (2.4\times10^{-3}\ \text{m})z^2 + 3.82\times10^{-10}\ \text{m}^3 = 0 \Rightarrow z = 0.537\ \text{mm}, 1.014\ \text{mm}.$

d) Stable equilibrium occurs if a slight displacement from equilibrium yields a force back toward the equilibrium point. If one evaluates the forces at small displacements from the equilibrium positions above, the 1.014 mm separation is seen to be stable, but not the 0.537 mm separation.

24.77: a) $C_0 = \frac{\varepsilon_0}{D}((L-x)\,L + xKL) = \frac{\varepsilon_0 L}{D}(L + (K-1)x).$

b) $\Delta U = \frac{1}{2}(\Delta C)V^2$ where $C = C_0 + \frac{\varepsilon_0 L}{D}(-dx + dxK)$

$$\Rightarrow \Delta U = \frac{1}{2}\left(\frac{\varepsilon_0 L\,dx}{D}(K-1)\right)V^2 = \frac{(K-1)\varepsilon_0 V^2 L}{2D}dx.$$

c) If the charge is kept constant on the plates, then:

$$Q = \frac{\varepsilon_0 LV}{D}(L + (K-1)x), \text{ and } U = \frac{1}{2}CV^2 = \frac{1}{2}C_0V^2\left(\frac{C}{C_0}\right)$$

$$\Rightarrow U \approx \frac{C_0V^2}{2}\left(1 - \frac{\varepsilon_0 L}{DC_0}(K-1)dx\right) \Rightarrow \Delta U = U - U_0 = -\frac{(K-1)\varepsilon_0 V^2 L}{2D}dx.$$

d) Since $dU = -Fdx = -\frac{(K-1)\varepsilon_0 V^2 L}{2D}dx$, then the force is in the opposite direction to the motion dx, meaning that the slab feels a force pushing it out.

24.78: a) For a normal spherical capacitor: $C_0 = 4\pi\varepsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right)$ Here we have, in effect, two parallel capacitors, C_L and C_U.

$$C_L = \frac{KC_0}{2} = 2\pi K\varepsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right) \text{and } C_U = \frac{C_0}{2} = 2\pi\varepsilon_0\left(\frac{r_a r_b}{r_b - r_a}\right).$$

b) Using a hemispherical Gaussian surface for each respective half:

$$E_L\frac{4\pi r^2}{2} = \frac{Q_L}{K\varepsilon_0} \Rightarrow E_L = \frac{Q_L}{2\pi K\varepsilon_0 r^2} \text{ and } E_U\frac{4\pi r^2}{2} = \frac{Q_U}{\varepsilon_0} \Rightarrow E_U = \frac{Q_U}{2\pi\varepsilon_0 r^2}.$$

But $Q_L = VC_L$ and $Q_U = VC_U$, $Q_L + Q_U = Q$.

So: $Q_L = \frac{VC_0K}{2} = KQ_U \Rightarrow Q_U(1+K) = Q \Rightarrow Q_U = \frac{Q}{1+K}$ and $Q_L = \frac{KQ}{1+K}$.

$$\Rightarrow E_L = \frac{KQ}{1+K}\frac{1}{2\pi K\varepsilon_0 r^2} = \frac{2}{1+K}\frac{Q}{4\pi\varepsilon_0 r^2} \text{ and } E_U = \frac{Q}{1+K}\frac{1}{2\pi K\varepsilon_0 r^2} = \frac{2}{1+K}\frac{Q}{4\pi K\varepsilon_0 r^2}.$$

c) The free charge density on upper and lower hemispheres are:

$$(\sigma_{f_{r_a}})_U = \frac{Q_U}{4\pi r_a^2} = \frac{Q}{4\pi r_a^2(1+K)} \text{ and } (\sigma_{f_{r_a}})_U = \frac{Q_u}{4\pi r_b^2} = \frac{Q}{4\pi r_b^2(1+K)}.$$

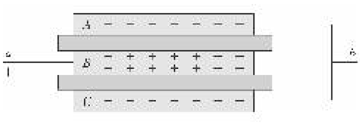
$$(\sigma_{f_{r_a}})_L = \frac{Q_L}{4\pi r_a^2} = \frac{KQ}{4\pi r_a^2(1+K)} \text{ and } (\sigma_{f_{r_a}})_L = \frac{Q_L}{4\pi r_b^2} = \frac{KQ}{4\pi r_b^2(1+K)}.$$

d) $\sigma_{i_{r_a}} = \sigma_{f_{r_a}}(1 - 1/K) = \frac{(K-1)}{K}\frac{Q}{4\pi r_a^2}\frac{K}{K+1} = \frac{K-1}{K+1}\frac{Q}{4\pi r_a^2}.$

$\sigma_{i_{r_b}} = \sigma_{f_{r_b}}(1 - 1/K) = \frac{(K-1)}{K}\frac{Q}{4\pi r_a^2}\frac{K}{K+1} = \frac{K-1}{K+1}\frac{Q}{4\pi r_b^2}.$

e) There is zero bound charge on the flat surface of the dielectric-air interface, or else that would imply a circumferential electric field, or that the electric field changed as we went around the sphere.

24.79: a)



b) $C = 2\left(\dfrac{\varepsilon A}{d}\right) = \dfrac{2(4.2)\varepsilon_0(0.120\text{ m})^2}{4.5\times 10^{-4}\text{ m}} \Rightarrow C = 2.38\times 10^{-9}\text{ F}.$

24.80: a) The capacitors are in parallel so:

$$C = \frac{\varepsilon_{eff} WL}{d} = \frac{\varepsilon_0 W(L-h)}{d} + \frac{K\varepsilon_0 Wh}{d} = \frac{\varepsilon_0 WL}{d}\left(1 + \frac{Kh}{L} - \frac{h}{L}\right) \Rightarrow K_{eff}$$
$$= \left(1 + \frac{Kh}{L} - \frac{h}{L}\right).$$

b) For gasoline, with $K = 1.95$:

$\frac{1}{4}$ full: $K_{eff}\left(h = \frac{L}{4}\right) = 1.24$; $\frac{1}{2}$ full: $K_{eff}\left(h = \frac{L}{2}\right) = 1.48$;

$\frac{3}{4}$ full: $K_{eff}\left(h = \frac{3L}{4}\right) = 1.71.$

c) For methanol, with $K = 33$:

$\frac{1}{4}$ full: $K_{eff}\left(h = \frac{L}{4}\right) = 9$; $\frac{1}{2}$ full: $K_{eff}\left(h = \frac{L}{2}\right) = 17$;

$\frac{3}{4}$ full: $K_{eff}\left(h = \frac{3L}{4}\right) = 25.$

d) This kind of fuel tank sensor will work best for methanol since it has the greater range of K_{eff} values.

25.1: $Q = It = (3.6\text{ A})(3)(3600\text{ s}) = 3.89\times 10^4\text{ C}.$

25.2: a) Current is given by $I = \frac{Q}{t} = \frac{420\text{ C}}{80(60\text{ s})} = 8.75\times 10^{-2}\text{ A}.$

b) $I = nqv_d A$

$$\Rightarrow v_d = \frac{I}{nqA} = \frac{8.75\times 10^{-2}\text{ A}}{(5.8\times 10^{28})(1.6\times 10^{-19}\text{ C})(\pi(1.3\times 10^{-3}\text{ m})^2)}$$
$$= 1.78\times 10^{-6}\text{ m/s}.$$

25.3: a) $v_d = \dfrac{I}{nqA} = \dfrac{4.85\text{ A}}{(8.5\times 10^{28})(1.6\times 10^{-19}\text{ C})(\pi/4)(2.05\times 10^{-3}\text{ m})^2}$

$= 1.08\times 10^{-4}\text{ m/s}$

$\Rightarrow$ travel time $= \frac{d}{v_d} = \frac{0.71\text{ m}}{1.08\times 10^{-4}\text{ m/s}} = 6574\text{ s} = 110\text{ min}$

b) If the diameter is now 4.12 mm, the time can be calculated using the formula above or comparing the ratio of the areas, and yields a time of 26542 s =442 min.

c) The drift velocity depends on the diameter of the wire as an inverse square relationship.

25.4: The cross-sectional area of the wire is

$$A = \pi r^2 = \pi(2.06 \times 10^{-3}\ \text{m})^2 = 1.333 \times 10^{-5}\ \text{m}^2.$$

The current density is

$$J = \frac{I}{A} = \frac{8.00\text{A}}{1.333 \times 10^{-5}\ \text{m}^2} = 6.00 \times 10^5\ \text{A/m}^2$$

We have $v_d = J/ne$; Therefore

$$n = \frac{J}{v_d e} = \frac{6.00 \times 10^5\ \text{A/m}^2}{(5.40 \times 10^{-5}\ \text{m/s})(1.60 \times 10^{-19}\ \text{C/electron})} = 6.94 \times 10^{28}\ \frac{\text{electrons}}{\text{m}^3}$$

25.5: $J = n|q|v_\text{d}$, so J/v_d is constant.

$J_1/v_{d1} = J_2/v_{d2}$,

$v_{d2} = v_{d1}(J_2/J_1) = v_{d1}(I_2/I_1) = (1.20 \times 10^{-4}\ \text{m/s})(6.00/1.20) = 6.00 \times 10^{-4}\ \text{m/s}$

25.6: The atomic weight of copper is $63.55\ \text{g/mole}$, and its density is $8.96\ \text{g/cm}^3$. The number of copper atoms in $1.00\ \text{m}^3$ is thus

$$\frac{(8.96\ \text{g/cm}^3)(1.00 \times 10^6\ \text{cm}^3/\text{m}^3)(6.023 \times 10^{23}\ \text{atoms/mole})}{63.55\ \text{g/mole}}$$

$$= 8.49 \times 10^{28}\ \text{atoms/m}^3$$

Since there are the same number of free electrons/m^3 as there are atoms of copper/m^3 (see Ex. 25.1), The number of free electrons per copper atom is one.

25.7: Consider $1\ \text{m}^3$ of silver.

density $= 10.5 \times 10^3\ \text{kg/m}^3$, so $m = 10.5 \times 10^3\ \text{kg}$

$M = 107.868 \times 10^{-3}\ \text{kg/mol}$, so $n = m/M = 9.734 \times 10^4$ mol and

$N = nN_\text{A} = 5.86 \times 10^{28}\ \text{atoms/m}^3$

If there is one free electron per m^3, there are 5.86×10^{28} free electrons/m^3. This agrees with the value given in Exercise 25.2.

25.8: a) $Q_{total} = (n_\text{Cl} + n_\text{Na})e = (3.92 \times 10^{16} + 2.68 \times 10^{16})(1.60 \times 10^{-19}\ \text{C}) = 0.0106\ \text{C}$

$$\Rightarrow I = \frac{Q_{total}}{t} = \frac{0.0106\ \text{C}}{1.00\ \text{s}} = 0.0106\text{A} = 10.6\ \text{mA}.$$

b) Current flows, by convention, in the direction of positive charge. Thus, current flows with Na^+ toward the negative electrode.

25.9: a) $Q = \int_0^8 I\,dt = \int_0^8 (55 - 0.65\,t^2)\,dt = 55t\Big|_0^8 + \frac{0.65}{3}t^3\Big|_0^8 = 329\ \mathrm{C}.$

b) The same charge would flow in 10 seconds if there was a constant current of: $I = Q/t = (329\ \mathrm{C})/(8\ \mathrm{s}) = 41.1\ \mathrm{A}.$

25.10: a) $J = \frac{I}{A} = \frac{3.6\ \mathrm{A}}{(2.3\times10^{-3}\ \mathrm{m})^2} = 6.81\times10^5\ \mathrm{A/m^2}.$

b) $E = \rho J = (1.72\times10^{-8}\ \Omega\cdot\mathrm{m})(6.81\times10^5\ \mathrm{A/m^2}) = 0.012\ \mathrm{V/m}.$

c) Time to travel the wire's length:

$$t = \frac{l}{v_d} = \frac{l\,nqA}{I} = \frac{(4.0\ \mathrm{m})(8.5\times10^{28}/\mathrm{m^3})(1.6\times10^{-19}\ \mathrm{C})(2.3\times10^{-3}\ \mathrm{m})^2}{3.6\ \mathrm{A}} = 8.0\times10^4\ \mathrm{s}$$

$= 1333\ \mathrm{min} \approx 22\ \mathrm{hrs!}$

25.11: $R = \frac{\rho L}{A} = \frac{(1.72\times10^{-8}\ \Omega\cdot\mathrm{m})(24.0\ \mathrm{m})}{(\pi/4)(2.05\times10^{-3}\ \mathrm{m})^2} = 0.125\ \Omega$

25.12: $R = \frac{\rho L}{A} \Rightarrow L = \frac{RA}{\rho} = \frac{(1.00\ \Omega)(\pi/4)(0.462\times10^{-3}\ \mathrm{m})^2}{1.72\times10^{-8}\ \Omega\cdot\mathrm{m}} = 9.75\ \mathrm{m}.$

25.13: a) tungsten:

$$E = \rho J = \frac{\rho I}{A} = \frac{(5.25\times10^{-8}\ \Omega/\mathrm{m^3})(0.820\ \mathrm{A})}{(\pi/4)(3.26\times10^{-3}\ \mathrm{m})^2} = 5.16\times10^{-3}\ \mathrm{V/m}.$$

b) aluminum:

$$E = \rho J = \frac{\rho I}{A} = \frac{(2.75\times10^{-8}\ \Omega/\mathrm{m^3})(0.820\ \mathrm{A})}{(\pi/4)(3.26\times10^{-3}\ \mathrm{m})^2} = 2.70\times10^{-3}\ \mathrm{V/m}.$$

25.14: $R_{Al} = R_{Cu} \Rightarrow \frac{\rho_{Al}L}{A_{Al}} = \frac{\rho_{Cu}L}{A_{Cu}} \Rightarrow \frac{\pi d_{Al}^{\ 2}}{4\rho_{Al}} = \frac{\pi d_{Cu}^{\ 2}}{4\rho_{Cu}} \Rightarrow d_{Cu} = d_{Al}\sqrt{\frac{\rho_{Cu}}{\rho_{Al}}}$

$$\Rightarrow d_{Al} = (3.26\ \mathrm{mm})\sqrt{\frac{1.72\times10^{-8}\ \Omega\cdot\mathrm{m}}{2.75\times10^{-8}\ \Omega\cdot\mathrm{m}}} = 2.6\ \mathrm{mm}$$

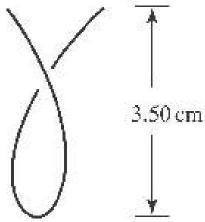
25.15: Find the volume of one of the wires:

$R = \frac{\rho L}{A}$ so $A = \frac{\rho L}{R}$ and

$$\text{volume} = AL = \frac{\rho L^2}{R} = \frac{1.72\times10^{-8}\ \mathrm{Ohm\cdot m})(3.50\mathrm{m})^2}{0.125\mathrm{Ohm}} = 1.686\times10^{-6}\,\mathrm{mcb}$$

$$m = (\text{density})V = (8.9\times10^{3}\ \text{kg/m}^{3})(1.686\times10^{-6}\ \text{m}^{3}) = 15\ \text{g}$$

25.16:



$$r_1 = \frac{3.5\ \text{cm}}{2} = 1.75\ \text{cm}$$

$$r_2 = \frac{3.25\ \text{mm}}{2} = 1.625\ \text{mm}$$

$$R = \rho \frac{l}{A}$$

$$l = 2\pi r_1 (\text{per coil}) \times 75 \text{ coils}$$

$$A = \pi r_2^2$$

$$\rho = \frac{RA}{l} = \frac{R\pi r_2^2}{(2\pi r_1)75} = \frac{R r_2^2}{150 r_1}$$

$$= \frac{(1.74\ \Omega)(1.625 \times 10^{-3}\ \text{m})^2}{150(1.75 \times 10^{-2}\ \text{m})}$$

$$= 1.75 \times 10^{-6}\ \Omega \cdot \text{m}$$

25.17: a) From Example 25.1, an 18-gauge wire has $A = 8.17 \times 10^{-3}\ \text{cm}^2$

$I = JA = (1.0 \times 10^5\ \text{A/cm}^2)(8.17 \times 10^{-3}\ \text{cm}^2) = 820\ \text{A}$

b) $A = I/J = (1000\ \text{A})/(1.0 \times 10^6\ \text{A/cm}^2) = 1.0 \times 10^{-3}\ \text{cm}^2$

$A = \pi r^2$ so $r = \sqrt{A/\pi} = \sqrt{(1.0 \times 10^{-3}\ \text{cm}^2/\pi} = 0.0178\ \text{cm}$

$d = 2r = 0.36\ \text{mm}$

25.18: Assuming linear variation of the resistivity with temperature:

$$\rho = \rho_0[1 + \alpha(T - T_0)]$$

$$= \rho_0[1 + (4.5 \times 10^{-3}/°\text{C})(320 - 20)°\text{C}]$$

$$= 2.35\rho_0$$

Since $\rho = E/J$, the electric field required to maintain a given current density is proportional to the resistivity. Thus $E = (2.35)(0.0560\ \text{V/m}) = 0.132\ \text{V/m}$

25.19: $R = \frac{\rho L}{A} = \frac{\rho L}{L^2} = \frac{\rho}{L} = \frac{2.75 \times 10^{-8} \Omega \cdot \text{m}}{1.80\text{m}} = 1.53 \times 10^{-8} \Omega$

25.20: The ratio of the current at $20°\text{C}$ to that at the higher temperature is $(0.860\ \text{A})/(0.220\ \text{A}) = 3.909.$ Since the current density for a given field is inversely proportional to $\rho (\rho = E/J)$, The resistivity must be a factor of 3.909 higher at the higher temperature.

$$\frac{\rho}{\rho_0} = 1 + \alpha (T - T_0)$$

$$T = T_0 + \frac{\frac{\rho}{\rho_0} - 1}{\alpha} = 20°\text{C} + \frac{3.909 - 1}{4.5 \times 10^{-3}/°\text{C}} = 666°\text{C}$$

25.21: $R = \frac{V}{I} = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} \Rightarrow r = \sqrt{\frac{I \rho L}{\pi V}} = \sqrt{\frac{(6.00\ \text{A})(2.75 \times 10^{-8}\ \Omega \cdot \text{m})(1.20\ \text{m})}{\pi (1.50\ \text{V})}}$

$= 2.05 \times 10^{-4}\ \text{m}.$

25.22: $\rho = \frac{RA}{L} = \frac{VA}{IL} = \frac{(4.50\ \text{V})\pi (6.54 \times 10^{-4}\ \text{m})^2}{(17.6\ \text{A})(2.50\ \text{m})} = 1.37 \times 10^{-7}\ \Omega \cdot \text{m}.$

25.23: a) $I = JA = \frac{EA}{\rho} = \frac{(0.49\ \text{V/m})(\pi/4\,(0.84 \times 10^{-3}\ \text{m})^2)}{(2.44 \times 10^{-8}\ \Omega \cdot \text{m})} = 11.1\ \text{A}.$

b) $V = IR = \frac{I \rho L}{A} = \frac{(11.1\ \text{A})(2.44 \times 10^{-8}\ \Omega \cdot \text{m})(6.4\ \text{m})}{(\pi/4)(0.84 \times 10^{-3}\ \text{m})^2} = 3.13\ \text{V}.$

c) $R = \frac{V}{I} = \frac{3.13\ \text{V}}{11.1\text{A}} = 0.28\ \Omega.$

25.24: Because the density does not change, volume stays the same, so $LA = (2L)(A/2)$ and the area is halved. So the resistance becomes:

$$R = \frac{\rho (2L)}{A/2} = 4 \frac{\rho L}{A} = 4R_0.$$

That is, four times the original resistance.

25.25: a) $E = \rho J = \dfrac{RAJ}{L} = \dfrac{RI}{L} = \dfrac{V}{L} = \dfrac{0.938\text{ V}}{0.75\text{ m}} = 1.25\text{ V/m}.$

b) $\rho = \dfrac{RA}{L} = \dfrac{V}{JL} = \dfrac{0.938\text{ V}}{(4.40 \times 10^{7}\text{ A/m}^2)(0.75\text{ m})} = 2.84 \times 10^{-8}\ \Omega \cdot \text{m}.$

25.26: $\dfrac{R-R_0}{R_0}=\alpha(T_f-T_i)$

$$\Rightarrow \alpha=\frac{R-R_0}{(T_f-T_i)R_0}=\frac{1.512\ \Omega-1.484\ \Omega}{(34.0^\circ\text{C}-20.0^\circ\text{C})(1.484\ \Omega)}=1.35\times10^{-3}\ {}^\circ\text{C}^{-1}.$$

25.27: a) $R_f-R_i=R_i\alpha(T_f-T_i)\Rightarrow R_f=100\ \Omega-100\ \Omega(0.0004^\circ\text{C}^{-1})(11.5^\circ\text{C})=99.54\ \Omega.$

b) $R_f-R_i=R_i\alpha(T_f-T_i)\Rightarrow R_f=0.0160\ \Omega+0.0160\ \Omega(-0.0005^\circ\text{C}^{-1})(25.8^\circ\text{C})=$ $0.0158\ \Omega.$

25.28: $T_f-T_i=\dfrac{R_f-R_i}{\alpha R_i};\quad T_f=T_i+\dfrac{R_f-R_i}{\alpha R_i}$

$$=\frac{215.8\ \Omega-217.3\ \Omega}{(-0.0005^\circ\ \text{C}^{-1})(217.3\ \Omega)}+4^\circ\text{C}=17.8^\circ\ \text{C}.$$

25.29: a) If 120 strands of wire are placed side by side, we are effectively increasing the area of the current carrier by 120. So the resistance is smaller by that factor:

$$R=5.60\times10^{-6}\ \Omega/120=4.67\times10^{-8}\ \Omega.$$

b) If 120 strands of wire are placed end to end, we are effectively increasing the length of the wire by 120, and so $R=(5.60\times10^{-6}\ \Omega)120=6.72\times10^{-4}\ \Omega.$

25.30: With the $4.0\ \Omega$ load, where r = internal resistance

$$12.6\text{ V}=(r+4.0\ \Omega)I$$

Change in terminal voltage:

$$\Delta V_T=rI=12.6\text{ V}-10.4\text{ V}=2.2\text{ V}$$

$$I=\frac{2.2\text{ V}}{r}$$

Substitute for I: $\quad 12.6\text{ V}=(r+4.0\ \Omega)\left(\dfrac{2.2\text{ V}}{r}\right)$

Solve for r: $\quad r=0.846\ \Omega$

25.31: a) $R=\dfrac{\rho L}{A}=\dfrac{1.72\times10^{-8}\ \Omega\text{m})(100\times10^3\text{m})}{\pi(0.050\text{m})^2}=0.219\Omega$

$V=IR=(125\text{A})(0.219\Omega)=27.4\text{V}$

b) $P=VI=(27.4\text{ V})(125\text{ A})=3422\text{ W}=3422\text{ J/s}$

Energy $=Pt=(3422\text{ J/s})(3600\text{ s})=1.23\times10^7\text{ J}$

25.32: a) $V_r = \mathcal{E} - V_{ab} = 24.0\ \text{V} - 21.2\ \text{V} = 2.8\ \text{V} \Rightarrow r = 2.8\ \text{V}/4.00\ \text{A} = 0.700\ \Omega$.

b) $V_R = 21.2\ \text{V} \Rightarrow R = 21.2\ \text{V}/4.00\ \text{A} = 5.30\ \Omega$

25.33: a) An ideal voltmeter has infinite resistance, so there would be NO current through the $2.0\ \Omega$ resistor.

b) $V_{ab} = \mathcal{E} = 5.0\ \text{V}$; since there is no current there is no voltage lost over the internal resistance.

c) The voltmeter reading is therefore 5.0 V since with no current flowing, it measures the terminal voltage of the battery.

25.34: a) A voltmeter placed over the battery terminals reads the emf: $\mathcal{E} = 24.0\ \text{V}$.

b) There is no current flowing, so $V_r = 0$.

c) The voltage reading over the switch is that over the battery: $V_s = 24.0\ \text{V}$.

d) Having closed the switch:

$I = 24.0\ \text{V}/5.88\ \Omega = 4.08\ \text{A} \Rightarrow V_{ab} = 24.0\ \text{V} - (4.08\ \text{A})(0.28\ \Omega) = 22.9\ \text{V}$.

$V_r = IR = (4.08\ \text{A})(5.60\ \Omega) = 22.9\ \text{V}$.

$V_s = 0$, since all the voltage has been "used up" in the circuit. The resistance of the switch is zero so $V_s = IR = 0$.

25.35: a) When there is no current flowing, the voltmeter reading is simply the emf of the battery: $\mathcal{E} = 3.08\ \text{V}$.

b) The voltage over the internal resistance is:

$$V_r = 3.08\ \text{V} - 2.97\ \text{V} = 0.11\ \text{V} \Rightarrow r = \frac{V}{I} = \frac{0.11\ \text{V}}{1.65\ \text{A}} = 0.067\ \Omega$$

c) $V_R = 2.97\ \text{V} = (1.65\ \text{A})R$

$$R = \frac{2.97\ \text{V}}{1.65\ \text{A}} = 1.8\ \Omega$$

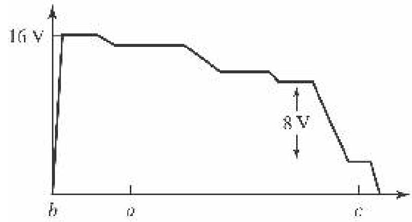
25.36: a) The current is counterclockwise, because the 16 V battery determines the direction of current flow. Its magnitude is given by:

$$I = \frac{\sum \mathcal{E}}{\sum R} = \frac{16.0\ \text{V} - 8.0\ \text{V}}{1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega} = 0.47\ \text{A}.$$

b) $V_{ab} = 16.0\ \text{V} - (1.6\ \Omega)(0.47\ \text{A}) = 15.2\ \text{V}$.

c) $V_{ac} = (5.0\ \Omega)(0.47\ \text{A}) + (1.4\ \Omega)(0.47\ \text{A}) + 8.0\ \text{V} = 11.0\ \text{V}$.

d)


16 V
8 V
b
a
c

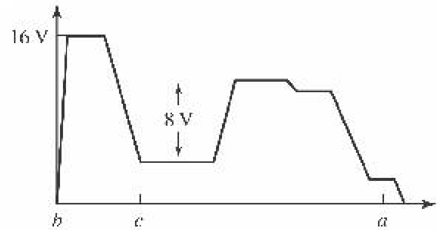
25.37: a) Now the current flows clockwise since both batteries point in that direction:

$$I = \frac{\sum \mathcal{E}}{\sum R} = \frac{16.0\ \text{V} + 8.0\ \text{V}}{1.6\ \Omega + 5.0\ \Omega + 1.4\ \Omega + 9.0\ \Omega} = 1.41\ \text{A}.$$

b) $V_{ab} = -16.0\ \text{V} + (1.6\ \Omega)(1.41\ \text{A}) = -13.7\ \text{V}.$

c) $V_{ac} = -(5.0\ \Omega)(1.41\ \text{A}) - (1.4\ \Omega)(1.41\ \text{A}) + 8.0\ \text{V} = -1.0\ \text{V}.$

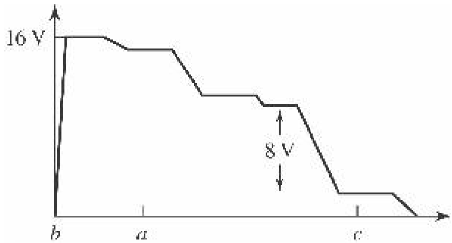
d)



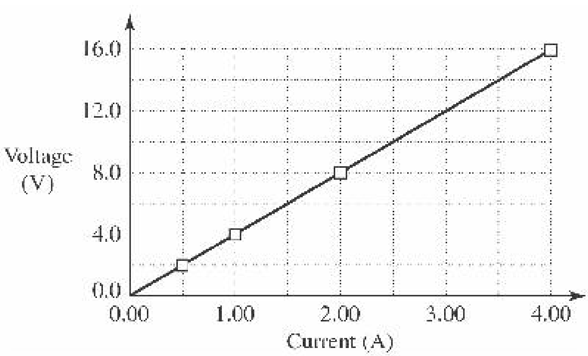
25.38: a) $V_{bc} = 1.9\ \text{V} \Rightarrow I = V_{bc} / R_{bc} = 1.9\ \text{V} / 9.0\ \ \Omega = 0.21\ \text{A}.$

b) $\sum \mathcal{E} = \sum IR \Rightarrow 8.0\ \text{V} = ((1.6 + 9.0 + 1.4 + R)\Omega)(0.21\ \text{A}) \Rightarrow R = \dfrac{5.48}{0.21} = 26.1\ \Omega$

c)



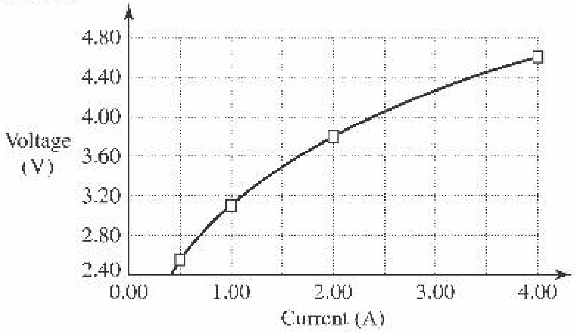
25.39: a) Nichrome wire:



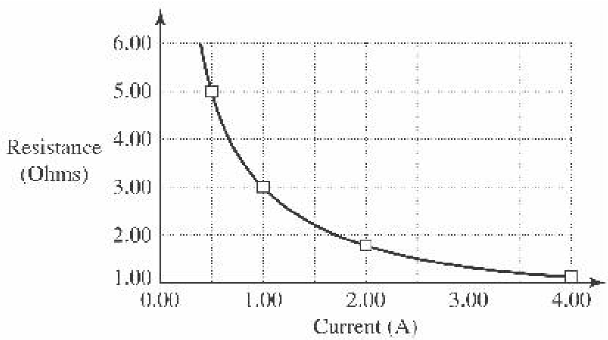
b) The Nichrome wire does obey Ohm's Law since it is a straight line.

c) The resistance is the voltage divided by current which is $3.88\ \Omega$.

25.40: a) Thyrite resistor:



b) The Thyrite is non-Ohmic since the plot is curved.
c) Calculating the resistance at each point by voltage divided by current:



25.41: a) $r = \mathcal{E}/I = 1.50\ \text{V}/14.8\ \text{A} = 0.101\ \Omega$.
b) $r = \mathcal{E}/I = 1.50\ \text{V}/6.8\ \text{A} = 0.22\ \Omega$.
c) $r = \mathcal{E}/I = 12.6\ \text{V}/1000\ \text{A} = 0.0126\ \Omega$.

25.42: a) $P = V^2/R \Rightarrow R = V^2/P = (15\ \text{V})^2/327\ \text{W} = 0.688\ \Omega$.
b) $V = IR \Rightarrow I = \dfrac{V}{R} = \dfrac{15\ \text{V}}{0.688\ \Omega} = 21.8\ \text{A}$.

25.43: $P = VI = (650\ \text{V})(0.80\ \text{A}) = 520\ \text{W}$.

25.44: $W = Pt = IVt = (0.13\ \text{A})(9\ \text{V})(1.5)(3600\ \text{s}) = 6318\ \text{J}$.

25.45: a) $P = I^2R \Rightarrow p = \dfrac{P}{\text{vol}} = \dfrac{I^2R}{AL} = \dfrac{J^2A^2R}{AL} = \dfrac{J^2A(\rho L/A)}{L} = J^2\rho \Rightarrow p = JE$ since $E = \rho J$.

b) From (a) $p = J^2\rho$.

c) Since $J = E/\rho$, (a) becomes $p = E^2/\rho$.

25.46: a) $I = \sum \mathcal{E}/R_{total} = 8.0\ \text{V}/17\ \Omega = 0.47\ \text{A} \Rightarrow P_{5\Omega} = I^2R = (0.47\ \text{A})^2(5.0\ \Omega) =$ 1.1 W and $P_{9\Omega} = I^2R = (0.47\ \text{A})^2(9.0\ \Omega) = 2.0\ \text{W}$.

b) $P_{16V} = \mathcal{E}I - I^2r = (16\ \text{V})(0.47\ \text{A}) - (0.47\ \text{A})^2(1.6\ \Omega) = 7.2\ \text{W}$.

c) $P_{8V} = \mathcal{E}I + Ir^2 = (8.0\ \text{V})(0.47\ \text{A}) + (0.47\ \text{A})^2(1.4\Omega) = 4.1\ \text{W}$.

d) (b) = (a) + (c)

25.47: a) $W = Pt = IVt = (60\ \text{A})(12\ \text{V})(3600\ \text{s}) = 2.59 \times 10^6\ \text{J}$.

b) To release this much energy we need a volume of gasoline given by:

$$m = \frac{2.59 \times 10^6\ \text{J}}{46{,}000\ \text{J/g}} = 56.0\ \text{g} \Rightarrow \text{vol} = \frac{m}{\rho} = \frac{0.056\ \text{kg}}{900\ \text{kg/m}^3} = 6.22 \times 10^{-5}\ \text{m}^3 = 0.062\ \text{liters.}$$

c) To recharge the battery:

$$t = (Wh)/P = (720\ \text{Wh})/(450\ \text{W}) = 1.6\ \text{h}.$$

25.48: a) $I = \mathcal{E}/(R+r) = 12\ \text{V}/10\ \Omega = 1.2\ \text{A} \Rightarrow P = \mathcal{E}I = (12\ \text{V})(1.2\ \text{A}) = 14.4\ \text{W}$. This is less than the previous value of 24 W.

b) The work dissipated in the battery is just: $P = I^2r = (1.2\ A)^2(2.0\ \Omega) = 2.9\ \text{W}$. This is less than 8 W, the amount found in Example (25.9).

c) The net power output of the battery is $14.4\ \text{W} - 2.9\ \text{W} = 11.5\ \text{W}$. This is less than 16 W, the amount found in Example (25.9).

25.49: a) $I = V/R = 12\ \text{V}/6\ \Omega = 2.0\ \text{A} \Rightarrow P = \mathcal{E}I = (12\ \text{V})(2.0\ \text{A}) = 24\ \text{W}$.

b) The power dissipated in the battery is $P = I^2r = (2.0\ \text{A})^2(1.0\ \Omega) = 4.0\ \text{W}$.

c) The power delivered is then $24\ \text{W} - 4\ \text{W} = 20\ \text{W}$.

25.50: a) $I = \sum \mathcal{E}/R = 3.0\ \text{V}/17\ \Omega = 0.18\ \text{A} \Rightarrow P = I^2R = 0.529\ \text{W}$.

b) $W = Pt = IVt = (0.18\ \text{A})(3.0\ \text{V})(5.0)(3600\ \text{s}) = 9530\ \text{J}$.

c) Now if the power to the bulb is 0.27 W,

$$P = I^2R \Rightarrow 0.27\ \text{W} = \left(\frac{3.0\ \text{V}}{17\ \Omega + R}\right)^2 (17\ \Omega) \Rightarrow (17\ \Omega + R)^2 = 567\ \Omega^2 \Rightarrow R = 6.8\ \Omega.$$

25.51: a) $P = V^2/R \Rightarrow R = V^2/P = (120\ \text{V})^2/540\ \text{W} = 26.7\ \Omega$.

b) $I = V/R = 120\ \text{V}/26.7\ \Omega = 4.5\ \text{A}$.

c) If the voltage is just 110 V, then $I = 4.13\ \text{A} \Rightarrow P = VI = 454\ \text{W}$.

d) Greater. The resistance will be less so the current drawn will increase, increasing the power.

25.52: From Eq. (25.24), $\rho = \dfrac{m}{ne^2\tau}$.

$$\Rightarrow \tau = \frac{m}{ne^2\rho} = \frac{9.11\times10^{-31}\ \text{kg}}{(1.0\times10^{16}\ \text{m}^{-3})\,(1.60\times10^{-19}\ \text{C})^2\ (2300\ \Omega\cdot\text{m})} = 1.55\times10^{-12}\ \text{s}.$$

b) The number of free electrons in copper $(8.5\times10^{28}\ \text{m}^{-3})$ is much larger than in pure silicon $(1.0\times10^{16}\ \text{m}^{-3})$.

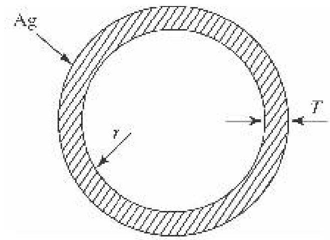
25.53: a) $\rho = \dfrac{RA}{L} = \dfrac{(0.104\ \Omega)\,(\pi/4)\,(2.50\times10^{-3}\text{m})^2}{14.0\ \text{m}} = 3.65\times10^{-8}\ \Omega\cdot\text{m}.$

b) $I = JA = \dfrac{EA}{\rho} = \dfrac{(1.28\ \text{V/m})\,(\pi/4)\,(2.50\times10^{-3}\ \text{m})^2}{3.65\times10^{-8}\ \Omega\cdot\text{m}} = 172\ \text{A}.$

c) $v_d = \dfrac{J}{nq} = \dfrac{E}{\rho nq} = \dfrac{1.28\ \text{V/m}}{(3.65\times10^{-8}\ \Omega\cdot\text{m})\,(8.5\times10^{28}\ \text{m}^{-3})\,(1.6\times10^{-19}\ \text{C})}$

$= 2.58\times10^{-3}\ \text{m/s}.$

25.54: r = 2.00 cm
T = 0.100 mm



$$I = \frac{V}{R} = \frac{V}{\rho l/A} = \frac{VA}{\rho l} = \frac{V(2\pi rT)}{\rho l}$$

$$= \frac{(12\ \text{V})\,(2\pi)(2.00\times10^{-2}\ \text{m})\,(0.100\times10^{-3}\ \text{m})}{(1.47\times10^{-8}\ \Omega\cdot\text{m})\,(25.0\ \text{m})}$$

$= 410\ \text{A}$

25.55: With the voltmeter connected across the terminals of the battery there is no current through the battery and the voltmeter reading is the battery emf; $\varepsilon = 12.6\ \text{V}$.

With a wire of resistance R connected to the battery current I flows and $\varepsilon - Ir - IR = 0$

Call the resistance of the 20.0-m piece R_1; then the resistance of the 40.0-m piece is $R_2 = 2R_1$.

$\varepsilon - I_1 r - I_1 R_1 = 0; \qquad 12.6\text{ V} - (7.00\text{ A})r - (7.00\text{ A})R_1 = 0$
$\varepsilon - I_2 r - I_2(2R_1) = 0;\ 12.6\text{ V} - (4.20\text{ A})r - (4.20\text{ A})(2R_1) = 0$

Solving these two equations in two unknowns gives $R_1 = 1.20\Omega$. This is the resistance of 20.0 m, so the resistance of one meter is $[1.20\Omega/(20.0\text{m})](1.00\text{m}) = 0.060\Omega$

25.56: a) $I = \dfrac{V}{R} = \dfrac{V}{R_{Cu} + R_{Ag}}$

and

$$R_{Cu} = \frac{\rho_{Cu} L_{Cu}}{A_{Cu}} = \frac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})\ (0.8\text{ m})}{(\pi/4)\ (6.0 \times 10^{-4}\text{m})^2} = 0.049\ \Omega,$$

and

$$R_{Ag} = \frac{\rho_{Ag} L_{Ag}}{A_{Ag}} = \frac{(1.47 \times 10^{-8}\ \Omega \cdot \text{m})\ (1.2\text{ m})}{(\pi/4)\ (6.0 \times 10^{-4}\text{ m})^2} = 0.062\ \Omega$$

$$\Rightarrow I = \frac{5.0\text{ V}}{0.049\ \Omega + 0.062\ \Omega} = 45\text{ A}.$$

So the current in the copper wire is 45 A.

b) The current in the silver wire is 45 A, the same as that in the copper wire or else charge would build up at their interface.

c) $E_{Cu} = J\rho_{Cu} = \dfrac{IR_{Cu}}{L_{Cu}} = \dfrac{(45\text{ A})\ (0.049\ \Omega)}{0.8\text{ m}} = 2.76\text{ V/m}.$

d) $E_{Ag} = J\rho_{Ag} = \dfrac{IR_{Ag}}{L_{Ag}} = \dfrac{(45\text{ A})\ (0.062\ \Omega)}{1.2\text{ m}} = 2.33\ \text{ V/m}.$

e) $V_{Ag} = IR_{Ag} = (45\text{ A})\ (0.062\ \Omega) = 2.79\text{ V}.$

25.57: a) The current must be the same in both sections of the wire, so the current in the thin end is 2.5 mA.

b) $E_{1.6\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72 \times 10^{-8}\ \Omega \cdot \text{m})\ (2.5 \times 10^{-3}\text{ A})}{(\pi/4)(1.6 \times 10^{-3}\text{ A})^2} = 2.14\ \times 10^{-5}\text{ V/m}.$

c) $E_{0.8\text{mm}} = \rho J = \dfrac{\rho I}{A} = \dfrac{(1.72\ \times 10^{-8}\ \Omega \cdot \text{m})\ (2.5 \times 10^{-3}\text{ A})}{(\pi/4)\ (0.80 \times 10^{-3}\text{ A})^2}$

$= 8.55 \times 10^{-5}\text{ V/m } (= 4E_{1.6\text{mm}}).$

d) $V = E_{1.6\text{ mm}} L_{1.6\text{ mm}} + E_{0.8\text{ mm}} L_{0.8\text{ mm}}$

$\Rightarrow V = (2.14 \times 10^{-5}\text{ V/m})\ (1.20\text{ m}) + (8.55 \times 10^{-5}\text{ V/m})\ (1.80\text{ m}) = 1.80 \times 10^{-4}\text{ V}.$

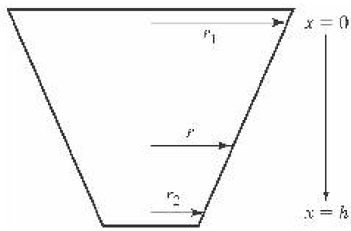
25.58: a) $\dfrac{K}{\text{volume}} = n\left(\dfrac{1}{2}mv_d^{\ 2}\right)$

$\Rightarrow \dfrac{K}{\text{volume}} = \dfrac{1}{2}(8.5\times 10^{28}\ \text{m}^{-3})\,(9.11\times 10^{-31}\ \text{kg})\,(1.5\times 10^{-4}\ \text{m/s})^2$

$= 8.7\times 10^{-10}\ \text{J/m}^3.$

b) $U = qV = ne(\text{volume})V = (8.5\times 10^{28}\text{m}^{-3})\,(1.6\times 10^{-19}\text{C})\,(10^{-6}\ \text{m}^3)\,(1.0\ \text{V}) = 13600\ \text{J}.$
And the kinetic energy in 1.0 cm^3 is $K = (8.7\times 10^{-10}\ \text{J/m}^3)\,(10^{-6}\text{m}) =$
8.7×10^{-16} J. So $\dfrac{U}{K} = \dfrac{13600\ \text{J}}{8.7\times 10^{-16}\ \text{J}} = 1.6\times 10^{19}.$

25.59: a)



$$dR = \frac{\rho L}{A} = \frac{\rho dx}{\pi r^2} \text{ where } r = r_1 - \left(\frac{r_1 - r_2}{h}\right)x.$$

$$\Rightarrow R = \int_0^h \frac{\rho\, dx}{\pi\left(r_1 - \left(\frac{r_1-r_2}{h}\right)x\right)^2} = -\frac{\rho h}{\pi(r_1 - r_2)}\int_{r_1}^{r_2}\frac{du}{u^2}$$

$$= \frac{\rho h}{\pi(r_1 - r_2)}\left.\frac{1}{u}\right|_{r_1}^{r_2} \Rightarrow R = \frac{\rho h}{\pi}\left(\frac{1}{r_1 r_2}\right).$$

b) When $r_1 = r_2 = r$, $R = \dfrac{\rho h}{\pi r^2} = \dfrac{\rho L}{A}.$

25.60: a) $dR = \dfrac{\rho dr}{4\pi r^2} \Rightarrow R = \dfrac{\rho}{4\pi}\displaystyle\int_a^b \dfrac{dr}{r^2} = -\dfrac{\rho}{4\pi}\left.\dfrac{1}{r}\right|_a^b = \dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right).$

b) $I = \dfrac{V_{ab}}{R} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)} \Rightarrow J = \dfrac{I}{A} = \dfrac{V_{ab}4\pi ab}{\rho(b-a)4\pi r^2} = \dfrac{V_{ab}ab}{\rho(b-a)r^2}.$

c) If the thickness of the shells is small, we have the resistance given by:

$$R = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\rho(b-a)}{4\pi ab} \approx \frac{\rho L}{4\pi a^2} = \frac{\rho L}{A}, \text{ where } L = b - a.$$

25.61: $E = \rho J$ and $E = \frac{\sigma}{K\epsilon_0} = \frac{Q}{AK\epsilon_0} \Rightarrow \rho J = \frac{Q}{AK\epsilon_0} \Rightarrow AJ = I = \frac{Q}{K\epsilon_0 \rho} =$ leakage current.

25.62: a) $I = \frac{V}{R} \Rightarrow J = \frac{I}{A} = \frac{V}{RA} = \frac{V}{(\rho L/A)A} = \frac{V}{\rho L}$. So to make the current density a maximum, we need the length between faces to be as small as possible, which means $L = d$. So the potential difference should be applied to those faces which are a distance d apart. This maximum current density is $J_{MAX} = \frac{V}{\rho d}$.

b) For a maximum current $I = \frac{V}{R} = \frac{VA}{\rho L} = JA$ must be a maximum. The maximum area is presented by the faces that are a distance d apart, and these two faces also have the greatest current density, so again, the potential should be placed over the faces a distance d apart. This maximum current is

$$I_{MAX} = 6\frac{Vd}{\rho}.$$

25.63: a) $R = \dfrac{\rho L}{A} = \dfrac{(9.5 \times 10^{-7}\ \Omega \cdot \text{m})\ (0.12\ \text{m})}{(\pi/4)\ (0.0016\ \text{m})^2} = 0.057\ \Omega$

b) $\rho(T) = \rho_0(1 + \alpha \Delta T) \Rightarrow \rho(60°\ \text{C}) = (9.5 \times 10^{-7}\ \Omega \cdot \text{m})\ (1 + (0.00088(\text{C}°)^{-1})\ (40°\text{C})$

$\Rightarrow \rho(60°\text{C}) = 9.83 \times 10^{-7}\ \Omega \cdot \text{m} \Rightarrow \Delta\rho = 3.34 \times 10^{-8}\ \Omega \cdot \text{m}.$

c) $\Delta V = \beta V_0 \Delta T \Rightarrow A\Delta L = A(\beta L_0 \Delta T) \Rightarrow \Delta L = \beta L_0 \Delta T = (18 \times 10^{-5}\ (\text{C}°)^{-1}) \times$ $(0.12\ \text{m})\ (40°\text{C}) \Rightarrow \Delta L = 8.64 \times 10^{-4}\ \text{m} = 0.86\ \text{mm}$. The volume of the fluid remains constant. As the fluid expands the container, outward expansion "becomes" upward expansion due to surface effects.

d) $R = \dfrac{\rho L}{A} \Rightarrow \Delta R = \dfrac{\Delta\rho L}{A} + \dfrac{\rho \Delta L}{A}$

$$\Rightarrow \Delta R = \frac{(3.34 \times 10^{-8}\ \Omega \cdot \text{m})\ (0.12\ \text{m})}{(\pi/4)\ (0.0016\ \text{m})^2} + \frac{(95 \times 10^{-8}\ \Omega \cdot \text{m})\ (0.86 \times 10^{-3}\ \text{m})}{(\pi/4)\ (0.0016\ \text{m})^2}$$

$= 2.40 \times 10^{-3}\ \Omega$

e) From Equation (25.12), $\alpha = \frac{1}{\Delta T}\left(\frac{R}{R_0} - 1\right) = \frac{1}{40°\text{C}}\left(\frac{(0.057\ \Omega + 2.40 \times 10^{-3}\ \Omega)}{0.057\ \Omega} - 1\right) =$ $1.1 \times 10^{-3}\ (\text{C}°)^{-1}$. This value is greater than the temperature coefficient of resistivity and therefore is an important change caused by the length increase.

25.64: a) $I = \dfrac{\sum \mathcal{E}}{\sum R} = \dfrac{8.0\ \text{V} - 4.0\ \text{V}}{24.0\ \Omega} = 0.167\ \text{A}$

$\Rightarrow V_{ad} = 8.00\ \text{V} - (0.167\ \text{A})\ (8.50\ \Omega) = 6.58\ \text{V}.$

b) The terminal voltage is

$V_{bc} = +\,4.00\ \text{V} + (0.167\ \text{A})\,(0.50\ \Omega) = +\,4.08\ \text{V}.$

c) Adding another battery at point d in the opposite sense to the 8.0 V battery:

$$I = \frac{\sum \mathcal{E}}{\sum R} = \frac{10.3\ \text{V} - 8.0\ \text{V} + 4.0\ \text{V}}{24.5\ \Omega} = 0.257\ \text{A, and so}$$
$$\Rightarrow V_{bc} = 4.00\ \text{V} - (0.257\ \text{A})\,(0.50\ \Omega) = 3.87\ \text{V}.$$

25.65: a) $V_{ab} = \mathcal{E} - Ir \Rightarrow 8.4\ \text{V} = \mathcal{E} - (1.50\ \text{A})\,r$ and $9.4\ \text{V} = \mathcal{E} + (3.50\ \text{A})\,r$

$$\Rightarrow 9.4\ \text{V} = (8.4\ \text{V} + (1.50\ \text{A})r) + (3.50\ \text{A})r$$
$$\Rightarrow r = \frac{9.4\ \text{V} - 8.4\ \text{V}}{5.00\ \text{A}} = 0.2\ \Omega$$

b) $\mathcal{E} = 8.4\ \text{V} + (1.50\ \text{A})\,(0.20\ \Omega) = 8.7\ \text{V}.$

25.66: a) $I = V/R = 14\ \text{kV}/(10\ \text{k}\Omega + 2\ \text{k}\Omega) = 1.17\ \text{A}.$

b) $P = I^2R = (1.17\ \text{A})^2\ (10{,}000\ \Omega) = 13.7\ \text{kW}.$

c) If we want the current to be 1.0 mA, then the internal resistance must be:

$$R + r = \frac{14{,}000\ \text{V}}{0.001\ \text{A}} = 1.4 \times 10^7 \Omega \Rightarrow R = 14\ \text{M}\Omega - 10\ \text{k}\Omega \approx 14\ \text{M}\Omega.$$

25.67: a) $R = \dfrac{\rho L}{A} = \dfrac{(5.0\ \Omega\cdot\text{m})\,(0.10\ \text{m})}{\pi(0.050\ \text{m})^2} = 1000\ \Omega.$

b) $V = IR = (100 \times 10^{-3}\ \text{A})\,(1000\ \Omega) = 100\ \text{V}.$

c) $P = VI = (100\text{V})\,(100 \times 10^{-3}\ \text{A}) = 10\ \text{W}.$

25.68: a) $V = 2.50I + 0.360I^2 = 4.0\ \text{V}$. Solving the quadratic equation yields $I = 1.34\ \text{A}$ or $-8.29\ \text{A}$, so the appropriate current through the semiconductor is $I = 1.34\ \text{A}.$

b) If the current $I = 2.68\ \text{A}$,

$$\Rightarrow V = (2.50\ \text{V/A})\,(2.68\ \text{A}) + (0.36\ \text{V/A}^2)\,(2.68\ \text{A})^2 = 9.3\ \text{V}.$$

25.69: $V = IR + V(I) = IR + \alpha I + \beta I^2 = (\alpha + R)\,I + \beta I^2$

$$\Rightarrow \beta I^2 + (R+\alpha)\,I - V = 0$$
$$\Rightarrow (1.3)\,I^2 + (3.8+3.2)\,I - 12.6 = 0 \Rightarrow I = 1.42\ \text{A}.$$

25.70: a) $r = \dfrac{\mathcal{E}}{I} = \dfrac{7.86\ \text{V}}{9.25\ \text{A}} = 0.85\ \Omega \Rightarrow I = \dfrac{\mathcal{E}}{R+r} = \dfrac{7.86\ \text{V}}{0.85\ \Omega + 2.4\ \Omega} = 2.42\ \text{A}.$

b) $\beta I^2 + (\alpha + r)\,I - \mathcal{E} = 0 \Rightarrow 0.36 I^2 + (2.50 + 0.85)\,I - 7.86 = 0$
$$\Rightarrow I = 1.94\ \text{A}$$

c) The terminal voltage at this current is
$$V_{ab} = \mathcal{E} - Ir = 7.86\ \text{V} - (1.94\ \text{A})\,(0.85\ \Omega) = 6.21\ \text{V}.$$

25.71: a) With an ammeter in the circuit:
$$I = \frac{\mathcal{E}}{r + R + R_A} \Rightarrow \mathcal{E} = I_A(r + R + R_A).$$
So with no ammeter:
$$I = \frac{\mathcal{E}}{r+R} = I_A\left(\frac{r + R + R_A}{r+R}\right) = I_A\left(1 + \frac{R_A}{r+R}\right).$$

b) We want:
$$\frac{I}{I_A} = \left(1 + \frac{R_A}{r+R}\right) \approx 1.01 \Rightarrow \frac{R_A}{r+R} \approx 0.01 \Rightarrow R_A\ (0.01)\,(0.45\ \Omega + 3.8\ \Omega)$$
$$= 0.0425\ \Omega.$$

c) This is a maximum value, since any larger resistance makes the current even less that it would be without it. That is, since the ammeter is in series, ANY resistance it has increases the circuit resistance and makes the reading less accurate.

25.72: a) With a voltmeter in the circuit:
$$I = \frac{\mathcal{E}}{r + R_V} \Rightarrow V_{ab} = \mathcal{E} - Ir = \mathcal{E}\left(1 - \frac{r}{r + R_V}\right).$$

b) We want:
$$\frac{V_{ab}}{\mathcal{E}} = \left(1 - \frac{r}{r + R_V}\right) \approx 0.99 \Rightarrow \frac{r}{r + R_V} \approx 0.01$$
$$\Rightarrow R_V \approx \frac{r - 0.01r}{0.01} = 99r = 99 \cdot 045\ \Omega = 44.6\ \Omega.$$

c) This is the minimum resistance necessary—any greater resistance leads to less current flow and hence less potential loss over the battery's internal resistance.

25.73: a) The line voltage, current to be drawn, and wire diameter are what must be considered in household wiring.

b) $P = VI \Rightarrow I = \dfrac{P}{V} = \dfrac{4200\ \text{W}}{120\ \text{V}} = 35\ \text{A}$, so the 8-gauge wire is necessary, since it can carry up to 40 A.

c) $P = I^2R = \dfrac{I^2\rho L}{A} = \dfrac{(35\ \text{A})^2\ (1.72\times10^{-8}\ \Omega\cdot\text{m})\,(42.0\ \text{m})}{(\pi/4)\,(0.00326\ \text{m})^2} = 106\ \text{W}.$

d) If 6-gauge wire is used,

$$P = \frac{I^2\rho L}{A} = \frac{(35\ \text{A})^2\ (1.72\times10^{-8}\ \Omega\cdot\text{m})\,(42\ \text{m})}{(\pi/4)\ )\,(0.00412\ \text{m})^2} = 66\ \text{W}$$

$\Rightarrow \Delta E = \Delta Pt = (40\ \text{W})\,(365)\,(12\ \text{h}) = 175\ \text{kWh}$

$\Rightarrow$ Savings $= (175\ \text{kWh})\,(\$0.11/\text{kWh}) = \$19.25.$

25.74: Initially: $R_0 = V/I_0 = (120\ \text{V})/(1.35\ \text{A}) = 88.9\ \Omega$

Finally: $R_f = V/I_f = (120\ \text{V})/(1.23\ \text{A}) = 97.6\ \Omega.$

And $\dfrac{R_f}{R_0} = 1 + \alpha\,(T_f - T_0) \Rightarrow (T_f - T_0) = \dfrac{1}{\alpha}\left(\dfrac{R_f}{R_0} - 1\right) = \dfrac{1}{4.5\times10^{-4}\,{}^\circ\text{C}^{-1}}\left(\dfrac{97.6\ \Omega}{88.9\ \Omega} - 1\right)$

$\Rightarrow T_f - T_0 = 217^\circ\text{C} \Rightarrow T_f = 217^\circ\text{C} + 20^\circ\text{C} = 237^\circ\text{C}.$

b) (i) $P_0 = VI_0 = (120\ \text{V})\,(1.35\ \text{A}) = 162\ \text{W}$

(ii) $P_f = VI_f = (120\ \text{V})\,(1.23\ \text{A}) = 148\ \text{W}$

25.75: a) $I = \dfrac{\Sigma\,\mathcal{E}}{\Sigma\,R} = \dfrac{12.0\ \text{V} - 8.0\ \text{V}}{10.0\ \Omega} = 0.40\ \text{A}.$

b) $P_{total} = I^2\,R_{total} = (0.40\ \text{A})^2\ (10\ \Omega) = 1.6\ \text{W}.$

c) Power generated in $\mathcal{E}_1$, $P = \mathcal{E}_1\ I = (12.0\ \text{V})\,(0.40\ \text{A}) = 4.8\ \text{W}.$

d) Rate of electrical energy transferred to chemical energy in $\mathcal{E}_2$ $P = \mathcal{E}_2 I = (8.0\ \text{V}) \times (0.40\ \text{A}) = 3.2\ \text{W}.$

e) Note $(\text{c}) = (\text{b}) + (\text{d})$, and so the rate of creation of electrical energy equals its rate of dissipation.

25.76: a) $R_{steel} = \dfrac{\rho L}{A} = \dfrac{(2.0\times10^{-7}\,\Omega\cdot\text{m})\,(2.0\ \text{m})}{(\pi/4)\,(0.018\ \text{m})^2} = 1.57\times10^{-3}\ \Omega$

$R_{Cu} = \dfrac{\rho L}{A} = \dfrac{(1.72\times10^{-8}\ \Omega\cdot\text{m})\,(35\ \text{m})}{(\pi/4)\,(0.008\ \text{m})^2} = 0.012\ \Omega$

$\Rightarrow V = IR = I\ (R_{steel} + R_{Cu}) = (15000\ \text{A})\,(1.57\times10^{-3}\ \Omega + 0.012\ \Omega) = 204\ \text{V}.$

b) $E = Pt = I^2Rt = (15000\ \text{A})^2(0.0136\ \Omega)\,(65\times10^{-6}\ \text{s}) = 199\ \text{J}.$

25.77: a) $\Sigma F = ma = |q|E \Rightarrow \dfrac{|q|}{m} = \dfrac{a}{E}.$

b) If the electric field is constant, $V_{bc} = EL \Rightarrow \dfrac{|q|}{m} = \dfrac{aL}{V_{bc}}.$

c) The free charges are "left behind" so the left end of the rod is negatively charged, while the right end is positively charged. Thus the right end is at the higher potential.

d) $a = \dfrac{V_{bc}|q|}{mL} = \dfrac{(1.0\times10^{-3}\ \text{V})\,(1.6\times10^{-19}\ \text{C})}{(9.11\times10^{-31}\ \text{kg})\,(0.50\ \text{m})} = 3.5\times10^{8}\ \text{m/s}^2.$

e) Performing the experiment in a rotational way enables one to keep the experimental apparatus in a localized area—whereas an acceleration like that obtained in (d), if linear, would quickly have the apparatus moving at high speeds and large distances.

25.78: a) We need to heat the water in 6 minutes, so the heat and power required are:

$$Q = mc_v\Delta T = (0.250\ \text{kg})\,(4190\ \text{J/kg°C})\,(80\text{°C}) = 83800\ \text{J}$$

$$\Rightarrow P = \frac{Q}{t} = \frac{83800\ \text{J}}{6(60\ \text{s})} = 233\ \text{W}.$$

$$\text{But } P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{233\ \text{W}} = 61.8\ \Omega.$$

b) $R = \dfrac{\rho L}{A} = \dfrac{\rho L^2}{\text{vol}} \Rightarrow L = \sqrt{\dfrac{R\cdot\text{vol}}{\rho}} = \sqrt{\dfrac{(61.8\ \Omega)\,(2.5\times10^{-5}\ \text{m}^3)}{1.00\times10^{-6}\ \Omega\cdot\text{m}}} = 39\ \text{m}.$

Now the radius of the wire can be calculated from the volume:

$$\text{vol} = L(\pi r^2) \Rightarrow r = \sqrt{\frac{\text{vol}}{\pi L}} = \sqrt{\frac{2.5\times10^{-5}\ \text{m}^3}{\pi(39\ \text{m})}} = 4.5\times10^{-4}\ \text{m}.$$

25.79: a) $V_{ab} = \mathcal{E} - Ir = 12.0\ \text{V} - (-10.0\ \text{A})\,(0.24\ \Omega) = 14.4\ \text{V}.$

b) $E = Pt = IVt = (10\ \text{A})\,(14.4\ \text{V})\,(5)\,(3600\ \text{s}) = 2.59\times10^{6}\ \text{J}.$

c) $E_{diss} = P_{diss}t = I^2rt = (10\ \text{A})^2\ (0.24\ \Omega)\,(5)\,(3600\ \text{s}) = 4.32\times10^{5}\ \text{J}.$

d) Discharged at 10 A:

$$I = \frac{\mathcal{E}}{r+R} \Rightarrow R = \frac{\mathcal{E} - Ir}{I} = \frac{12.0\ \text{V} - (10\ \text{A})\,(0.24\ \Omega)}{10\ \text{A}} = 0.96\ \Omega.$$

e) $E = Pt = IVt = (10\ \text{A})\,(9.6\ \text{V})\,(5)\,(3600\ \text{s}) = 1.73\times10^{6}\ \text{J}.$

f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{diss} = 4.32\times10^{5}\ \text{J}.$

g) The energy originally supplied went into the battery and some was also lost over the internal resistance. So the stored energy was less than was needed to charge it. Then when discharging, even more energy is lost over the internal resistance, and what is left is dissipated over the external resistor.

25.80: a) $V_{ab} = \mathcal{E} - Ir = 12.0\ \text{V} - (-30\ \text{A})\ (0.24\ \Omega) = 19.2\ \text{V}.$

b) $E = Pt = IVt = (30\ \text{A})\ (19.2\ \text{V})\ (1.7)\ (3600\ \text{s}) = 3.53 \times 10^6\ \text{J}.$

c) $E_{diss} = P_{diss}t = I^2Rt = (30\ \text{A})^2\ (0.24\ \Omega)\ (1.7)\ (3600\ \text{s}) = 1.32 \times 10^6\ \text{J}.$

d) Discharged at 30 A:

$$I = \frac{\mathcal{E}}{r+R} \Rightarrow R = \frac{\mathcal{E} - Ir}{I} = \frac{12.0\ \text{V} - (30\ \text{A})\ (0.24\ \Omega)}{30\ \text{A}} = 0.16\ \Omega.$$

e) $E = Pt = I^2Rt = (30\ \text{A})^2\ (0.16\ \Omega)\ (1.7)\ (3600) = 8.81 \times 10^5\ \text{J}.$

f) Since the current through the internal resistance is the same as before, there is the same energy dissipated as in (c): $E_{diss} = 1.32 \times 10^6\ \text{J}.$

g) Again, the energy originally supplied went into the battery and some was also lost over the internal resistance. So the stored energy was less than was needed to charge it. Then when discharging, even more energy is lost over the internal resistance, and what is left is dissipated over the external resistor. This time, at a higher current, much more energy is lost over the internal resistance.

25.81: a) $\alpha = \frac{1}{\rho}\left(\frac{d\rho}{dT}\right) = -\frac{n}{T} \Rightarrow \frac{ndT}{T} = \frac{d\rho}{\rho} \Rightarrow \ln\,(T^{-n}) = \ln(\rho) \Rightarrow \rho = \frac{a}{T^n}.$

b) $n = -\alpha T = -(-5 \times 10^{-4}\ (\text{K})^{-1})\ (293\ \text{K}) = 0.15.$

$$\rho = \frac{a}{T^n} \Rightarrow a = \rho T^n = (3.5 \times 10^{-5}\ \Omega \cdot \text{m})\ (293\ \text{K})^{0.15} = 8.0 \times 10^{-5}\ \Omega \cdot \text{m} \cdot \text{K}^{0.15}.$$

c) $T = -196°\text{C} = 77\ \text{K} : \rho = \dfrac{8.0 \times 10^{-5}}{(77\ \text{K})^{0.15}} = 4.3 \times 10^{-5}\ \Omega \cdot \text{m}.$

$$T = -300°\text{C} = 573\ \text{K} : \rho = \frac{8.0 \times 10^{-5}}{(573\ \text{K})^{0.15}} = 3.2 \times 10^{-5}\ \Omega \cdot \text{m}.$$

25.82: a) $\mathcal{E} = IR + IR_d \Rightarrow 2.00\ \text{V} = I(1.0\ \Omega) + V \Rightarrow 2 = I_s[\exp(eV/kT) - 1] + V.$

b) $I_s = 1.50 \times 10^{-3}\ \text{A}, T = 293\ \text{K} \Rightarrow 1333 = \exp\,[39.6\,V - 667] + 667\,V.$

Trial and error shows that the right-hand side (rhs) above, for specific V values, equals 1333 V, when $V = 0.179$ V. The current then is just

$$I = I_s\ \exp[39.6\,V - 1] = (1.5 \times 10^{-3}\ \text{A})\exp[39.6\,(0.179) - 1] = 1.80\ \text{A}.$$

25.83: a) $R = \dfrac{\rho L}{A} \Rightarrow dR = \dfrac{\rho\, dx}{A} = \dfrac{\rho_0 \exp[-x/L]\, dx}{A} \Rightarrow R = \dfrac{\rho_0}{A}\displaystyle\int_0^L \exp[-x/L]\, dx$

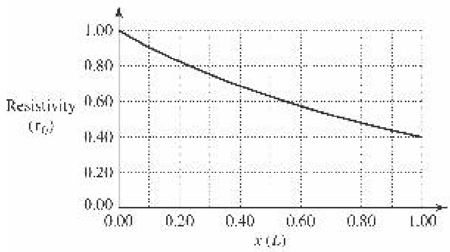
$$\Rightarrow R = \frac{\rho_0}{A}[-L\exp[-x/L]]_0^L = \frac{\rho_0 L}{A}(1-e^{-1}) \Rightarrow I = \frac{V_0}{R} = \frac{V_0 A}{\rho_0 L(1-e^{-1})}.$$

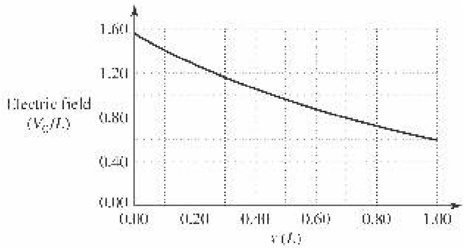
b) $E(x) = -\dfrac{\partial V}{\partial x} = -\dfrac{\partial (IR)}{\partial x} = -\dfrac{\partial}{\partial x}\left(\dfrac{I\rho_0 L e^{-x/L}}{A}\right) = \dfrac{I\rho_0 e^{-x/L}}{A} = \dfrac{V_0 e^{-x/L}}{L(1-e^{-1})}.$

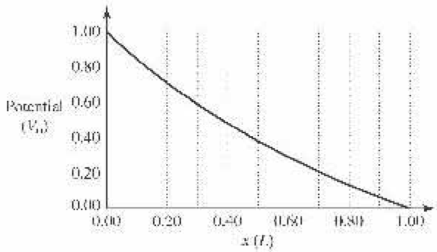
c) $V(x) = V_0 \dfrac{e^{-x/L}}{(1-e^{-1})} + C \Rightarrow V(0) = V_0 = \dfrac{V_0}{(1-e^{-1})} + C \Rightarrow C = \dfrac{-V_0 e^{-1}}{L(1-e^{-1})}$

$$\Rightarrow V(x) = V_0 \frac{(e^{-x/L} - e^{-1})}{(1-e^{-1})}.$$

d) Graphs of resistivity, electric field and potential from $x = 0$ to L.







25.84: a) $I = \dfrac{\mathcal{E}}{r+R} \Rightarrow P = \mathcal{E}I - I^2 r \Rightarrow \dfrac{dP}{dI} = \mathcal{E} - 2Ir = 0$ for maximum power output.

$$\Rightarrow I_{P\max} = \frac{1}{2}\frac{\mathcal{E}}{r} = \frac{1}{2} I_{\text{short circuit}}.$$

b) For the maximum power output of (a), $I = \dfrac{\mathcal{E}}{r+R} = \dfrac{1}{2}\dfrac{\mathcal{E}}{r} \Rightarrow +R = 2r \Rightarrow R = r.$

Then, $P = I^2 R = \left(\dfrac{\mathcal{E}}{2r}\right)^2 r = \dfrac{\mathcal{E}^2}{4r}.$

26.1: a) $R_{eq} = \left(\frac{1}{32} + \frac{1}{20}\right)^{-1} = 12.3\,\Omega.$

b) $I = \frac{V}{R_{eq}} = \frac{240\text{ V}}{12.3\,\Omega} = 19.5\text{ A}.$

c) $I_{32\Omega} = \frac{V}{R} = \frac{240\text{ V}}{32\,\Omega} = 7.5\text{ A}; I_{20\Omega} = \frac{V}{R} = \frac{240\text{ V}}{20\,\Omega} = 12\text{ A}.$

26.2: $R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{R_1 + R_2}{R_1 R_2}\right)^{-1} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}.$

$\Rightarrow R_{eq} = R_1 \frac{R_2}{R_1 + R_2} < R_1$ and $R_{eq} = R_2 \frac{R_1}{R_1 + R_2} < R_2.$

26.3: For resistors in series, the currents are the same and the voltages add. a) true. b) false. c) $P = I^2 R$. i same, R different so P different; false. d) true. e) $V = IR$. I same, R different; false. f) Potential drops as move through each resistor in the direction of the current; false. g) Potential drops as move through each resistor in the direction of the current, so $V_b > V_c$; false. h) true.

26.4: a) False, current divides at junction a.

b) True by charge conservation.

c) True. $V_1 = V_2$, so $I \propto \frac{1}{R}$

d) False. $P = IV. V_1 = V_2$, but $I_1 \neq I_2$, so $P_1 \neq P_2$.

e) False. $P = IV = \frac{V^2}{R}$. Since $R_2 > R_1, P_2 < P_1$.

f) True. Potential is independent of path.

g) True. Charges lose potential energy (as heat) in R_1.

h) False. See answer to (g).

i) False. They are at the *same* potential.

26.5: a) $R_{eq} = \left(\frac{1}{2.4\,\Omega} + \frac{1}{1.6\,\Omega} + \frac{1}{4.8\,\Omega}\right)^{-1} = 0.8\,\Omega.$

b) $I_{2.4} = \varepsilon/R_{2.4} = (28\text{ V})/(2.4\,\Omega) = 11.67\text{ A}; I_{1.6} = \varepsilon/R_{1.6} = (28\text{ V})/(1.6\,\Omega) = 17.5\text{ A};$ $I_{4.8} = \varepsilon/R_{4.8} = (28\text{ V})/(4.8\,\Omega) = 5.83\text{ A}.$

c) $I_{total} = \varepsilon/R_{total} = (28\text{ V})/(0.8\,\Omega) = 35\text{ A}.$

d) When in parallel, all resistors have the same potential difference over them, so here all have V = 28 V.

e) $P_{2.4} = I^2 R_{2.4} = (11.67\ \text{A})^2 (2.4\ \Omega) = 327\ \text{W}$; $P_{1.6} = I^2 R_{1.6} = (17.5\ \text{A})^2 (1.6\ \Omega) = 490\ \text{W}$; $P_{4.8} = I^2 R_{4.8} = (5.83\ \text{A})^2 (4.8\ \Omega) = 163\ \text{W}$.

f) For resistors in parallel, the most power is dissipated through the resistor with the least resistance since $P = I^2 R = \dfrac{V^2}{R}$, with V = constant.

26.6: a) $R_{eq} = \Sigma R_i = 2.4\ \Omega + 1.6\ \Omega + 4.8\ \Omega = 8.8\ \Omega$.

b) The current in each resistor is the same and is $I = \dfrac{\varepsilon}{R_{eq}} = \dfrac{28\ \text{V}}{8.8\ \Omega} = 3.18\ \text{A}$.

c) The current through the battery equals the current of (b), 3.18 A.

d) $V_{2.4} = IR_{2.4} = (3.18\ \text{A})(2.4\ \Omega) = 7.64\ \text{V}$; $V_{1.6} = IR_{1.6} = (3.18\ \text{A})(1.6\ \Omega) = 5.09\ \text{V}$; $V_{4.8} = IR_{4.8} = (3.18\ \text{A})(4.8\ \Omega) = 15.3\ \text{V}$.

e) $P_{2.4} = I^2 R_{2.4} = (3.18\ \text{A})^2 (2.4\ \Omega) = 24.3\ \text{W}$; $P_{1.6} = I^2 R_{1.6} = (3.18\ \text{A})^2 (1.6\ \Omega) = 16.2\ \text{W}$; $P_{4.8} = I^2 R_{4.8} = (3.18\ \text{A})^2 (4.8\ \Omega) = 48.5\ \text{W}$.

f) For resistors in series, the most power is dissipated by the resistor with the greatest resistance since $P = I^2 R$ with I constant.

26.7: a) $P = \dfrac{V^2}{R} \Rightarrow V = \sqrt{PR} = \sqrt{(5.0\ \text{W})(15{,}000\ \Omega)} = 274\ \text{V}$.

b) $P = \dfrac{V^2}{R} = \dfrac{(120\ \text{V})^2}{9{,}000\ \Omega} = 1.6\ \text{W}$.

26.8: $R_{eq} = \left(\left(\dfrac{1}{3.00\ \Omega} + \dfrac{1}{6.00\ \Omega} \right)^{-1} + \left(\dfrac{1}{12.0\ \Omega} + \dfrac{1}{4.00\ \Omega} \right)^{-1} \right) = 5.00\ \Omega$.

$I_{total} = \varepsilon / R_{total} = (6.00\ \text{V})/(5.00\ \Omega) = 12.0\ \text{A}$

$I_{12} = \dfrac{4}{12+4}(12.0) = 3.00\ \text{A}$; $I_4 = \dfrac{12}{12+4}(12.0) = 9.00\ \text{A}$;

$I_3 = \dfrac{6}{3+6}(12.0) = 8.00\ \text{A}$; $I_6 = \dfrac{3}{3+6}(12.0) = 4.00\ \text{A}$.

26.9: $R_{eq} = \left(\dfrac{1}{3.00\ \Omega + 1.00\ \Omega} + \dfrac{1}{5.00\ \Omega + 7.00\ \Omega} \right)^{-1} = 3.00\ \Omega$.

$I_{total} = \varepsilon / R_{total} = (48.0\ \text{V})/(3.00\ \Omega) = 16.0\ \text{A}$.

$$I_5 = I_7 = \frac{4}{4+12}(16.0) = 4.00 \text{ A}; I_1 = I_3 = \frac{12}{4+12}(16.0) = 12.0 \text{ A}.$$

26.10: a) The three resistors R_2, R_3 and R_4 are in parallel, so:

$$R_{234} = \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = \left(\frac{1}{8.20\ \Omega} + \frac{1}{1.50\ \Omega} + \frac{1}{4.50\ \Omega}\right)^{-1} = 0.99\ \Omega$$

$$\Rightarrow R_{eq} = R_1 + R_{234} = 3.50\ \Omega + 0.99\ \Omega = 4.49\ \Omega.$$

b) $I_1 = \frac{\varepsilon}{R_{eq}} = \frac{6.0 \text{ V}}{4.49\ \Omega} = 1.34 \text{ A} \Rightarrow V_1 = I_1 R_1 = (1.34 \text{ A})\ (3.50\ \Omega) = 4.69 \text{ V}.$

$$\Rightarrow V_{R_{234}} = I_1 R_{234} = (1.34 \text{ A})\ (0.99\ \Omega) = 1.33 \text{ V} \Rightarrow I_2 = \frac{V_{R_{234}}}{R_2} = \frac{1.33\ V}{8.20\ \Omega} = 0.162 \text{ A},$$

$$I_3 = \frac{V_{R_{234}}}{R_3} = \frac{1.33 \text{ V}}{1.50\ \Omega} = 0.887 \text{ A and } I_4 = \frac{V_{R_{234}}}{R_4} = \frac{1.33 \text{ V}}{4.50\ \Omega} = 0.296 \text{ A}.$$

26.11: Using the same circuit as in Problem 27.10, with all resistances the same:

$$R_{eq} = R_1 + R_{234} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)^{-1} = 4.50\ \Omega + \left(\frac{3}{4.50\ \Omega}\right)^{-1} = 6.00\ \Omega.$$

a) $I_1 = \frac{\varepsilon}{R_{eq}} = \frac{9.00 \text{ V}}{6.00\ \Omega} = 1.50 \text{ A}, I_2 = I_3 = I_4 = \frac{1}{3} I_1 = 0.500 \text{ A}.$

b) $P_1 = I_1{}^2 R_1 = (1.50 \text{ A})^2 (4.50\ \Omega) = 10.13 \text{ W}, P_2 = P_3 = P_4 = \frac{1}{9} P_1 = 1.125 \text{ W}.$

c) If there is a break at R_4, then the equivalent resistance increases:

$$R_{eq} = R_1 + R_{23} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 4.50\ \Omega + \left(\frac{2}{4.50\ \Omega}\right)^{-1} = 6.75\ \Omega.$$

And so:

$$I_1 = \frac{\varepsilon}{R_{eq}} = \frac{9.00 \text{ V}}{6.75\ \Omega} = 1.33 \text{ A}, I_2 = I_3 = \frac{1}{2} I_1 = 0.667 \text{ A}.$$

d) $P_1 = I_1{}^2 R_1 = (1.33 \text{ A})^2 (4.50\ \Omega) = 7.96 \text{ W}, P_2 = P_3 = \frac{1}{4} P_1 = 1.99 \text{ W}.$

e) So R_2 and R_3 are brighter than before, while R_1 is fainter. The amount of current flow is all that determines the power output of these bulbs since their resistances are equal.

26.12: From Ohm's law, the voltage drop across the 6.00 Ω resistor is $V = IR =$ (4.00 A)(6.00 Ω) = 24.0 V. The voltage drop across the 8.00 Ω resistor is the same, since these two resistors are wired in parallel. The current through the 8.00 Ω resistor is

then $I = V/R = 24.0\ \text{V}/8.00\ \Omega = 3.00\ \text{A}$. The current through the 25.0 Ω resistor is the sum of these two currents: 7.00 A. The voltage drop across the 25.0 Ω resistor is $V = IR$ = (7.00 A)(25.0 Ω) = 175 V, and total voltage drop across the top branch of the circuit is 175 + 24.0 = 199 V, which is also the voltage drop across the 20.0 Ω resistor. The current through the 20.0 Ω resistor is then $I = V/R = 199\ \text{V}/20\ \Omega = 9.95\ \text{A}$.

26.13: Current through 2.00-Ω resistor is 6.00 A. Current through 1.00-Ω resistor also is
6.00 A and the voltage is 6.00 V. Voltage across the 6.00-Ω resistor is 12.0 V + 6.0 V = 18.0 V. Current through the 6.00-Ω resistor is $(18.0\text{V})/(6.00\Omega) = 3.00\ \text{A}$. The battery voltage is 18.0 V.

26.14: a) The filaments must be connected such that the current can flow through each separately, and also through both in parallel, yielding three possible current flows. The parallel situation always has less resistance than any of the individual members, so it will give the highest power output of 180 W, while the other two must give power outputs of 60 W and
120 W.

$$60\ \text{W} = \frac{V^2}{R_1} \Rightarrow R_1 = \frac{(120\ \text{V})^2}{60\ \text{W}} = 240\ \Omega, \text{ and } 120\ \text{W} = \frac{V^2}{R_2} \Rightarrow R_2 = \frac{(120\ \text{V})^2}{120\ \text{W}} = 120\ \Omega.$$

Check for parallel: $P = \dfrac{V^2}{(\frac{1}{R_1} + \frac{1}{R_2})^{-1}} = \dfrac{(120\ \text{V})^2}{(\frac{1}{240\ \Omega} + \frac{1}{120\ \Omega})^{-1}} = \dfrac{(120\ \text{V})^2}{80\ \Omega} = 180\ \text{W}.$

b) If R_1 burns out, the 120 W setting stays the same, the 60 W setting does not work and the 180 W setting goes to 120 W: brightnesses of zero, medium and medium.

c) If R_2 burns out, the 60 W setting stays the same, the 120 W setting does not work, and the 180 W setting is now 60 W: brightnesses of low, zero and low.

26.15: a) $I = \dfrac{\varepsilon}{R} = \dfrac{120\ \text{V}}{(400\ \Omega + 800\ \Omega)} = 0.100\ \text{A}.$

b) $P_{400} = I^2R = (0.100\ \text{A})^2(400\ \Omega) = 4.0\ \text{W}; P_{800} = I^2R = (0.100\ \text{A})^2(800\ \Omega) =$ $8.0\ \text{W} \Rightarrow P_{total} = 4\ \text{W} + 8\ \text{W} = 12\ \text{W}.$

c) When in parallel, the equivalent resistance becomes:

$$R_{eq} = \left(\frac{1}{400\ \Omega} + \frac{1}{800\ \Omega}\right)^{-1} = 267\ \Omega \Rightarrow I_{total} = \frac{\varepsilon}{R_{eq}} = \frac{120\ \text{V}}{267\ \Omega} = 0.449\ \text{A}.$$

$$I_{400} = \frac{800}{400 + 800}(0.449\ \text{A}) = 0.30\ \text{A}; I_{800} = \frac{400}{400 + 800}(0.449\ \text{A}) = 0.150\ \text{A}.$$

d) $P_{400} = I^2R = (0.30\ \text{A})^2(400\ \Omega) = 36\ \text{W}; P_{800} = I^2R = (0.15\ \text{A})^2(800\ \Omega) = 18\ \text{W}$

$$\Rightarrow P_{total} = 36\text{ W} + 18\text{ W} = 54\text{ W}.$$

e) The 800 Ω resistor is brighter when the resistors are in series, and the 400 Ω is brighter when in parallel. The greatest total light output is when they are in parallel.

26.16: a) $R_{60\text{W}} = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{60\text{ W}} = 240\ \Omega;\ R_{200\text{W}} = \dfrac{V^2}{P} = \dfrac{(120\text{ V})^2}{200\text{ W}} = 72\ \Omega.$

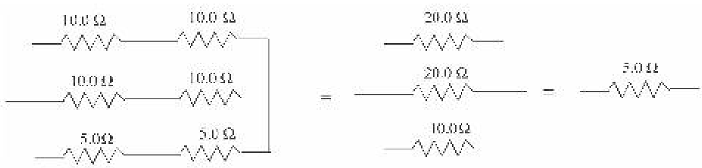
$$\Rightarrow I_{60\text{W}} = I_{200\text{W}} = \frac{\varepsilon}{R} = \frac{240\text{ V}}{(240\ \Omega + 72\ \Omega)} = 0.769\text{ A}.$$

b)

$P_{60\text{W}} = I^2R = (0.769\text{ A})^2(240\ \Omega) = 142\text{ W};\ P_{200\text{W}} = I^2R = (0.769\text{ A})^2(72\ \Omega) = 42.6\text{ W}.$

c) The 60 W bulb burns out quickly because the power it delivers (142 W) is 2.4 times its rated value.

26.17:



$30.0\text{ V} - I\,(20.0\ \Omega + 5.0\ \Omega + 5.0\ \Omega) = 0; \qquad I = 1.00\text{ A}$

For the $20.0\text{-}\Omega$ resistor thermal energy is generated at the rate $P = I^2R = 20.0\text{ W}.$

$Q = Pt$ and $Q = mc\Delta T$ gives

$$t = \frac{mc\Delta T}{P} = \frac{(0.100\text{ kg})\,(4190\text{ J/kg}\cdot\text{K})\,(40.0\text{ C°})}{20.0\text{ W}} = 1.01\times10^3\text{s}$$

26.18: a)

$$P_1 = I_1^2R_1$$
$$20\text{ W} = (2\text{A})^2R_1 \rightarrow R_1 = 5.00\Omega$$

R_1 and $10\ \Omega$ in parallel:

$$(10\ \Omega)I_{10} = (5\ \Omega)\,(2\text{ A})$$
$$I_{10} = 1\text{ A}$$

So $I_2 = 0.50$ A. R_1 and R_2 are in parallel, so

$$(0.50\text{ A})R_2 = (2\text{ A})\,(5\ \Omega)$$
$$R_2 = 20.0\ \Omega$$

b) $\varepsilon = V_1 = (2\text{A})(5\ \Omega) = 10.0\text{ V}$

c) From (a): $I_2 = 0.500\text{ A},\ I_{10} = 1.00\text{ A}$

d) $\qquad P_1 = 20.0\text{ W (given)}$

$$P_2 = i_2^2\ R_2 = (0.50\ \text{A})^2 (20\ \Omega) = 5.00\ \text{W}$$
$$P_{10} = i_{10}^2\ R_{10} = (1.0\ \text{A})^2 (10\ \Omega) = 10.0\ \text{W}$$
$$P_{\text{Resist}} = 20\ \text{W} + 5\ \text{W} + 10\ \text{W} = 35.0\ \text{W}$$
$$P_{\text{Battery}} = I\,\varepsilon = (3.50\ \text{A})\ (10.0\ \text{V}) = 35.0\ \text{W}$$
$P_{\text{Resist}} = P_{\text{Battery},}$ which agrees with the conservation of energy.

26.19: a) $I_R = 6.00\ \text{A} - 4.00\ \text{A} = 2.00\ \text{A}.$

b) Using a Kirchhoff loop around the outside of the circuit:

$$28.0\ \text{V} - (6.00\ \text{A})\ (3.00\ \Omega) - (2.00\ \text{A})\ R = 0 \Rightarrow R = 5.00\ \Omega.$$

c) Using a counterclockwise loop in the bottom half of the circuit:

$$\varepsilon - (6.00\ \text{A})\ (3.00\ \Omega) - (4.00\ \text{A})\ (6.00\ \Omega) = 0 \Rightarrow \varepsilon = 42.0\ \text{V}.$$

d) If the circuit is broken at point x, then the current in the 28 V battery is:

$$I = \frac{\sum \varepsilon}{\sum R} = \frac{28.0\ \text{V}}{3.00\ \Omega + 5.00\ \Omega} = 3.50\ \text{A}.$$

26.20: From the given currents in the diagram, the current through the middle branch of the circuit must be 1.00 A (the difference between 2.00 A and 1.00 A). We now use Kirchoff's Rules, passing counterclockwise around the top loop:

$$20.0\ \text{V} - (1.00\ \text{A})\ (6.00\ \Omega + 1.00\ \Omega) + (1.00\ \text{A})(4.00\ \Omega + 1.00\ \Omega) - \varepsilon_1 = 0 \Rightarrow \varepsilon_1 = 18.0\ \text{V}.$$

Now traveling around the external loop of the circuit:

$$20.0\ \text{V} - (1.00\ \ \text{A})(6.00\,\Omega + 1.00\ \Omega) - (2.00\ \text{A})(1.00\,\Omega + 2.00\,\Omega) - \varepsilon_2 = 0 \Rightarrow \varepsilon_2 = 7.0\ \text{V}.$$

And

$$\text{V}_{ab} = -(1.00\ \text{A})(4.00\ \Omega + 1.00\ \Omega) + 18.0\ \text{V} = +13.0\ \text{V, so } V_{ba} = -13.0\ \text{V}.$$

26.21: a) The sum of the currents that enter the junction below the $3\text{-}\Omega$ resistor equals $3.00\ \text{A} + 5.00\ \text{A} = 8.00\ \text{A}.$

b) Using the lower left loop:

$$\varepsilon_1 - (4.00\ \Omega)(3.00\ \text{A}) - (3.00\ \Omega)(8.00\ \text{A}) = 0$$
$$\Rightarrow \varepsilon_1 = 36.0\ \text{V}.$$

Using the lower right loop:

$$\varepsilon_2 - (6.00\ \Omega)(5.00\ \text{A}) - (3.00\ \Omega)(8.00\ \text{A}) = 0$$
$$\Rightarrow \varepsilon_2 = 54.0\ \text{V}.$$

c) Using the top loop:

$$54.0\ \text{V} - R(2.00\ \text{A}) - 36.0\ \text{V} = 0 \Rightarrow R = \frac{18.0\ \text{V}}{2.00\ \text{A}} = 9.00\ \Omega.$$

26.22: From the circuit in Fig. 26.42, we use Kirchhoff's Rules to find the currents, I_1

to the left through the 10 V battery, I_2 to the right through 5 V battery, and I_3 to the right through the 10 Ω resistor:

Upper loop:

$$10.0\text{ V} - (2.00\,\Omega + 3.00\,\Omega)I_1 - (1.00\,\Omega + 4.00\,\Omega)I_2 - 5.00\text{ V} = 0$$

$$\Rightarrow 5.0\text{ V} - (5.00\,\Omega)I_1 - (5.00\,\Omega)I_2 = 0 \Rightarrow I_1 + I_2 = 1.00\text{ A}.$$

Lower loop: $5.00\text{ V} + (1.00\,\Omega + 4.00\,\Omega)I_2 - (10.0\,\Omega)I_3 = 0$

$$\Rightarrow 5.00\text{ V} + (5.00\,\Omega)I_2 - (10.0\,\Omega)I_3 = 0 \Rightarrow I_2 - 2I_3 = -1.00\text{ A}$$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = 0.800\text{ A}, I_2 = 0.200\text{ A}, I_3 = 0.600\text{ A}.$$

b) $V_{ab} = -(0.200\text{ A})(4.00\,\Omega) - (0.800\text{ A})(3.00\,\Omega) = -3.20\text{ V}.$

26.23: After reversing the polarity of the 10-V battery in the circuit of Fig. 26.42, the only change in the equations from Problem 26.22 is the upper loop where the 10 V battery is:

Upper loop: $-10.0\text{ V} - (2.00\,\Omega + 3.00\,\Omega)I_1 - (1.00\,\Omega + 4.00\,\Omega)I_2 - 5.00\text{ V} = 0$

$$\Rightarrow -15.0\text{ V} - (5.00\,\Omega)I_1 - (5.00\,\Omega)I_2 = 0 \Rightarrow I_1 + I_2 = -3.00\text{ A}.$$

Lower loop: $5.00\text{ V} + (1.00\,\Omega + 4.00\,\Omega)I_2 - (10.0\,\Omega)I_3 = 0$

$$\Rightarrow 5.00\text{ V} + (5.00\,\Omega)I_2 - (10.0\,\Omega)I_3 = 0 \Rightarrow I_2 - 2I_3 = -1.00\text{ A}.$$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = -1.60\text{ A}, I_2 = -1.40\text{ A}, I_3 = -0.200\text{ A}.$$

b) $V_{ab} = +(1.40\text{ A})(4.00\,\Omega) + (1.60\text{ A})(3.00\,\Omega) = 10.4\text{ V}.$

26.24: After switching the 5-V battery for a 20-V battery in the circuit of Fig. 26.42, there is a change in the equations from Problem 26.22 in both the upper and lower loops:

Upper loop: $10.0\text{ V} - (2.00\,\Omega + 3.00\,\Omega)I_1 - (1.00\,\Omega + 4.00\,\Omega)I_2 - 20.00\text{ V} = 0$

$$\Rightarrow -10.0\text{ V} - (5.00\,\Omega)I_1 - (5.00\,\Omega)I_2 = 0 \Rightarrow I_1 + I_2 = -2.00\text{ A}.$$

Lower loop: $20.00\text{ V} + (1.00\,\Omega + 4.00\,\Omega)I_2 - (10.0\,\Omega)I_3 = 0$

$$\Rightarrow 20.00\text{ V} + (5.00\,\Omega)I_2 - (10.0\,\Omega)I_3 = 0 \Rightarrow I_2 - 2I_3 = -4.00\text{ A}.$$

Along with $I_1 = I_2 + I_3$, we can solve for the three currents and find:

$$I_1 = -0.4\text{ A}, I_2 = -1.6\text{ A}, I_3 = +1.2\text{ A}.$$

b) $I_2(4\,\Omega) - I_1(3\,\Omega) = (1.6\text{ A})(4\,\Omega) + (0.4\text{ A})(3\,\Omega) = 7.6\text{ V}$

26.25: The total power dissipated in the four resistors of Fig. 26.10a is given by the sum of:

$$P_2 = I^2R_2 = (0.5\text{ A})^2(2\,\Omega) = 0.5\text{ W}, P_3 = I^2\,R_3 = (0.5\text{ A})^2(3\,\Omega) = 0.75\text{ W},$$

$$P_4 = I^2R_4 = (0.5\text{ A})^2(4\,\Omega) = 1\text{ W}, P_7 = I^2\,R_7 = (0.5\text{ A})^2(7\,\Omega) = 1.8\text{ W}.$$

$\Rightarrow P_{total} = P_2 + P_3 + P_4 + P_7 = 4 \text{ W}.$

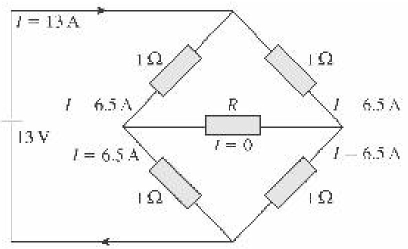
26.26: a) If the 12-V battery is removed and then replaced with the opposite polarity, the current will flow in the clockwise direction, with magnitude;

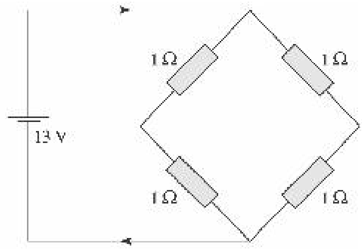
$$I = \frac{\sum \varepsilon}{\sum R} = \frac{12 \text{ V} + 4 \text{ V}}{16\ \Omega} = 1 \text{ A}.$$

b) $V_{ab} = -(R_4 + R_7)I + \varepsilon_4 = -(4\ \Omega + 7\ \Omega)(1 \text{ A}) + 4 \text{ V} = -7 \text{ V}.$

26.27: a) Since all the external resistors are equal, the current must be symmetrical through them. That is, there can be no current through the resistor R for that would imply an imbalance
in currents through the other resistors.

With no current going through R, the circuit is like that shown below at right.





So the equivalent resistance of the circuit is

$$R_{eq} = \left(\frac{1}{2\ \Omega} + \frac{1}{2\ \Omega}\right)^{-1} = 1\ \Omega \Rightarrow I_{total} = \frac{13 \text{ V}}{1\ \Omega} = 13 \text{ A}.$$

$$\Rightarrow I_{\text{each leg}} = \frac{1}{2} I_{total} = 6.5 \text{ A}, \text{ and no current passes through } R.$$

b) As worked out above, $R_{eq} = 1\ \Omega$.

c) $V_{ab} = 0$, since no current flows.

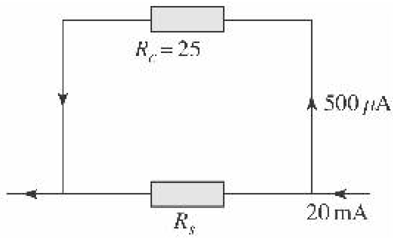
d) R does not show up since no current flows through it.

26.28: Given that the full-scale deflection current is 500 μA and the coil resistance is $25.0\ \Omega$:

a) For a 20-mA ammeter, the two resistances are in parallel:

$$V_c = V_s \Rightarrow I_c R_c = I_s R_s \Rightarrow (500 \times 10^{-6} \text{ A})(25.0\ \Omega) = (20 \times 10^{-3} \text{ A} - 500 \times 10^{-6} \text{ A})R_s$$

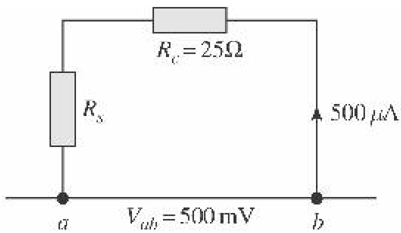
$$\Rightarrow R_s = 0.641\ \Omega$$



b) For a 500-m voltmeter, the resistances are in series:

$$V_{ab} = I(R_c + R_s) \Rightarrow R_s \Rightarrow \frac{V_{ab}}{I} - R_c$$

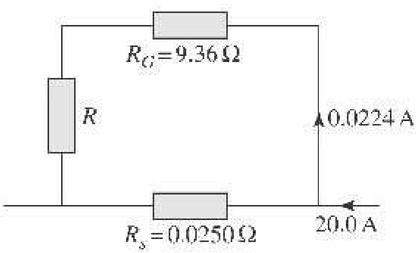
$$\Rightarrow R_s = \frac{500 \times 10^{-3}\ \text{V}}{500 \times 10^{-6}\ \text{A}} - 25.0\ \Omega = 975\ \Omega$$



26.29: The full-scale deflection current is 0.0224 A, and we wish a full-scale reading for 20.0 A.

$$(0.0224\ \text{A})(9.36\ \Omega + R) = (20.0\ \text{A} - 0.0224\ \text{A})(0.0250\ \Omega)$$

$$\Rightarrow R = \frac{0.499\ \Omega\text{A}}{0.0224\ \text{A}} - 9.36\ \Omega = 12.9\ \Omega$$



26.30: a) $I = \dfrac{\varepsilon}{R_{total}} = \dfrac{90\ \text{V}}{(8.23\ \Omega + 425\ \Omega)} = 0.208\ \text{A}$

$$\Rightarrow V = \varepsilon - Ir = 90\ \text{V} - (0.208\ \text{A})(8.23\ \Omega) = 88.3\ \Omega.$$

b) $V = \varepsilon - Ir = \varepsilon - \dfrac{\varepsilon r}{r + R_V} = \dfrac{\varepsilon R_V}{r + R_V} = \dfrac{\varepsilon}{(r / R_V) + 1} \Rightarrow \dfrac{r}{R_V} = \dfrac{\varepsilon}{V} - 1.$

Now if V is to be off by no more than 4% it requires: $\dfrac{r}{R_V} = \dfrac{90}{86.4} - 1 = 0.0416.$

26.31: a) When the galvanometer reading is zero:

$$\varepsilon_2 = IR_{cb} \text{ and } \varepsilon_1 = IR_{ab} \Rightarrow \varepsilon_2 = \varepsilon_1 \frac{R_{cb}}{R_{ab}} = \varepsilon_1 \frac{x}{l}.$$

b) The value of the galvanometer's resistance is unimportant since no current flows through it.

c) $\varepsilon_2 = \varepsilon_1 \dfrac{x}{l} = (9.15\ \text{V})\dfrac{0.365\ \text{m}}{1.000\ \text{m}} = 3.34\ \text{V}.$

26.32: Two voltmeters with different resistances are connected in series across a 120-V line. So the current flowing is $I = \dfrac{V}{R_{total}} = \dfrac{120\ \text{V}}{100 \times 10^3\ \Omega} = 1.20 \times 10^{-3}\ \text{A}$. But the current required for full-scale deflection for each voltmeter is:

$$I_{fsd(10\ \text{k}\Omega)} = \frac{150\ \text{V}}{10{,}000\ \Omega} = 0.0150\ \text{A and } I_{fsd(90\ \text{k}\Omega)} = \frac{150\ \text{V}}{90{,}000\ \Omega} = 1.67 \times 10^{-3}\ \text{A}.$$

So the readings are:

$$V_{10\ \text{k}\Omega} = 150\ \text{V}\left(\frac{1.20 \times 10^{-3}\ \text{A}}{0.0150\ \text{A}}\right) = 12\ \text{V and } V_{90\ \text{k}\Omega} = 150\ \text{V}\left(\frac{1.20 \times 10^{-3}\ \text{A}}{1.67 \times 10^{-3}\ \text{A}}\right) = 108\ \text{V}.$$

26.33: A half-scale reading occurs with $R = 600\ \Omega$. So the current through the galvanometer is half the full-scale current.

$$\Rightarrow \varepsilon = I\,R_{total} \Rightarrow 1.50\ \text{V} = \left(\frac{3.60 \times 10^{-3}\ \text{A}}{2}\right)(15.0\ \Omega + 600\ \Omega + R_s) \Rightarrow R_s = 218\ \Omega.$$

26.34: a) When the wires are shorted, the full-scale deflection current is obtained:
$\varepsilon = IR_{total} \Rightarrow 1.52\ \text{V} = (2.50 \times 10^{-3}\ \text{A})(65.0\ \Omega + R) \Rightarrow R = 543\ \Omega.$

b) If the resistance $R_x = 200\ \Omega : I = \dfrac{V}{R_{total}} = \dfrac{1.52\ \text{V}}{65.0\ \Omega + 543\ \Omega + R_x} = 1.88\ \text{mA}.$

c) $I_x = \dfrac{\varepsilon}{R_{total}} = \dfrac{1.52\ \text{V}}{65.0\ \Omega + 543\ \Omega + R_x} \Rightarrow R_x = \dfrac{1.52\ \text{V}}{I_x} - 608\ \Omega.$

$$\text{So: } I_x = \frac{1}{4} I_{fsd} = 6.25 \times 10^{-4}\ \text{A} \Rightarrow R_x = \frac{1.52\ \text{V}}{6.25 \times 10^{-4}\ \text{A}} - 608\ \Omega = 1824\ \Omega.$$

$$I_x = \frac{1}{2} I_{fsd} = 1.25 \times 10^{-3}\ \text{A} \Rightarrow R_x = \frac{1.52\ \text{V}}{1.25 \times 10^{-3}\ \text{A}} - 608\ \Omega = 608\ \Omega.$$

$$I_x = \frac{3}{4} I_{fsd} = 1.875\times10^{-3}\ \text{A} \Rightarrow R_x = \frac{1.52\ \text{V}}{1.875\times10^{-3}\ \text{A}} - 608\ \Omega = 203\ \Omega.$$

26.35: $[RC] = \left[\frac{V}{I}\frac{Q}{V}\right] = \left[\frac{Q}{I}\right] = \left[\frac{Q}{Q/t}\right] = [t]$

26.36: An uncharged capacitor is placed into a circuit.

a) At the instant the circuit is completed, there is no voltage over the capacitor, since it has no charge stored.

b) All the voltage of the battery is lost over the resistor, so $V_R = \varepsilon = 125\ \text{V}$.

c) There is no charge on the capacitor.

d) The current through the resistor is $i = \frac{\varepsilon}{R_{total}} = \frac{125\ \text{V}}{7500\ \Omega} = 0.0167\ \text{A}$.

e) After a long time has passed:

The voltage over the capacitor balances the emf: $V_c = 125\ \text{V}$.

The voltage over the resister is zero.

The capacitor's charge is $q = Cv_c = (4.60\times10^{-6}\ \text{F})\,(125\ \text{V}) = 5.75\times10^{-4}\ \text{C}$.

The current in the circuit is zero.

26.37: a) $i = \frac{q}{RC} = \frac{6.55\times10^{-8}\ \text{C}}{(1.28\times10^{6}\ \Omega)\,(4.55\times10^{-10}\ \text{F})} = 1.12\times10^{-4}\ \text{A}$.

b) $\tau = RC = (1.28\times10^{6}\ \Omega)\,(4.55\times10^{-10}\ \text{F}) = 5.82\times10^{-4}\ \text{s}$.

26.38:

$$v = v_0 e^{-\tau/RC} \Rightarrow C = \frac{\tau}{R\ln(v_0/v)} = \frac{4.00\ \text{s}}{(3.40\times10^{6}\ \Omega)\,(\ln(12/3))} = 8.49\times10^{-7}\ \text{F}.$$

26.39: a) The time constant $RC = (0.895\times10^{6}\ \Omega)\,(12.4\times10^{-6}\ \text{F}) = 11.1\ \text{s}$. So at:

$t = 0\ \text{s}: q = C\varepsilon(1 - e^{-t/RC}) = 0.$

$t = 5\ \text{s}: q = C\varepsilon(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})\,(60.0\ \text{V})\,(1 - e^{-(5.0\ \text{s})/(11.1\ \text{s})})$

$= 2.70\times10^{-4}\ \text{C}.$

$t = 10\ \text{s}: q = C\varepsilon(1 - e^{-t/RC}) = (12.4\times10^{-6}\ \text{F})\,(60.0\ \text{V})\,(1 - e^{-(10.0\ \text{s})/(11.1\ \text{s})})$

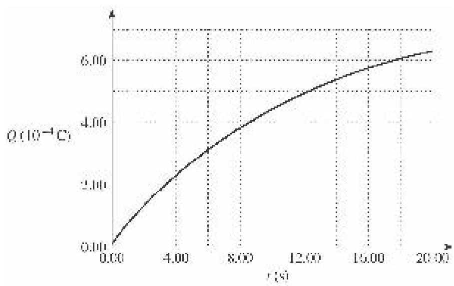
$= 4.42\times10^{-4}\ \text{C}.$

$$t = 20\text{ s}: q = C\varepsilon(1-e^{-t/RC}) = (12.4\times10^{-6}\text{ F})\,(60.0\text{ V})\,(1-e^{-(20.0\text{ s})/(11.1\text{s})})$$
$$= 6.21\times10^{-4}\text{ C.}$$
$$t = 100\text{ s}: q = C\varepsilon(1-e^{-t/RC}) = (12.4\times10^{-6}\text{ F})\,(60.0\text{ V})\,(1-e^{-(100\text{ s})/(11.1\text{s})})$$
$$= 7.44\times10^{-4}\text{ C.}$$

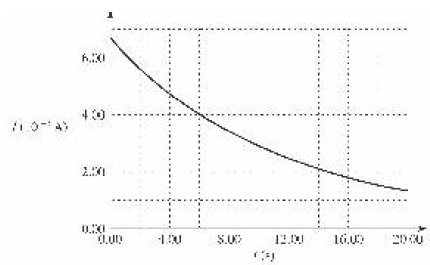
b) The current at time t is given by: $i = \frac{\varepsilon}{R}e^{-t/RC}$. So at :

$$t = 0\text{ s}: i = \frac{60.0\text{ V}}{8.95\times10^{5}\ \Omega}e^{-0/11.1} = 6.70\times10^{-5}\text{ A.}$$
$$t = 5\text{ s}: i = \frac{60.0\text{ V}}{8.95\times10^{5}\ \Omega}e^{-5/11.1} = 4.27\times10^{-5}\text{ A.}$$
$$t = 10\text{ s}: i = \frac{60.0\text{ V}}{8.95\times10^{5}\ \Omega}e^{-10/11.1} = 2.27\times10^{-5}\text{ A.}$$
$$t = 20\text{ s}: i = \frac{60.0\text{ V}}{8.95\times10^{5}\ \Omega}e^{-20/11.1} = 1.11\times10^{-5}\text{ A.}$$
$$t = 100\text{ s}: i = \frac{60.0\text{ V}}{8.95\times10^{5}\ \Omega}e^{-100/11.1} = 8.20\times10^{-9}\text{ A.}$$

c) Charge against time:



Current against time:



26.40: a) Originally, $\tau = RC = 0.870\text{ s}$. The combined capacitance of the two identical capacitors in series is given by

$$\frac{1}{C_{tot}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C};\ C_{tot} = \frac{C}{2}$$

The new time constant is thus $R\left(\frac{C}{2}\right) = \frac{0.870\ s}{2} = 0.435$ s.

b) With the two capacitors in parallel the new total capacitane is simply 2 C. Thus the time constant is $R(2C) = 2(0.870\ \text{s}) = 1.74$ s.

26.41: $\varepsilon - V_R - V_C = 0$

$\varepsilon = 120\ \text{V},\ V_R = IR = (0.900\ \text{A})\ (80.0\ \Omega) = 72\text{V},$ so $V_C = 48$ V

$Q = CV = (4.00 \times 10^{-6}\ \text{F})\ (48\text{V}) = 192\ \mu\text{C})$

26.42: a) $Q = CV = (5.90 \times 10^{-6}\ \text{F})\ (28.0\ \text{V}) = 1.65 \times 10^{-4}$ C.

b) $q = Q(1 - e^{-t/RC}) \Rightarrow e^{-t/RC} = 1 - \dfrac{q}{Q} \Rightarrow R = \dfrac{-t}{C \ln(1 - q/Q)}$.

After $t = 3 \times 10^{-3}\ \text{s}: R = \dfrac{-3 \times 10^{-3}\ \text{s}}{(5.90 \times 10^{-6}\ \text{F})\ (\ln(1 - 110/165))} = 463\ \Omega.$

c) If the charge is to be 99% of final value:

$$\frac{q}{Q} = (1 - e^{-t/RC}) \Rightarrow t = -RC \ln(1 - q/Q)$$

$$= -(463\ \Omega)\ (5.90 \times 10^{-6}\ \text{F}) \ln(0.01) = 0.0126\ \text{s}.$$

26.43: a) The time constant $RC = (980\ \Omega)\ (1.50 \times 10^{-5}\ \text{F}) = 0.0147$ s.

$t = 0.05\ \text{s}: q = C\varepsilon(1 - e^{-t/RC}) = (1.50 \times 10^{-5}\ \text{F})\ (18.0\ \text{V})\ (1 - e^{-0.010/0.0147}) = 1.33 \times 10^{-4}$ C.

b) $i = \dfrac{\varepsilon}{R} e^{-t/RC} = \dfrac{18.0\ \text{V}}{980\ \Omega} e^{-0.10/0.0147} = 9.30 \times 10^{-3}$ A.

$\Rightarrow V_R = IR = (9.30 \times 10^{-3}\ \text{A})\ (980\ \Omega) = 9.11$ V and $V_C = 18.0\ \text{V} - 9.11\ \text{V} = 8.89$ V.

c) Once the switch is thrown, $V_R = V_C = 8.89$ V.

d) After $t = 0.01\ \text{s}: q = Q_0 e^{-t/RC} = (1.50 \times 10^{-5}\ \text{F})\ (8.89\ \text{V}) e^{-0.01/0.0147} = 6.75 \times 10^{-5}$ C.

26.44: a) $I = \dfrac{P}{V} = \dfrac{4100\ \text{W}}{240\ \text{V}} = 17.1\ \text{A}.$ So we need at lest 14-gauge wire (good up to 18 A). 12 gauge is ok (good up to 25 A).

b) $P = \dfrac{V^2}{R} \Rightarrow R = \dfrac{V^2}{P} = \dfrac{(240\text{ V})^2}{4100\text{ W}} = 14\ \Omega$

c) At $11¢/\text{kWhr} \Rightarrow$ in 1 hour, cost $= (11¢/\text{kWhr})\,(1\text{ hr})(4.1\text{ kW}) = 45¢$.

26.45: We want to trip a 20-A circuit breaker:

$$I = \frac{1500\text{ W}}{120\text{ V}} + \frac{P}{120\text{ V}} \Rightarrow \text{With } P = 900\text{ W}: I = \frac{1500\text{ W}}{120\text{ V}} + \frac{900\text{ W}}{120\text{ V}}\ 20\text{ A}.$$

26.46: The current gets split evenly between all the parallel bulbs. A single bulb will draw $I = \dfrac{P}{V} = \dfrac{90\text{ W}}{120\text{ V}} = 0.75\text{ A} \Rightarrow$ Number of bulbs $\le \dfrac{20\text{ A}}{0.75\text{ A}} = 26.7$. So you can attach 26 bulbs safely.

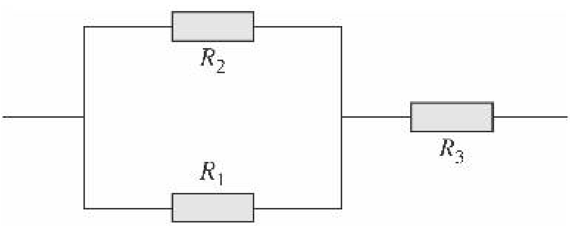
26.47: a) $I = \dfrac{V}{R} = \dfrac{120\text{ V}}{20\ \Omega} = 6.0\text{ A} \Rightarrow P = IV = (6.0\text{ A})\,(120\text{ V}) = 720\text{ W}.$

b) At $T = 280°\text{C}$, $R = R_0\,(1 + \alpha\Delta T) = 20\ \Omega\,(1 + (2.8\times10^{-3}\ (\text{C°})^{-1}\ (257°\text{C}))$

$$= 34.4\ \Omega.$$

$$\Rightarrow I = \frac{V}{R} = \frac{120\text{ V}}{34.4\text{ A}} = 3.49\text{ A} \Rightarrow P = (3.49\text{ A})\,(120\text{ V}) = 419\text{ W}.$$

26.48: a)

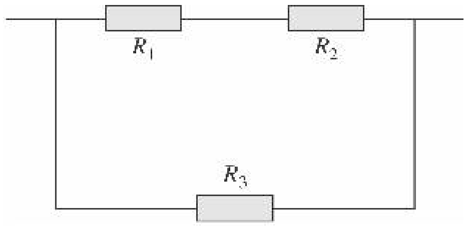


$$R_{eq} = R_3 + \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = R_3 + \left(\frac{R_1R_2}{R_1 + R_2}\right)$$

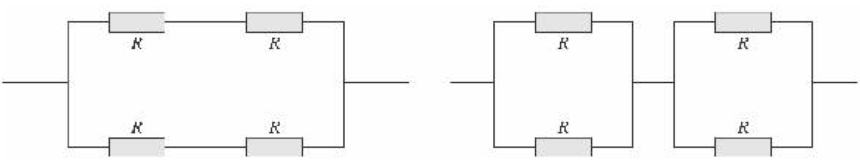
$$\text{If } R_{eq} = R_1 \Rightarrow R_3 = R_1 - \left(\frac{R_1R_2}{R_1 + R_2}\right) = \frac{R_1^2}{R_1 + R_2}.$$

b) $R_{eq} = \left(\dfrac{1}{R_1 + R_2} + \dfrac{1}{R_3} \right)^{-1} = \dfrac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$

If $R_{eq} = R_1 \Rightarrow R_1(R_1 + R_2 + R_3) = R_3(R_1 + R_2) \Rightarrow R_3 = R_1\ (R_1 + R_2)/\ R_2.$

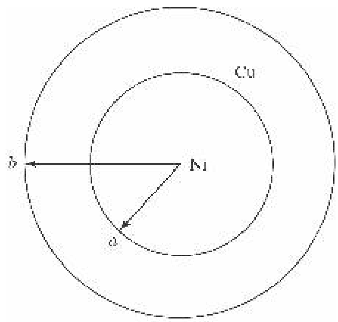


26.49: a) We wanted a total resistance of 400 Ω and power of 2.4 W from a combination of individual resistors of 400 Ω and 1.2 W power-rating.



b) The current is given by: $I = \sqrt{P/R} = \sqrt{2.4\ \text{W}/400\ \Omega} = 0.077$ A. In each leg half the current flows, so the power in each resistor in each resistor in each combination is the same: $P = (I/2)^2 R = (0.039\ \text{A})^2 (400\ \Omega) = 0.6$ W.

26.50: a) First realize that the Cu and Ni cables are in parallel.



$$\frac{1}{R_{\text{Cable}}} = \frac{1}{R_{\text{Ni}}} + \frac{1}{R_{\text{Cu}}}$$

$$R_{\text{Ni}} = \rho_{\text{Ni}} L/A = \rho_{\text{Ni}} \frac{L}{\pi a^2}$$

$$R_{\text{Cu}} = \rho_{\text{Cu}}\ L/A = \rho_{\text{Cu}} \frac{L}{\pi(b^2 - a^2)}$$

So: $$\frac{1}{R_{\text{cable}}} = \frac{\pi a^2}{\rho_{\text{Ni}} L} + \frac{\pi(b^2 - a^2)}{\rho_{\text{Cu}} L}$$

$$= \frac{\pi}{L}\left(\frac{a^2}{\rho_{\text{Ni}}} + \frac{b^2 - a^2}{\rho_{\text{Cu}}}\right)$$

$$= \frac{\pi}{20\text{m}}\left[\frac{(0.050\text{ m})^2}{7.8\times 10^{-8}\ \Omega\text{m}} + \frac{(0.100\text{ m})^2 - (0.050\text{ m})2}{1.72\times 10^{-8}\ \Omega\text{m}}\right]$$

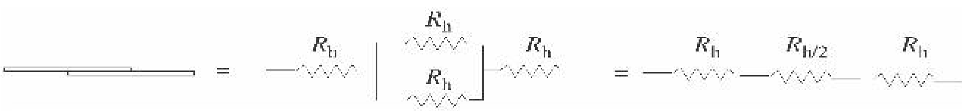
$$R_{\text{Cable}} = 13.6\times 10^{-6}\ \Omega = 13.6\ \mu\Omega$$

b) $R = \rho_{\text{eff}} \dfrac{L}{A} = \rho_{\text{eff}} \dfrac{L}{\pi b^2}$

$$\rho_{\text{eff}} = \frac{\pi b^2 R}{L} = \frac{\pi(0.10\text{ m})^2\ (13.6\times 10^{-6}\ \Omega)}{20\text{ m}}$$

$$= 2.14\times 10^{-8}\ \Omega\text{m}$$

26.51: Let $R = 1.00\ \Omega$, the resistance of one wire. Each half of the wire has $R_{\text{h}} = R/2$.



The equivalent resistance is $R_{\text{h}} + R_{\text{h}}/\ 2\ + R_{\text{h}} = 5R_{\text{h}}/2 = \frac{5}{2}(0.500\ \Omega)\ 1.25\ \Omega$

26.52: a) The equivalent resistance of the two bulbs is $1.0\ \Omega$. So the current is:

$I = \dfrac{V}{R_{total}} = \dfrac{8.0\text{ V}}{1.0\ \Omega + 0.80\ \Omega} = 4.4\text{ A} \Rightarrow$ the current through each bulb is 2.2 A.

$V_{\text{bulb}} = \varepsilon - Ir = 8.0\text{ V} - (4.4\text{ A})\,(0.80\ \Omega) = 4.4\text{ V} \Rightarrow P_{\text{bulb}} = IV = (2.2\text{ A})\,(4.4\text{ V}) = 9.9\text{ W}$

b) If one bulb burns out, then

$$I = \frac{V}{R_{total}} = \frac{8.0\ \text{V}}{2.0\ \Omega + 0.80\ \Omega} = 2.9\ \text{A} \Rightarrow P = I^2 R = (2.9\ \text{A})^2\ (2.0\ \Omega) = 16.3\ \text{W},$$

so the remaining bulb is brighter than before.

26.53: The maximum allowed power is when the total current is the maximum allowed value of $I = \sqrt{P/R} = \sqrt{36\ \text{W}/2.4\ \Omega} = 3.9\ \text{A}$. Then half the current flows through the parallel resistors and the maximum power is:

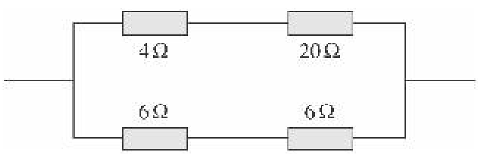
$$P_{\max} = (I/2)^2 R + (I/2)^2 R + I^2\ R = \frac{3}{2} I^2 R = \frac{3}{2}(3.9\ \text{A})^2\ (2.4\ \Omega) = 54\ \text{W}.$$

26.54: a) $R_{eq}(8, 16, 16) = \left(\frac{1}{8\ \Omega} + \frac{1}{16\ \Omega} + \frac{1}{16\ \Omega} \right)^{-1} = 4.0\ \Omega;$

$$R_{eq}(9, 18) = \left(\frac{1}{9\ \Omega} + \frac{1}{18\ \Omega} \right)^{-1} = 6.0\ \Omega.$$

So the circuit is equivalent to the one shown below. Thus:

$$R_{eq} = \left(\frac{1}{6\ \Omega + 6\ \Omega} + \frac{1}{20\ \Omega + 4\ \Omega} \right)^{-1} = 8.0\ \Omega$$



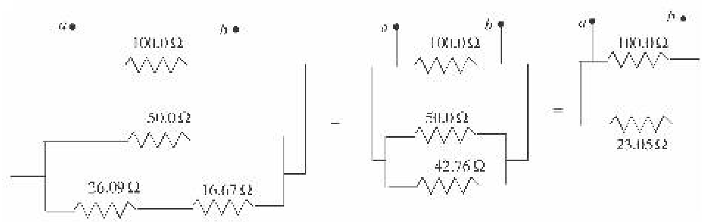
b) If the current through the $8\text{-}\Omega$ resistor is 2.4 A, then the top branch current is $I(8, 16, 16) = 2.4\ \text{A} + \frac{1}{2} 2.4\ \text{A} + \frac{1}{2} 2.4\ \text{A} = 4.8\ \text{A}$. But the bottom branch current is twice that of the top, since its resistance is half. Therefore the potential of point a relative to point x is $V_{ax} = -IR_{eq}(9, 18) = -(9.6\ \text{A})\ (6.00\ \Omega) = -58\ \text{V}.$

26.55: Circuit (a)

The 75.0 Ω and 40.0 Ω resistors are in parallel and have equivalent resistance 26.09 Ω

The 25.0 Ω and 50.0 Ω resistors are in parallel and have equivalent resistance 16.67 Ω.

The network is equivalent to



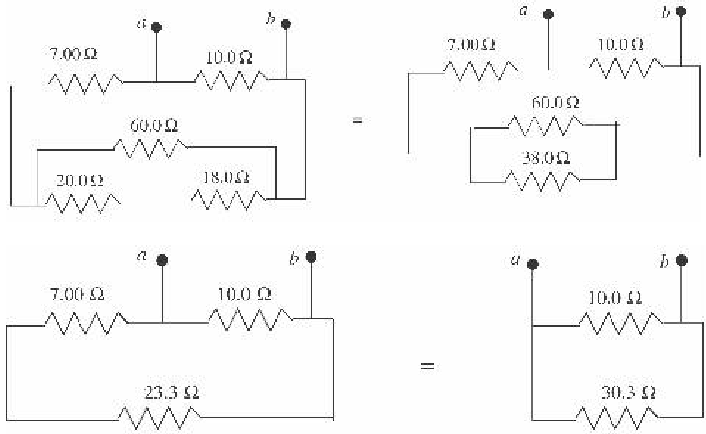
$$\frac{1}{R_{eq}} = \frac{1}{100.0\ \Omega} + \frac{1}{23.05\ \Omega} \text{ so } R_{eq} = 18.7\ \Omega$$

Circuit (b)

The

30.0 Ω and 45.0 Ω resistors are in parallel and have equivalent resistance 18.0 Ω.

The network is equivalent to



$$\frac{1}{R_{eq}} = \frac{1}{10.0\ \Omega} + \frac{1}{30.3\ \Omega} \quad \text{so} \quad R_{eq} = 7.5\ \Omega$$

26.56: Recognize that the ohmmeter measures the equivalent parallel resistance, not just X.

$$\frac{1}{20.2\,\Omega}=\frac{1}{X}+\frac{1}{115\,\Omega}+\frac{1}{130\,\Omega}+\frac{1}{85\,\Omega}$$
$$X=46.8\,\Omega$$

26.57: Top left loop: $12-5(I_2-I_3)-1I_2=0 \Rightarrow 12-6I_2+5I_3=0.$

Top right loop: $9-8(I_1+I_3)-1I_1=0 \Rightarrow 9-9I_1-8I_3=0.$
Bottom loop: $12-10I_3-9+1I_1-1I_2=0 \Rightarrow 3+I_1-I_2-10I_3=0.$

Solving these three equations for the currents yields:

$I_1=0.848\ A, I_2=2.14\ \text{A, and } I_3=0.171\ \text{A.}$

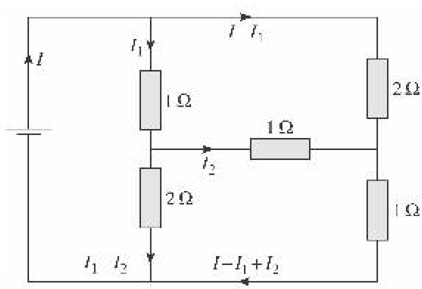
26.58: Outside loop: $24-7(1.8)-3(1.8-I_\varepsilon)=0 \Rightarrow I_\varepsilon=-2.0\ \text{A.}$
Right loop: $\varepsilon-7(1.8)-2(-2.0)=0 \Rightarrow \varepsilon=8.6\ \text{V.}$

26.59: Left loop: $20-14-2I_1+4(I_2-I_1)=0 \Rightarrow 6-6I_1+4I_2=0.$
Right loop: $36-5I_2-4(I_2-I_1)=0 \Rightarrow 36+4I_1-9I_2=0.$

Solving these two equations for the currents yields:

$I_1=5.21\ \text{A}=I_{2\Omega}, I_2=6.32\ \text{A}=I_{5\Omega}, \text{ and } I_{4\Omega}=I_2-I_1=1.11\ \text{A.}$

26.60: a) Using the currents as defined on the circuit diagram below we obtain three equations to solve for the currents:



$$\begin{aligned}
&\text{Left loop:} \quad 14 - I_1 - 2(I_1 - I_2) = 0 \\
&\qquad \Rightarrow 3I_1 - 2I_2 = 14. \\
&\text{Top loop:} \quad -2(I - I_1) + I_2 + I_1 = 0 \\
&\qquad \Rightarrow -2I + 3I_1 + I_2 = 0. \\
&\text{Bottom loop:} - (I - I_1 + I_2) + 2(I_1 - I_2) - I_2 = 0 \\
&\qquad \Rightarrow -I + 3I_1 - 4I_2 = 0.
\end{aligned}$$

Solving these equations for the currents we find:

$$I = I_{\text{battery}} = 10.0 \text{ A}; I_1 = I_{\text{R}_1} = 6.0 \text{ A}; I_2 = I_{R_3} = 2.0 \text{ A}.$$

So the other currents are:

$$I_{R_2} = I - I_1 = 4.0 \text{ A}; I_{R_4} = I_1 - I_2 = 4.0 \text{ A}; I_{R_5} = I - I_1 + I_2 = 6.0 \text{ A}.$$

b) $R_{\text{eq}} = \frac{V}{I} = \frac{14.0 \text{ V}}{10.0 \text{ A}} = 1.40\ \Omega.$

26.61: a) Going around the complete loop, we have:

$$\sum \varepsilon - \sum IR = 12.0 \text{ V} - 8.0 \text{ V} - I(9.0\ \Omega) = 0 \Rightarrow I = 0.44 \text{ A}.$$

$$\Rightarrow V_{ab} = \sum \varepsilon - \sum IR = 12.0 \text{ V} - 10.0 \text{ V} - (0.44 \text{ A})\,(2\ \Omega + 1\ \Omega + 1\ \Omega)$$
$$= +\,0.22 \text{ V}.$$

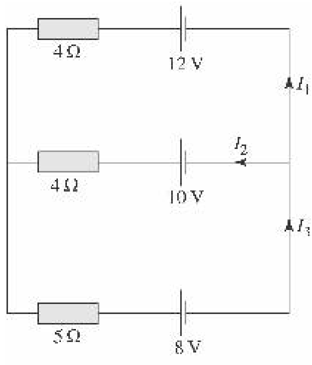
b) If now the points a and b are connected by a wire, the circuit becomes equivalent to the diagram shown below. The two loop equations for currents are (leaving out the units):

$$12 - 10 - 4I_1 + 4I_2 = 0 \Rightarrow I_2 = I_1 - 0.5$$

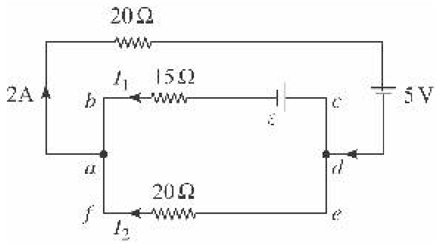
and

$$\begin{aligned}
&10 - 8 - 4I_2 - 5I_3 = 2 - 4I_2 - 5(I_1 + I_2) = 0 \\
&\Rightarrow 2 - (4I_1 - 2) - 5I_1 - 5I_1 + 2.5 = 0 \\
&\Rightarrow I_1 = 0.464 \text{ A}.
\end{aligned}$$

Thus the current through the 12-V battery is 0.464 A.



26.62: a) First do series/parallel reduction:



Now apply Kirchhoff's laws and solve for ε.

$$\Delta V_{\text{adefa}} = 0: -(20\ \Omega)(2\ \text{A}) - 5\ \text{V} - (20\ \Omega)I_2 = 0$$
$$I_2 = -2.25\ \text{A}$$
$$I_1 + I_2 = 2\ \text{A} \rightarrow I_1 = 2\ \text{A} - (-2.25\ \text{A}) = 4.25\ \text{A}$$

$$\Delta V_{\text{abcdefa}} = 0: (15\ \Omega)(4.25\ \text{A}) + \varepsilon - (20\ \Omega)(-2.25\ \text{A}) = 0$$
$$\varepsilon = -109\ \text{V; polarity should be reversed.}$$

b) Parallel branch has a $10\ \Omega$ resistance.

$$\Delta V_{\text{par}} = RI = (10\ \Omega)(2\text{A}) = 20\ \text{V}$$

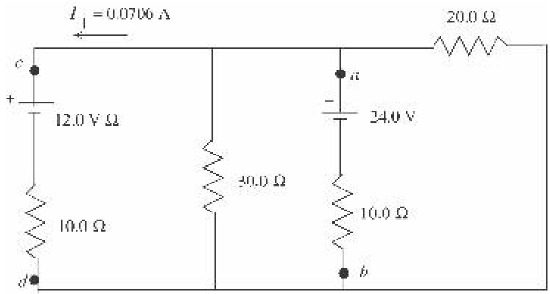
Current in upper part: $I = \frac{\Delta V}{R} = \frac{20\ \text{V}}{30\ \Omega} = \frac{2}{3}\ \text{A}$

$$Pt = U \rightarrow I^2\ Rt = U$$

$$\left(\frac{2}{3}\ \text{A}\right)^2 (10\ \Omega)t = 60\ \text{J}$$

$$t = 13.5\ \text{s}$$

26.63:



$$V_d + I_1(10.0\ \Omega) + 12.0\ \text{V} = V_c$$
$$V_c - V_d = 12.706\ \text{V};\quad V_a - V_b = V_c - V_d = 12.7\ \text{V}$$

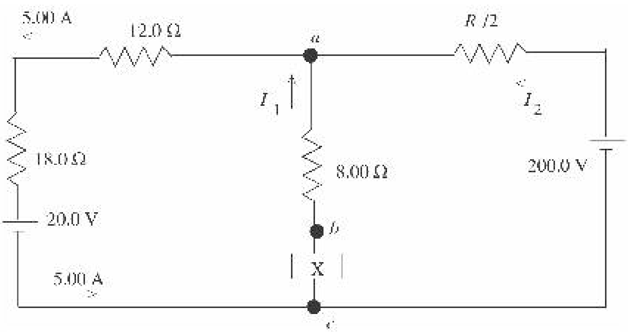
26.64: First recognize that if the 40 Ω resistor is safe, all the other resistors are also safe.

$$I^2R = P \rightarrow I^2(40\,\Omega) = 1\ \text{W}$$
$$\text{I} = 0.158\ \text{A}$$

Now use series / parallel reduction to simplify the circuit. The upper parallel branch is 6.38 Ω and the lower one is 25 Ω. The series sum is now 126 Ω. Ohm's law gives

$$\varepsilon = (126\ \Omega)(0.158\ \text{A}) = 19.9\ \text{V}$$

26.65: The 20.0-Ω and 30.0-Ω resistors are in parallel and have equivalent resistance 12.0 Ω. The two resistors *R* are in parallel and have equivalent resistance *R*/2. The circuit is equivalent to



a) $V_a - (5.00\text{ A})(12.0\ \Omega) - (5.00\text{ A})(18.0\ \Omega) - 20.0\text{ V} = V_c$

$V_c - V_a = 20.0\text{ V} + 90.0\text{ V} + 60.0\text{ V} = 170.0\text{ V}$

$V_b - V_a = V_{ba} = 16.0\text{ V}$

$X - V_{ba} = 170.0\text{ V}$ so $X = 186.0\text{ V}$, with the upper terminal +

b) $I_1 = (16.0\text{ V})/(8.00\ \Omega) = 2.00\text{ A}$

The junction rule applied to point a gives $I_2 + I_1 = 5.00\text{ A}$, so $I_2 = 3.00\text{ A}$. The current through the 200.0 V battery is in the direction from the $-$ to the $+$ terminal, as shown in the diagram.

c) $200.0\text{ V} - I_2(R/2) = 170.0\text{ V}$

$(3.00\text{ A})(R/2) = 30.0\text{ V}$ so $\text{R} = 20.0\ \Omega$

26.66: For three identical resistors in series, $P_s = \dfrac{V^2}{3R}$. If they are now in parallel over the same voltage, $P_p = \dfrac{V^2}{R_{eq}} = \dfrac{V^2}{R/3} = \dfrac{9V^2}{3R} = 9P_s = 9(27\text{ W}) = 243\text{ W}$.

26.67: $P_1 = \varepsilon^2/R_1$ so $R_1 = \varepsilon^2/P_1$

$P_2 = \varepsilon^2/R_2$ so $R_2 = \varepsilon^2/P_2$

a) When the resistors are connected in parallel to the emf, the voltage across each resistor is ε and the power dissipated by each resistor is the same as if only the one resistor were connected. $P_{\text{tot}} = P_1 + P_2$

b) When the resistors are connected in series the equivalent resistance is $R_{eq} = R_1 + R_2$

$$p_{\text{tot}} = \frac{\varepsilon^2}{R_1 + R_2} = \frac{\varepsilon^2}{\varepsilon^2/P_1 + \varepsilon^2/P_2} = \frac{P_1 P_2}{P_1 + P_2}$$

26.68: a) Ignoring the capacitor for the moment, the equivalent resistance of the two parallel resistors is

$$\frac{1}{R_{eq}} = \frac{1}{6.00\ \Omega} + \frac{1}{3.00\ \Omega} = \frac{3}{6.00\ \Omega};\ R_{eq} = 2.00\ \Omega$$

In the absence of the capacitor, the total current in the circuit (the current through the 8.00 Ω resistor) would be

$$i = \frac{\varepsilon}{R} = \frac{42.0\text{ V}}{8.00\ \Omega + 2.00\ \Omega} = 4.20\text{ A}$$

of which $2/3$, or 2.80 A, would go through the $3.00\ \Omega$ resistor and $1/3$, or 1.40 A, would go through the $6.00\ \Omega$ resistor. Since the current through the capacitor is given by

$$i = \frac{V}{R}e^{-t/RC},$$

at the instant $t = 0$ the circuit behaves as through the capacitor were not present, so the currents through the various resistors are as calculated above.

b) Once the capacitor is fully charged, no current flows through that part of the circuit. The $8.00\ \Omega$ and the $6.00\ \Omega$ resistors are now in series, and the current through them is $i = \varepsilon/R = (42.0\ \text{V})/(8.00\ \Omega + 6.00\ \Omega) = 3.00\ \text{A}$. The voltage drop across both the $6.00\ \Omega$ resistor and the capacitor is thus $V = iR = (3.00\ \text{A})(6.00\ \Omega) = 18.0\ \text{V}$. (There is no current through the $3.00\ \Omega$ resistor and so no voltage drop across it.) The change on the capacitor is

$$Q = CV = (4.00\times10^{-6}\ \text{farad})(18.0\ \text{V}) = 7.2\times10^{-5}\ \text{C}$$

26.69: a) When the switch is open, only the outer resistances have current through them. So the equivalent resistance of them is:

$$R_{eq} = \left(\frac{1}{6\ \Omega+3\ \Omega} + \frac{1}{3\ \Omega+6\ \Omega}\right)^{-1} = 4.50\ \Omega \Rightarrow I = \frac{V}{R_{eq}} = \frac{36.0\ \text{V}}{4.50\ \Omega} = 8.00\ \text{A}$$

$$\Rightarrow V_{ab} = \left(\frac{1}{2}8.00\ \text{A}\right)(3.00\ \Omega) - \left(\frac{1}{2}8.00\ \text{A}\right)(6.00\ \Omega) = -12.0\ \text{V}.$$

b) If the switch is closed, the circuit geometry and resistance ratios become identical to that of Problem 26.60 and the same analysis can be carried out. However, we can also use symmetry to infer the following:

$I_{6\Omega} = \frac{2}{3}I_{3\Omega}$, and $I_{switch} = \frac{1}{3}I_{3\Omega}$. From the left loop as in Problem 26.60:

$$36\ \text{V} - \left(\frac{2}{3}I_{3\Omega}\right)(6\ \Omega) - I_{3\Omega}(3\ \Omega) = 0 \Rightarrow I_{3\Omega} = 5.14\ \text{A} \Rightarrow I_{switch} = \frac{1}{3}I_{3\Omega} = 1.71\ \text{A}.$$

$$\text{(c)}\quad I_{battery} = \frac{2}{3}I_{3\Omega} + I_{3\Omega} = \frac{5}{3}I_{3\Omega} = 8.57\ \text{A} \Rightarrow R_{eq}\ \frac{\varepsilon}{I_{battery}} = \frac{36.0\ \text{V}}{8.57\ \text{A}} = 4.20\ \Omega$$

26.70: a) With an open switch: $V_{ab} = \varepsilon = 18.0\ \text{V}$, since equilibrium has been reached.

b) Point "a" is at a higher potential since it is directly connected to the positive terminal of the battery.

c) When the switch is closed:

$$18.0\ \text{V} = I(6.00\ \Omega + 3.00\ \Omega) \Rightarrow I = 2.00\ \text{A} \Rightarrow V_b = (2.00\ \text{A})(3.00\ \Omega) = 6.00\ \text{V}.$$

d) Initially the capacitor's charges were:

$$Q_3 = CV = (3.00\times10^{-6}\ \text{F})(18.0\ \text{V}) = 5.40\times10^{-5}\ \text{C}.$$

$$Q_6 = CV = (6.00\times10^{-6}\ \text{F})(18.0\ \text{V}) = 1.08\times10^{-4}\ \text{C}.$$

After the switch is closed:

$$Q_3 = CV = (3.00\times10^{-6}\ \text{F})(18.0\ \text{V} - 12.0\ \text{V}) = 1.80\times10^{-5}\ \text{C}.$$

$$Q_6 = CV = (6.00\times10^{-6}\ \text{F})(18.0\ \text{V} - 6.0\ \text{V}) = 7.20\times10^{-5}\ \text{C}.$$

So both capacitors lose 3.60×10^{-5} C.

26.71: a) With an open switch:

$$Q_3 = C_{eq}V = (2.00\times10^{-6}\ \text{F})(18.0\ \text{V}) = 3.60\times10^{-5}\ \text{C}.$$

Also, there is a current in the left branch:

$$I = \frac{18.0\ \text{V}}{6.00\ \Omega + 3.00\ \Omega} = 2.00\ \text{A}.$$

So, $V_{ab} = V_{6\mu F} - V_{6\Omega} = \dfrac{Q_{6\mu F}}{C} - IR_{6\Omega} = \dfrac{3.6\times10^{-5}\ \text{C}}{6.0\times10^{-6}\ \text{F}} - (2.0\ \text{A})(6.0\ \Omega) = -6.00\ \text{V}.$

b) Point "b" is at the higher potential.

c) If the switch is closed:

$$V_b = V_a = (2.00\ \text{A})(3.00\ \Omega) = 6.00\ \text{V}.$$

d) New charges are:

$$Q_3 = CV = (3.00\times10^{-6}\ \text{F})(6.0\ \text{V}) = 1.80\times10^{-5}\ \text{C}.$$

$$Q_6 = CV = (6.00\times10^{-6}\ \text{F})(-12.0\ \text{V}) = -7.20\times10^{-5}\ \text{C}.$$

$$\Rightarrow \Delta Q_3 = +3.60\times10^{-5}\ \text{C} - (1.80\times10^{-5}\ \text{C}) = +1.80\times10^{-5}\ \text{C}.$$

$$\Rightarrow \Delta Q_6 = -3.60\times10^{-5}\ \text{C} - (-7.20\times10^{-5}\ \text{C}) = +3.60\times10^{-5}\ \text{C}.$$

So the total charge flowing through the switch is 5.40×10^{-5} C.

26.72: The current for full-scale deflection is 0.02 A. From the circuit we can derive three equations:

(i) $(R_1 + R_2 + R_3)(0.100\ \text{A} - 0.02\ \text{A}) = 48.0\ \Omega(0.02\ \text{A})$

$$\Rightarrow R_1 + R_2 + R_3 = 12.0\ \Omega.$$

(ii) $(R_1 + R_2)(1.00\ \text{A} - 0.02\ \text{A}) = (48.0\ \Omega + R_3)(0.02\ \text{A})$

$$\Rightarrow R_1 + R_2 - 0.0204R_3 = 0.980\ \Omega.$$

(iii) $R_1(10.0\ \text{A} - 0.02\ \text{A}) = (48.0\ \Omega + R_2 + R_3)(0.02\ \text{A})$

$$\Rightarrow R_1 - 0.002R_2 - 0.002R_3 = 0.096\ \Omega.$$

From (i) and (ii) $\Rightarrow R_3 = 10.8\ \Omega$

From (ii) and (iii) $R_2 = 1.08\ \Omega$. And so $\Rightarrow R_1 = 0.12\ \Omega$

26.73: From the 3-V range:

$$(1.00\times10^{-3}\ \text{A})(40.0\ \Omega + R_1) = 3.00\ \text{V} \Rightarrow R_1 = 2960\ \Omega \Rightarrow R_{overall} = 3000\ \Omega.$$

From the 15-V range:

$$(1.00\times10^{-3}\ \text{A})(40.0\ \Omega + R_1 + R_2) = 15.0\ \text{V} \Rightarrow R_2 = 12000\ \Omega \Rightarrow R_{overall} = 15000\ \Omega.$$

From the 150-V range:

$$(1.00\times10^{-3}\ \text{A})(40.0\ \Omega + R_1 + R_2 + R_3) = 150\ \text{V} \Rightarrow R_2 = 135{,}000\ \Omega$$

$$\Rightarrow R_{overall} = 150\ \text{k}\Omega.$$

26.74: a) $R_{eq} = 100\ \text{k}\Omega + \left(\dfrac{1}{200\ \text{k}\Omega} + \dfrac{1}{50\ \text{k}\Omega}\right)^{-1} = 140\ \text{k}\Omega$

$$\Rightarrow I = \frac{0.400\ \text{kV}}{140\ \text{k}\Omega} = 2.86 \times 10^{-3}\ \text{A}.$$

$$\Rightarrow V_{200\text{k}\Omega} = I\,R = (2.86 \times 10^{-3}\ \text{A})\left(\frac{1}{200\ \text{k}\Omega} + \frac{1}{50\ \text{k}\Omega}\right)^{-1} = 114.4\ \text{V}.$$

b) *If* $V_R = 5.00 \times 10^6\ \Omega$, then we carry out the same calculations as above to find $R_{eq} = 292\ \text{k}\Omega \Rightarrow I = 1.37 \times 10^{-3}\ \text{A} \Rightarrow V_{200\text{k}\Omega} = 263\ \text{V}.$

c) If $V_R = \infty$, then we find $R_{eq} = 300\ \text{k}\Omega \Rightarrow I = 1.33 \times 10^{-3}\ \text{A} \Rightarrow V_{200\text{k}\Omega} = 266\ \text{V}.$

26.75: $I = \dfrac{110\ \text{V}}{(30\ \text{k}\Omega + R)} \Rightarrow V = 100\ \text{V} - \dfrac{(110\ \text{V})30\ \text{k}\Omega}{(30\ \text{k}\Omega + R)} = 68\ \text{V}.$

$$\Rightarrow (68\ \text{V})(30\ \text{k}\Omega + R) = (110\ \text{V})30\ \text{k}\Omega \Rightarrow R = 18.5\ \text{k}\Omega.$$

26.76: a) $V = IR + IR_A \Rightarrow R = \frac{V}{I} - R_A$. The true resistance R is always less than the reading because in the circuit the ammeter's resistance causes the current to be less then it should. Thus the smaller current requires the resistance R to be calculated larger than it should be.

b) $I = \frac{V}{R} + \frac{V}{R_V} \Rightarrow R = \frac{VR_V}{IR_V - V} = \frac{V}{I - V/R_V}$. Now the current measured is greater than that through the resistor, so $R = V/I_R$ is always greater than V/I.

c) (a): $P = I^2R = I^2(V/I - R_A) = IV - I^2R_A$.

(b): $P = V^2/R = V(I - V/R_V) = IV - V^2/R_V$.

26.77: a) When the bridge is balanced, no current flows through the galvanometer:

$$I_G = 0 \Rightarrow V_N = V_P \Rightarrow NI_{NM} = PI_{PX} \Rightarrow N\frac{(P + X)}{(P + X + N + M)} = P\frac{(N + M)}{(P + X + N + M)}$$

$$\Rightarrow N(P + X) = P(N + M) \Rightarrow NX = PM \Rightarrow X = \frac{MP}{N}.$$

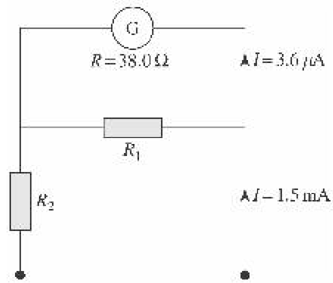
(b) $X = \dfrac{(8.50\,\Omega)(33.48\,\Omega)}{15.00\,\Omega} = 1897\,\Omega$

26.78: In order for the second galvanometer to give the same full-scale deflection and to have the same resistance as the first, we need two additional resistances as shown below. So:

$$(3.6\ \mu\text{A})(38.0\ \Omega) = (1.496\ \text{mA})R_1 \Rightarrow R_1 = 91.4\ \text{m}\Omega.$$

And for the total resistance to be 65 Ω:

$$65 = R_2 + \left(\frac{1}{38.0\ \Omega} + \frac{1}{0.0914\ \Omega}\right)^{-1} \Rightarrow R_2 = 64.9\ \Omega.$$



26.79: a) $I = \dfrac{90V}{(224\ \Omega + 589\ \Omega)} = 0.111\ \text{A}.$

$$\Rightarrow V_{224\Omega} = (0.111\ \text{A})(224\ \Omega) = 24.9\ \text{V}.$$
$$\Rightarrow V_{589\Omega} = (0.111\ \text{A})(589\ \Omega) = 65.4\ \text{V}.$$

b) $I = \dfrac{90\ \text{V}}{589\ \Omega + (\frac{1}{R_V} + \frac{1}{224\ \Omega})^{-1}}$ and $V_{224\Omega} = \varepsilon - IR_{589\Omega}$

$$\Rightarrow 23.8\ \text{V} = 90\ \text{V} - \frac{(90\ \text{V})(589\ \Omega)}{589\ \Omega + \left(\frac{1}{R_V} + \frac{1}{224\ \Omega}\right)^{-1}}$$

$$\Rightarrow \left(\frac{1}{R_V} + \frac{1}{224\ \Omega}\right)^{-1} = 211.8\ \Omega \Rightarrow R_V = 3874\ \Omega.$$

c) If the voltmeter is connected over the 589 - Ω resistor, then:

$$R_{eq} = 224\ \Omega + \left(\frac{1}{3874} + \frac{1}{589\ \Omega}\right)^{-1} = 735\ \Omega$$

$$\Rightarrow I = \frac{90\ \text{V}}{735\ \Omega} = 0.122\ \text{A} = I_V + I_{589\Omega} \text{ also } 3874 I_V = 589 I_{589\Omega}$$

$$\Rightarrow I_{589\Omega} = \frac{0.122\ \text{A}}{(1 + \frac{589}{3875})} = 0.106\ \text{A} \Rightarrow V_{589\Omega} = I_{589\Omega} R = (0.106\ \text{A})(589\Omega) = 62.4\ \text{V}.$$

d) No. From the equation in part (b) one can see that any voltmeter with finite resistance R_V placed in parallel with any other resistance will always decrease the measured voltage.

26.80: a) (i) $P_R = \dfrac{V^2}{R} = \dfrac{(120\ \text{V})^2}{4.26\ \Omega} = 3380\ \text{W}$ (ii) $P_C = \dfrac{dU}{dt} = \dfrac{1}{2C}\dfrac{d(q^2)}{dt} = \dfrac{iq}{C} = 0.$

(iii) $P_{\varepsilon} = \varepsilon I = (120\text{ V})\dfrac{120\text{ V}}{4.26\,\Omega} = 3380\text{ W}.$

b) After a long time, $i = 0 \Rightarrow P_R = 0,\ P_C = 0,\ P_{\varepsilon} = 0.$

c)

$$\text{When } q = \frac{1}{2}Q_f \Rightarrow e^{-t/RC} = 0.5 \Rightarrow i = \frac{1}{2}I_0 = \frac{120\text{ V}}{2(4.26\ \Omega)} = 14.08\text{ A}.$$

$$\Rightarrow P_R = I^2R = (14.08\text{ A})^2(4.26\ \Omega) = 845\text{ W}.$$

$$P_C = \frac{iq}{C} = \frac{iQ_f}{2C} = \frac{i\varepsilon}{2} = \frac{(14.08\text{ A})(120\text{ V})}{2} = 845\text{ W}.$$

And $P_\varepsilon = \varepsilon I = (120\text{ V})(14.08\text{ A}) = 1690\text{ W}.$

26.81: a) If the given capacitor was fully charged for the given emf, $Q_{max} = CV =$ $(3.4\times10^{-6}\text{ F})(180\text{ V}) = 6.12\times10^{-4}\text{ C}$. Since it has more charge than this *after* it was connected, this tells us the capacitor is discharging and so the current must be flowing toward the negative plate. The capacitor started with more charge than was "allowed" for the given emf. Let

$Q(t=0) = Q_0$ and $Q(t=\infty) = Q_f$. For all t, $Q(t) = (Q_0 - Q_f)e^{-t/RC} + Q_f$

We are given Q at some time $t = T; Q(t=T) = 8.15\times10^{-4}\text{C}$ and from above $Q_f = 6.12\times10^{-4}\text{C}$. The current $I(t) = \frac{dQ(t)}{dt} = \frac{(Q_0-Q_f)}{RC}e^{-t/RC}$. At $t = T$, $Q(T) =$ $(Q_0 - Q_f)e^{-T/RC} + Q_f$. So the current at $t = T$ is $I(T) = \frac{-(Q_0-Q_f)}{RC}(-e^{T/RC}) = \frac{-(Q(T)-Q_f)}{RC}$.

Thus $I(T) = \frac{-8.15\times10^{-4}\text{ C}+6.12\times10^{-4}\text{ C}}{(7.25\times10^{3}\ \Omega)(3.40\times10^{-6}\text{ F})} = -8.24\times10^{-3}\text{A}$ (toward the negative plate).

b) As time goes on, the capacitor will discharge to $6.12\times10^{-4}\text{ C}$ as calculated above.

26.82: For a charged capacitor, connected into a circuit:

$$I_0 = \frac{Q_0}{RC} \Rightarrow Q_0 = I_0RC = (0.620\text{ A})(5.88\text{ k}\Omega)(8.55\times10^{-10}\text{ F}) = 3.12\times10^{-6}\text{ C}.$$

26.83: $\varepsilon = I_0R \Rightarrow R = \frac{\varepsilon}{I_0} = \frac{110\text{ V}}{6.5\times10^{-5}\text{ A}} = 1.69\times10^{6}\ \Omega \Rightarrow$

$$C = \frac{\tau}{R} = \frac{6.2\text{ s}}{1.69\times10^{6}\ \Omega} = 3.67\times10^{-6}\text{ F}.$$

26.84: a) $U_0 = \frac{Q_0^{\ 2}}{2C} = \frac{(0.0081\text{ C})^2}{2(4.62\times10^{-6}\text{ F})} = 7.10\text{ J}.$

b) $P_0 = I_0^{\ 2}R = \left(\frac{Q_0}{RC}\right)^2 R = \frac{(0.0081\text{ C})^2}{(850\ \Omega)(4.62\times10^{-6}\text{ F})^2} = 3616\text{ W}.$

c) When $U = \frac{1}{2}U_0 = \frac{1}{2}\frac{Q_0^{\ 2}}{2C}$

$$\Rightarrow Q = \frac{Q_0}{\sqrt{2}} \Rightarrow P = \left(\frac{Q}{RC}\right)^2 R = \frac{1}{2}\left(\frac{Q_0}{RC}\right)^2 R = \frac{1}{2}P_0 = 1808 \text{ W}.$$

26.85: a) We will say that a capacitor is discharged if its charge is less than that of one electron, The time this takes is then given by:

$$q = Q_0 e^{-t/RC} \Rightarrow t = RC \ln(Q_0/e)$$

$$\Rightarrow t = (6.7\times 10^5\ \Omega)(9.2\times 10^{-7}\text{ F})\ln(7.0\times 10^{-6}\text{ C}/1.6\times 10^{-19}\text{ C}) = 19.36\text{ s},$$

or 31.4 time constants.

b) As shown in (a), $t = \tau \ln(Q_0/q)$, and so the number of time constants required to discharge the capacitor is independent of R and C, and depends only on the initial charge.

26.86: a) The equivalent capacitance and time constant are:

$$C_{eq} = \left(\frac{1}{3\ \mu\text{F}} + \frac{1}{6\ \mu\text{F}}\right)^{-1} = 2.00\mu\text{F} \Rightarrow \tau = R_{total}C_{eq} = (6.00\ \Omega)(2.00\ \mu\text{F}) = 1.20\times 10^{-5}\text{ s}.$$

b) After $t = 1.20\times 10^{-5}$ s, $q = Q_f(1 - e^{-t/RC_{eq}}) = C_{eq}\varepsilon(1 - e^{-t/RC_{eq}})$

$$\Rightarrow V_{3\mu\text{F}} = \frac{q}{C_{3\mu\text{F}}} = \frac{C_{eq}\varepsilon}{C_{3\mu\text{F}}}(1 - e^{-t/RC_{eq}}) = \frac{(2.0\ \mu\text{F})(12\text{ V})}{3.0\ \mu\text{F}}(1 - e^{-1}) = 5.06\text{ V}.$$

26.87: a) $E_{total} = \int_0^\infty P_\varepsilon\, dt = \int_0^\infty \varepsilon I\, dt = \frac{\varepsilon^2}{R}\int e^{-t/RC}\, dt = \varepsilon^2 C(1) = \varepsilon^2 C.$

b) $E_R = \int_0^\infty P_R\, dt = \int_0^\infty i^2 R\, dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC}\, dt = \frac{1}{2}\varepsilon^2 C.$

c) $U = \frac{Q_0^{\ 2}}{2C} = \frac{V^2 C}{2} = \frac{1}{2}\varepsilon^2 C = E_{total} - E_R.$

d) One half of the energy is stored in the capacitor, regardless of the sizes of the resistor.

26.88: $i = -\frac{Q_0}{RC}e^{-t/RC} \Rightarrow P = i^2 R = \frac{Q_0^{\ 2}}{RC^2}e^{-2t/RC} \Rightarrow E = \frac{Q_0^{\ 2}}{RC^2}\int_0^\infty e^{-2t/RC}\, dt$

$$= \frac{Q_0^{\ 2}}{RC^2}\frac{RC}{2} = \frac{Q_0^{\ 2}}{2C} = U_0.$$

26.89: a) Using Kirchhoff's Rules on the circuit we find:

Left loop: $92 - 140I_1 - 210I_2 + 55 = 0 \Rightarrow 147 - 140I_1 - 210I_2 = 0.$

Right loop: $57 - 35I_3 - 210I_2 + 55 = 0 \Rightarrow 112 - 210I_2 - 35I_3 = 0.$

Currents: $\Rightarrow I_1 - I_2 + I_3 = 0.$

Solving for the three currents we have:

$$I_1 = 0.300 \text{ A}, \quad I_2 = 0.500 \text{ A}, \ I_3 = 0.200 \text{ A}.$$

b) Leaving only the 92-V battery in the circuit:

Left loop: $92 - 140I_1 - 210I_2 = 0.$

Right loop: $-35I_3 - 210I_2 = 0.$

Currents: $I_1 - I_2 + I_3 = 0.$

Solving for the three currents:

$I_1 = 0.541 \text{ A}, \ I_2 = 0.077 \text{ A}, \ I_3 = -0.464 \text{ A}.$

c) Leaving only the 57-V battery in the circuit:

Left loop: $140I_1 + 210I_2 = 0.$

Right loop: $57 - 35I_3 - 210I_2 = 0.$

Currents: $I_1 - I_2 + I_3 = 0.$

Solving for the three currents:

$I_1 = -0.287 \text{ A}, \quad I_2 = 0.192 \text{ A}, \ I_3 = 0.480 \text{ A}.$

d) Leaving only the 55-V battery in the circuit:

Left loop: $55 - 140I_1 - 210I_2 = 0.$

Right loop: $55 - 35I_3 - 210I_2 = 0.$

Currents: $I_1 - I_2 + I_3 = 0.$

Solving for the three currents:

$$I_1 = 0.046 \text{ A}, \quad I_2 = 0.231 \text{ A}, \ I_3 = 0.185 \text{ A}.$$

e) If we sum the currents from the previous three parts we find:

$I_1 = 0.300 \text{ A}, \quad I_2 = 0.500 \text{ A}, \ I_3 = 0.200 \text{ A}$, just as in part (a).

f) Changing the 57-V battery for an 80-V battery just affects the calculation in part (c). It changes to:

Left loop: $140I_1 + 210I_2 = 0.$

Right loop: $80 - 35I_3 - 210I_2 = 0.$

Currents: $I_1 - I_2 + I_3 = 0.$

Solving for the three currents:

$$I_1 = -0.403 \text{ A}, \quad I_2 = 0.269 \text{ A}, \ I_3 = 0.672 \text{ A}.$$

So the total current for the full circuit is the sum of (b), (d) and (f) above:

$$I_1 = 0.184 \text{ A}, \quad I_2 = 0.576 \text{ A}, \ I_3 = 0.392 \text{ A}.$$

26.90: a) Fully charged:

$$Q = CV = (10.0 \times 10^{-12}\ \text{F})(1000\ \text{V}) = 1.00 \times 10^{-8}\ \text{C}.$$

b) $i_0 = \dfrac{\varepsilon - V_{C'}}{R} = \dfrac{\varepsilon}{R} - \dfrac{q}{RC'} \Rightarrow i(t) = \left(\dfrac{\varepsilon}{R} - \dfrac{q}{RC'}\right)e^{-t/RC'}$, where $C' = 1.1C$.

c) We need a resistance such that the current will be greater than 1 μA for longer than 200 μs.

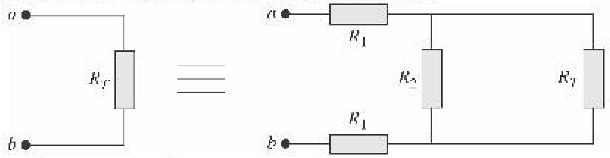
$$\Rightarrow i(200\ \mu s) = 1.0 \times 10^{-6}\ \text{A} = \frac{1}{R}\left(1000\ \text{V} - \frac{1.0 \times 10^{-8}\ \text{C}}{1.1(1.0 \times 10^{-11}\ \text{F})}\right)e^{-\frac{2.0 \times 10^{-4}\ \text{s}}{R(1.1 \times 10^{-11}\ \text{F})}}$$

$$\Rightarrow 1.0 \times 10^{-6}\ \text{A} = \frac{1}{R}(90.9)e^{-(1.8 \times 10^{7}\ \Omega)/R} \Rightarrow 18.3R - R\ln R - 1.8 \times 10^{7} = 0.$$

Solving for R numerically we find $7.15 \times 10^{6}\ \Omega \le R \le 7.01 \times 10^{7}\ \Omega$.

If the resistance is too small, then the capacitor discharges too quickly, and if the resistance is too large, the current is not large enough.

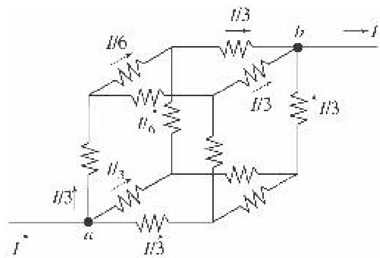
26.91: We can re-draw the circuit as shown below:



$$\Rightarrow R_T = 2R_1 + \left(\frac{1}{R_2} + \frac{1}{R_T}\right)^{-1} = 2R_1 + \frac{R_2 R_T}{R_2 + R_T} \Rightarrow R_T^{\,2} - 2R_1 R_T - 2R_1 R_2 = 0.$$

$$\Rightarrow R_T = R_1 \pm \sqrt{R_1^{\,2} + 2R_1 R_2} \text{ but } R_T > 0 \Rightarrow R_T = R_1 + \sqrt{R_1^{\,2} + 2R_1 R_2}.$$

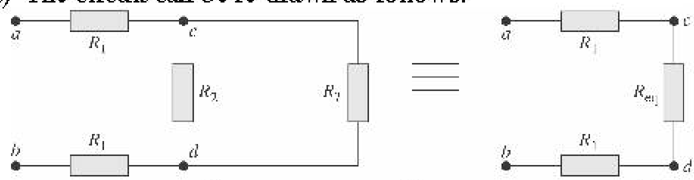
26.92:



Let current I enter at a and exit at b. At a there are three equivalent branches, so current is $I/3$ in each. At the next junction point there are two equivalent branches so each gets current $I/6$. Then at b there are three equivalent branches with current $I/3$ in each. The voltage drop from a to b then is $V = \left(\frac{I}{3}\right)R + \left(\frac{I}{6}\right)R + \left(\frac{I}{3}\right)R = \frac{5}{6}IR$.

This must be the same as $V = IR_{eq}$, so $R_{eq} = \dfrac{5}{6}R.$

26.93: a) The circuit can be re-drawn as follows:



Then $V_{cd} = V_{ab}\dfrac{R_{eq}}{2R_1 + R_{eq}} = V_{ab}\dfrac{1}{2R_1/R_{eq} + 1}$ and $R_{eq} = \dfrac{R_2 R_T}{R_2 + R_T}$.

But $\beta = \dfrac{2R_1(R_T + R_2)}{R_T R_2} = \dfrac{2R_1}{R_{eq}} \Rightarrow V_{cd} = V_{ab}\dfrac{1}{1+\beta}$.

b) Recall

$$V_1 = \frac{V_0}{(1+\beta)} \Rightarrow V_2 = \frac{V_1}{(1+\beta)} = \frac{V_0}{(1+\beta)^2} \Rightarrow V_n = \frac{V_{n-1}}{(1+\beta)} = \frac{V_0}{(1+\beta)^n}.$$

If $R_1 = R_2 \Rightarrow R_T = R_1 + \sqrt{R_1^{\ 2} + 2R_1R_1} = R_1(1+\sqrt{3})$ and $\beta = \dfrac{2(2+\sqrt{3})}{1+\sqrt{3}} = 2.73.$

So, for the nth segment to have 1% of the original voltage, we need:

$$\frac{1}{(1+\beta)^n} = \frac{1}{(1+2.73)^n} \le 0.01 \Rightarrow n = 4 : V_4 = 0.005V_0 .$$

c) $R_T = R_1 + \sqrt{R_1^{\ 2} + 2R_1R_2}$

$$\Rightarrow R_T = 6400\ \Omega + \sqrt{(6400\ \Omega)^2 + 2(6400\ \Omega)\,(8.0\times10^8\ \Omega)} = 3.2\times10^6\ \Omega.$$

$$\Rightarrow \beta = \frac{2(6400\ \Omega)\,(3.2\times10^6\ \Omega + 8.0\times10^8\ \Omega)}{(3.2\times10^6\ \Omega)\,(8.0\times10^8\ \Omega)} = 4.0\times10^{-3}.$$

d) Along a length of 2.0 mm of axon, there are 2000 segments each 1.0 μm long. The voltage therefore attenuates by:

$$V_{2000} = \frac{V_0}{(1+\beta)^{2000}} \Rightarrow \frac{V_{2000}}{V_0} = \frac{1}{(1+4.0\times10^{-3})^{2000}} = 3.4\times10^{-4}.$$

e) If $R_2 = 3.3\times10^{12}\ \Omega \Rightarrow R_T = 2.1\times10^8\ \Omega$ and $\beta = 6.2\times10^{-5}$.

$$\Rightarrow \frac{V_{2000}}{V_0} = \frac{1}{(1+6.2\times10^{-5})^{2000}} = 0.88.$$

27.1: a) $\vec{F} = q\vec{v}\times\vec{B}(-1.24\times10^{-8}\ \text{C})(-3.85\times10^4\ \text{m/s})(1.40\ \text{T})(\hat{j}\times\hat{i})$

$$\Rightarrow \vec{F} = -(6.68\times10^{-4}\ \text{N})\hat{k}.$$

b) $\vec{F} = q\vec{v}\times\vec{B}$

$$\Rightarrow \vec{F} = (-1.24\times10^{-8}\ \text{C})(1.40\ \text{T})[(-3.85\times10^4\ \text{m/s})(\hat{j}\times\hat{k}) + (4.19\times10^4\ \text{m/s})(\hat{i}\times\hat{k})]$$

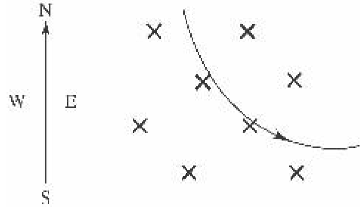
$$\Rightarrow \vec{F} = (6.68\times10^{-4}\ \text{N})\,\hat{i} + (7.27\times10^{-4}\ \text{N})\,\hat{j}.$$

27.2: Need a force from the magnetic field to balance the downward gravitational force. Its magnitude is:

$$qvB = mg \Rightarrow B = \frac{mg}{qv} = \frac{(1.95\times10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{(2.50\times10^{-8}\ \text{C})(4.00\times10^4\ \text{m/s})} = 1.91\ \text{T}.$$

The right-hand rule requires the magnetic field to be to the east, since the velocity is northward, the charge is negative, and the force is upwards.

27.3: By the right-hand rule, the charge is positive.



27.4: $\vec{F} = m\vec{a} = q\vec{v} \times \vec{B} \Rightarrow \vec{a} = \dfrac{q\vec{v} \times \vec{B}}{m}$

$$\Rightarrow \vec{a} = \frac{(1.22 \times 10^{-8}\ \text{C})(3.0 \times 10^{4}\ \text{m/s})(1.63\ \text{T})(\hat{j} \times \hat{i})}{1.81 \times 10^{-3}\ \text{kg}} = -\,(0.330\ \text{m/s}^2)\hat{k}.$$

27.5: See figure on next page. Let $F_0 = qvB$, then:

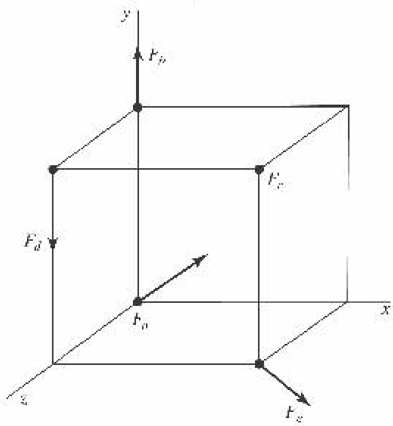
$F_a = F_0$ in the $-\hat{k}$ direction

$F_b = F_0$ in the $+\hat{j}$ direction

$F_c = 0$, since B and velocity are parallel

$F_d = F_0 \sin 45^\circ$ in the $-\hat{j}$ direction

$F_e = F_0$ in the $-(\hat{j} + \hat{k})$ direction



27.6: a) The smallest possible acceleration is zero, when the motion is parallel to the magnetic field. The greatest acceleration is when the velocity and magnetic field are at right angles:

$$a = \frac{qvB}{m} = \frac{(1.6 \times 10^{-19}\ \text{C})(2.50 \times 10^{6}\ \text{m/s})(7.4 \times 10^{-2}\ \text{T})}{(9.11 \times 10^{-31}\ \text{kg})} = 3.25 \times 10^{16}\ \text{m/s}^2.$$

b) If $a = \dfrac{1}{4}(3.25 \times 10^{16}\ \text{m/s}^2) = \dfrac{qvB \sin\phi}{m} \Rightarrow \sin\phi = 0.25 \Rightarrow \phi = 14.5^\circ.$

27.7: $F = |q| v B \sin\phi \Rightarrow v = \dfrac{F}{|q| B \sin\phi} = \dfrac{4.60 \times 10^{-15}\ \text{N}}{(1.6 \times 10^{-19}\ \text{C})(3.5 \times 10^{-3}\ \text{T}) \sin 60^\circ}$

$= 9.49 \times 10^{6}\ \text{m/s}.$

27.8: a) $\vec{F} = q\vec{v} \times \vec{B} = qB_z[v_x(\hat{i} \times \hat{k}) + v_y(\hat{j} \times \hat{k}) + v_z(\hat{k} \times \hat{k})] = qB_z[v_x\ (-\hat{j}) + v_y\ (\hat{i})].$

Set this equal to the given value of $\vec{F}$ to obtain:

$$v_x = \frac{F_y}{-qB_z} = \frac{(7.40 \times 10^{-7}\ \text{N})}{-(-5.60 \times 10^{-9}\ \text{C})(-1.25\ \text{T})} = -106\ \text{m/s}$$

$$v_y = \frac{F_x}{qB_z} = \frac{-(3.40 \times 10^{-7}\ \text{N})}{(-5.60 \times 10^{-9}\ \text{C})(-1.25\ \text{T})} = -48.6\ \text{m/s}.$$

b) The value of v_z is indeterminate.

c) $\vec{v} \cdot \vec{F} = v_x F_x + v_y F_y + v_z F_z = \dfrac{F_y}{-qB_z} F_x + \dfrac{F_x}{qB_z} F_y = 0; \theta = 90^\circ.$

27.9: $\vec{F} = q\vec{v} \times \vec{B}, \vec{v} = v_y \hat{j}$ *with* $v_y = -3.80 \times 10^{3}\ \text{m/s}$

$F_x = +7.60 \times 10^{-3}\ \text{N}, F_y = 0, \text{and } F_z = -5.20 \times 10^{-3}\ \text{N}$

$F_x = q(v_y B_z - v_z B_y) = q v_y B_z$

$B_z = F_x / q v_y = (7.60 \times 10^{-3}\ \text{N}) / ([7.80 \times 10^{-6}\ \text{C})(-3.80 \times 10^{3}\ \text{m/s})] = -0.256\ \text{T}$

$F_y = q(v_z B_x - v_x B_z) = 0,$ which is consistent with $\vec{F}$ as given in the problem. No force component along the direction of the velocity.

$F_z = q(v_x B_y - v_y B_x) = -q v_y B_x$

$B_x = -F_z / q v_y = -0.175\ \text{T}$

b) B_y is not determined. No force due to this component of $\vec{B}$ along $\vec{v}$; measurement of the force tells us nothing about B_y.

c) $\vec{B} \cdot \vec{F} = B_x F_x + B_y F_y + B_z F_z = (-0.175\ \text{T})(+7.60 \times 10^{-3}\ \text{N}) +$

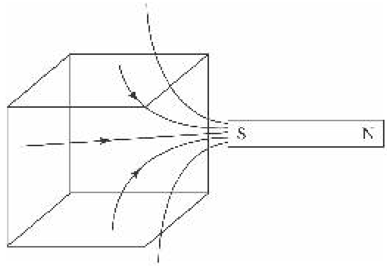
$(-0.256\ \text{T})(-5.20 \times 10^{-3}\ \text{N})$

$\vec{B} \cdot \vec{F} = 0$; $\vec{B}$ and $\vec{F}$ are perpendicular (angle is 90°)

27.10: a) The total flux must be zero, so the flux through the remaining surfaces must be -0.120 Wb.

b) The shape of the surface is unimportant, just that it is closed.

c)



27.11: a) $\Phi_B = \vec{B} \cdot \vec{A} = (0.230\ \text{T})\pi(0.065\ \text{m})^2 = 3.05 \times 10^{-3}\ \text{Wb}.$

b) $\Phi_B = \vec{B} \cdot \vec{A} = (0.230\ \text{T})\pi(0.065\ \text{m})^2 \cos 53.1° = 1.83 \times 10^{-3}\ \text{Wb}.$

c) $\Phi_B = 0$ since $\vec{B} \perp \vec{A}$.

27.12: a) $\Phi_B(abcd) = \vec{B} \cdot \vec{A} = 0.$

b) $\Phi_B(befc) = \vec{B} \cdot \vec{A} = -(0.128\text{T})(0.300\ \text{m})(0.300\ \text{m}) = -0.0115\ \text{Wb}.$

c) $\Phi_B(aefd) = \vec{B} \cdot \vec{A} = BA\cos\phi = \dfrac{3}{5}(0.128\ \text{T})(0.500\ \text{m})(0.300\ \text{m}) = +0.0115\ \text{Wb}.$

d) The net flux through the rest of the surfaces is zero since they are parallel to the x-axis so the total flux is the sum of all parts above, which is zero.

27.13: a) $\vec{B} = [(\beta - \gamma y^2)]\hat{j}$ and we can calculate the flux through each surface. Note that there is no flux through any surfaces parallel to the y-axis. Thus, the total flux through the closed surface is:

$$\Phi_B(abe) = \vec{B} \cdot \vec{A} = ([-(0.300\ \text{T} - 0)] + [0.300\ \text{T} - (2.00\ \text{T/m}^2)(0.300\ \text{m})^2]) \times \frac{1}{2}(0.400\ \text{m})(0.300\ \text{m})$$
$$= -0.0108\ \text{Wb}.$$

b) The student's claim is implausible since it would require the existence of a magnetic monopole to result in a net non-zero flux through the closed surface.

27.14: a) $p = mv = m\left(\dfrac{RqB}{m}\right) = RqB = (4.68 \times 10^{-3}\ \text{m})(6.4 \times 10^{-19}\ \text{C})(1.65\ \text{T})$

$= 4.94 \times 10^{-21}\ \text{kg m/s}.$

b) $L = Rp = R^2qB = (4.68 \times 10^{-3}\ \text{m})^2(6.4 \times 10^{-19}\ \text{C})(1.65\ \text{T}) = 2.31 \times 10^{-23}\ \text{kg m}^2/\text{s}.$

27.15: a) $B = \dfrac{mv}{|q|R} = \dfrac{(9.11\times10^{-31}\ \text{kg})(1.41\times10^{6}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.0500\ \text{m})} = 1.61\times10^{-4}\ \text{T}.$

The direction of the magnetic field is into the page (the charge is negative).

b) The time to complete half a circle is just the distance traveled divided by the velocity:

$$t = \frac{D}{v} = \frac{\pi R}{v} = \frac{\pi(0.0500\ \text{m})}{1.41\times10^{6}\ \text{m/s}} = 1.11\times10^{-7}\ \text{s}.$$

27.16: a) $B = \dfrac{mv}{qR} = \dfrac{(1.67\times10^{-27}\ \text{kg})(1.41\times10^{6}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.0500\ \text{m})} = 0.294\ \text{T}$

The direction of the magnetic field is out of the page (the charge is positive).

b) The time to complete half a circle is unchanged:

$$t = 1.11\times10^{-7}\ \text{s}.$$

27.17: $K_1 + U_1 = K_2 + U_2$

$U_1 = K_2 = 0$, so $K_1 = U_2$; $\dfrac{1}{2}mv^2 = ke^2/r$

$$v = e\sqrt{\frac{2k}{mr}} = (1.602\times10^{-19}\ \text{C})\sqrt{\frac{2k}{(3.34\times10^{-27}\ \text{kg})(1.0\times10^{-15}\ \text{m})}} = 1.2\times10^{7}\ \text{m/s}$$

b) $\sum \vec{F} = m\vec{a}$ gives $qvB = mv^2/r$

$$B = \frac{mv}{qr} = \frac{(3.34\times10^{-27}\ \text{kg})(1.2\times10^{7}\ \text{m/s})}{(1.602\times10^{-19}\ \text{C})(2.50\ \text{m})} = 0.10\ \text{T}$$

27.18: a) $F = qvB\sin\theta$

$$B = \frac{F}{qv\sin\theta} = \frac{0.00320\times10^{-9}\ \text{N}}{8(1.60\times10^{-19}\ \text{C})(500,000\ \text{m/s})\sin 90^\circ}$$

$B = 5.00$ T. If the angle θ is less than 90°, a larger field is needed to produce the same force. The direction of the field must be toward the south so that $\vec{v}\times\vec{B}$ can be downward.

b) $F = qvB\sin\theta$

$$v = \frac{F}{qB\sin\theta} = \frac{4.60\times10^{-12}\ \text{N}}{(1.60\times10^{-19}\ \text{C})(2.10\ \text{T})\sin 90^\circ}$$

$v = 1.37\times10^{7}\ \text{m/s}$. If θ is less than 90°, the speed would have to be larger to have the same force. The force is upward, so $\vec{v}\times\vec{B}$ must be downward since the electron is negative, so the velocity must be toward the south.

27.19: $q = (4.00 \times 10^{8})(-1.602 \times 10^{-19}\ \text{C}) = 6.408 \times 10^{-11}\ \text{C}$

speed at bottom of shaft: $\frac{1}{2}mv^2 = mgy;\ v = \sqrt{2gy} = 49.5\ \text{m/s}$

$\vec{v}$ is downward and $\vec{B}$ is west, so $\vec{v} \times \vec{B}$ is north. Since $q < 0$, $\vec{F}$ is south.

$F = qvB \sin\theta = (6.408 \times 10^{-11}\ \text{C})(49.5\ \text{m/s})(0.250\ \text{T}) \sin 90° = 7.93 \times 10^{-10}\ \text{N}$

27.20: (a) $$R = \frac{mv}{qB}$$

$$v = \frac{qBR}{m} = \frac{3(1.60 \times 10^{-19}\ \text{C})(0.250\ \text{T})(\frac{0.950}{2}\ \text{m})}{12(1.67 \times 10^{-27}\ \text{kg})}$$

$$v = 2.84 \times 10^{6}\ \text{m/s}$$

Since $\vec{v} \times \vec{B}$ is to the left but the charges are bent to the right, they must be negative.

b) $F_{\text{grav}} = mg = 12(1.67 \times 10^{-27}\ \text{kg})(9.80\text{m/s}^2) = 1.96 \times 10^{-25}\ \text{N}$

$F_{\text{magnetic}} = qvB = 3(1.6 \times 10^{-19}\ \text{C})(2.84 \times 10^{6}\ \text{m/s})(0.250\ \text{T})$

$= 3.41 \times 10^{-13}\ \text{N}$

Since $F_{\text{magn}} \approx 10^{12} \times F_{\text{grav}},$ we can safely neglect gravity.

c) The speed does not change since the magnetic force is perpendicular to the velocity and therefore does not do work on the particles.

27.21: a) $$v = \frac{qRB}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(6.96 \times 10^{-3}\ \text{m})(2.50\ \text{T})}{(3.34 \times 10^{-27}\ \text{kg})} = 8.34 \times 10^{5}\ \text{m/s}.$$

b) $$t = \frac{D}{v} = \frac{\pi R}{v} = \frac{\pi(6.96 \times 10^{-3}\text{m})}{8.34 \times 10^{5}\ \text{m/s}} = 2.62 \times 10^{-8}\ \text{s}.$$

c) $$\frac{1}{2}mv^2 = qV \Rightarrow V = \frac{mv^2}{2q} = \frac{(3.34 \times 10^{-27}\ \text{kg})(8.34 \times 10^{5}\ \text{m/s})^2}{2(1.60 \times 10^{-19}\ \text{C})} = 7260\ \text{V}.$$

27.22: $$R = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31}\ \text{kg})(2.8 \times 10^{6}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(0.0877\ \text{T})} = 1.82 \times 10^{-4}\ \text{m}.$$

27.23: a) $$B = \frac{m2\pi f}{|q|} = \frac{(9.11 \times 10^{-31}\ \text{kg})2\pi(3.00 \mp \times 10^{12}\ \text{Hz})}{(1.60 \times 10^{-19}\ \text{C})} = 107\ \text{T}.$$

This is about 2.4 times the greatest magnitude yet obtained on earth.

b) Protons have a greater mass than the electrons, so a greater magnetic field would be required to accelerate them with the same frequency, so there would be no advantage in using them.

27.24: The initial velocity is all in the y-direction, and we want the pitch to equal the radius of curvature

$$\Rightarrow d_x = v_x T = \frac{mv_y}{qB} = R.$$

But $$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}.$$

$$\Rightarrow \frac{2\pi m v_x}{qB} = \frac{mv_y}{qB} \Rightarrow \frac{v_y}{v_x} = 2\pi = \tan\theta \Rightarrow \theta = 81.0^\circ.$$

27.25: a) The radius of the path is unaffected, but the pitch of the helix varies with time as the proton is accelerated in the x-direction.

b) $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi m}{qB} = \dfrac{2\pi(1.67\times10^{-27}\ \text{kg})}{(1.60\times10^{-19}\ \text{C})(0.500\ \text{T})} = 1.31\times10^{-7}\ \text{s}, t = T/2,$ and

$$a_x = \frac{F}{m} = \frac{qE}{m} = \frac{(1.6\times10^{-19}\ \text{C})(2.00\times10^4\ \text{V/m})}{1.67\times10^{-27}\ \text{kg}} = 1.92\times10^{12}\ \text{m/s}^2.$$

$$d_x = v_{0x}t + \frac{1}{2}a_x t^2 = (1.5\times10^5\ \text{m/s})(6.56\times10^{-8}\ \text{s}) + \frac{(1.92\times10^{12}\ \text{m/s}^2)(6.56\times10^{-8}\ \text{s})^2}{2}$$

$$\Rightarrow d_x = 0.014\ \text{m}.$$

27.26: $\dfrac{1}{2}mv^2 = qV \Rightarrow v = \sqrt{\dfrac{2qV}{m}} = \sqrt{\dfrac{2(1.6\times10^{-19}\ \text{C})(220\ \text{V})}{(1.16\times10^{-26}\ \text{kg})}} = 7.79\times10^4\ \text{m/s}.$

$$\Rightarrow R = \frac{mv}{qB} = \frac{(1.16\times10^{-26}\ \text{kg})(7.79\times10^4\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.723\ \text{T})} = 7.81\times10^{-3}\ \text{m}.$$

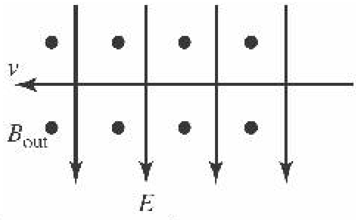
27.27: $\dfrac{1}{2}mv^2 = |q|\Delta V \Rightarrow v = \sqrt{\dfrac{2|q|\Delta V}{m}} = \sqrt{\dfrac{2(1.6\times10^{-19}\ \text{C})\,(2.0\times10^3\ \text{V})}{(9.11\times10^{-31}\ \text{kg})}}$

$$= 2.65\times10^7\ \text{m/s}.$$

$$\Rightarrow B = \frac{mv}{|q|R} = \frac{(9.11\times10^{-31}\ \text{kg})(2.65\times10^7\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.180\ \text{m})} = 8.38\times10^{-4}\ \text{T}.$$

27.28: a) $v = E/B = (1.56\times10^4\ \text{V/m})/(4.62\times10^{-3}\ \text{T}) = 3.38\times10^6\ \text{m/s}.$

b)



c) $R = \dfrac{mv}{|q|B} = \dfrac{(9.11\times10^{-31}\ \text{kg})(3.38\times10^6\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(4.62\times10^{-3}\ \text{T})}$

$\Rightarrow R = 4.17\times10^{-3}\ \text{m}.$

$T = \dfrac{2\pi m}{|q|B} = \dfrac{2\pi R}{v} = \dfrac{2\pi(4.17\times10^{-3}\ \text{m})}{(3.38\times10^6\ \text{m/s})} = 7.74\times10^{-9}\ \text{s}.$

27.29: a) $F_B = F_E$ so $|q|vB = |q|E;\quad B = E/v = 0.10\,\text{T}$

Forces balance for either sign of q.

b) $E = V/d$ so $v = E/B = V/dB$

smallest v:

largest V, smallest B, $v_{\min} = \dfrac{120\ \text{V}}{(0.0325\ \text{m})(0.180\ \text{T})} = 2.1\times10^4\ \text{m/s}$

largest v:

smallest V, largest B, $v_{\min} = \dfrac{560\ \text{V}}{(0.0325\ \text{m})(0.054\ \text{T})} = 3.2\times10^5\ \text{m/s}$

27.30: To pass undeflected in both cases, $E = vB = (5.85\times10^3\ \text{m/s})(1.35\ \text{T}) = 7898\ \text{N/C}.$

a) If $q = 0.640\times10^{-9}$ C, the electric field direction is given by $-(\hat{\boldsymbol{j}}\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the opposite direction to the magnetic force.

b) If $q = -0.320\times10^{-9}$ C, the electric field direction is given by $((-\hat{\boldsymbol{j}})\times(-\hat{\boldsymbol{k}})) = \hat{\boldsymbol{i}}$, since it must point in the same direction as the magnetic force, which has swapped from part (a). The electric force will now point opposite to the magnetic force for this negative charge using $\vec{\boldsymbol{F}}_e = q\vec{\boldsymbol{E}}$.

27.31: $R = \dfrac{mv}{qB} = \dfrac{mE}{qB^2} \Rightarrow m = \dfrac{RqB^2}{E} = \dfrac{(0.310\ \text{m})(1.60\times10^{-19}\ \text{C})(0.540\ \text{T})^2}{(1.12\times10^5\ \text{V/m})}$

$= 1.29\times10^{-25}\ \text{kg}$

$$\Rightarrow m(\text{amu}) = \frac{1.29 \times 10^{-25} \text{ kg}}{1.66 \times 10^{-27} \text{ kg}} = 78 \text{ atomic mass units.}$$

27.32: a) $E = vB = (1.82 \times 10^{6}\,\text{m/s})(0.650\ \text{T}) = 1.18 \times 10^{6}\,\text{V/m}$.

b) $E = V/d \Rightarrow V = Ed = (1.18 \times 10^{6}\,\text{V/m})(5.20 \times 10^{-3}\ \text{m}) = 6.14\ \text{kV}$.

27.33: a) For minimum magnitude, the angle should be adjusted so that $(\overrightarrow{B})$ is parallel to the ground, thus perpendicular to the current. To counter gravity, $ILB = mg$, so $B = \frac{mg}{IL}$.

b) We want the magnetic force to point up. With a northward current, a westward B field will accomplish this.

27.34: a) $F = Ilb = (1.20\ \text{A})\ (0.0100\ \text{m})\ (0.588\ \text{T}) = 7.06 \times 10^{-3}\ \text{N}$, and by the righthand rule, the easterly magnetic field results in a southerly force.

b) If the field is southerly, then the force is to the west, and of the same magnitude as part (a), $F = 7.06 \times 10^{-3}\ \text{N}$.

c) If the field is $30°$ south of west, the force is $30°$ west of north ($90°$ counterclockwise from the field) and still of the same magnitude, $F = 7.60 \times 10^{-6}\ \text{N}$.

27.35: $I = \dfrac{F}{lB} = \dfrac{0.13\ \text{N}}{(0.200\ \text{m})\ (0.067\ \text{T})} = 9.7\ \text{A}$.

27.36: $F = IlB = (10.8\ \text{A})(0.050\,\text{m})(0.550\,\text{T}) = 0.297\ \text{N}$.

27.37: The wire lies on the x-axis and the force on 1 cm of it is

a) $\vec{F} = I\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})(-0.65\ \text{T})(\hat{i} \times \hat{j}) = +(0.023\ \text{N})\ \hat{k}$.

b) $\vec{F} = I\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\,\text{m})(+0.56\ \text{T})(\hat{i} \times \hat{k}) = +(0.020\ \text{N})\ \hat{j}$.

c) $\vec{F} = I\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})(-0.31\ \text{T})(\hat{i} \times \hat{i}) = 0$.

d) $\vec{F} = I\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})(-0.28\ \text{T})(\hat{i} \times \hat{k}) = (-9.8 \times 10^{-3}\ \text{N})\ \hat{j}$.

e) $\vec{F} = I\vec{l} \times \vec{B} = (-3.50\ \text{A})(0.010\ \text{m})[0.74\ \text{T}\ (\hat{i} \times \hat{j}) - 0.36\ \text{T}\ (\hat{i} \times \hat{k})]$

$= -(0.026\ \text{N})\hat{k} - (0.013\ \text{N})\hat{j}$.

27.38: $$\vec{F} = I\vec{l} \times \vec{B}$$

Between the poles of the magnet, the magnetic field points to the right. Using the fingertips of your right hand, rotate the current vector by $90°$ into the direction of the

magnetic field vector. Your thumb points downward–which is the direction of the magnetic force.

27.39: a) $F_I = mg$ when bar is just ready to levitate.

$$IlB = mg,\ I = \frac{mg}{lB} = \frac{(0.750\ \text{kg})(9.80\text{m/s}^2)}{(0.500\ \text{m})(0.450\ \text{T})} = 32.67\ \text{A}$$
$$\varepsilon = IR = (32.67\ \text{A})(25.0\ \Omega) = 817\ \text{V}$$

b) $R = 2.0\ \Omega,\ I = \varepsilon/R = (816.7\ \text{V})/(2.0\ \Omega) = 408\ \text{A}$

$F_I = IlB = 92\ \text{N}$

$a = (F_I - mg)/a = 113\ \text{m/s}^2$

27.40: (a) The magnetic force on the bar must be upward so the current through it must be to the right. Therefore a must be the positive terminal.

(b) For balance, $F_{\text{magn}} = mg$

$$IlB\ \sin\theta = mg$$
$$m = \frac{IlB\ \sin\theta}{g}$$
$$I = \varepsilon/R = 175\ \text{V}/5.00\ \Omega = 35.0\ \text{A}$$
$$m = \frac{(35.0\ \text{A})(0.600\ \text{m})(1.50\ \text{T})}{9.80\ \text{m/s}^2} = 3.21\ \text{kg}$$

27.41: a) The force on the straight section along the $-x$-axis is zero.

For the half of the semicircle at negative x the force is out of the page. For the half of the semicircle at positive x the force is into the page. The net force on the semicircular section is zero.

The force on the straight section that is perpendicular to the plane of the figure is in the $-y$-direction and has magnitude $F = ILB$.

The total magnetic force on the conductor is ILB, in the $-y$-direction.

b) If the semicircular section is replaced by a straight section along the x-axis, then the magnetic force on that straight section would be zero, the same as it is for the semicircle.

27.42: a) $\tau = IBA = (6.2\ \text{A})(0.19\ \text{T})(0.050\ \text{m})(0.080\ \text{m}) = 4.71\times 10^{-3}\ \text{N}\cdot\text{m}$.

b) $\mu = IA = (6.2\ \text{A})(0.050\ \text{m})(0.080\ \text{m}) = 0.025\ \text{A}\cdot\text{m}^2$.

c) Maximum torque will occur when the area is largest, which means a circle:

$$2\pi R = 2(0.050\ \text{m} + 0.080\ \text{m}) \Rightarrow R = 0.041\ \text{m}.$$
$$\Rightarrow \tau_{\max} = IBA = (6.2\ \text{A})(0.19\ \text{T})\pi(0.04041\ \text{m})^2 = 6.22\times 10^{-3}\ \text{N}\cdot\text{m}.$$

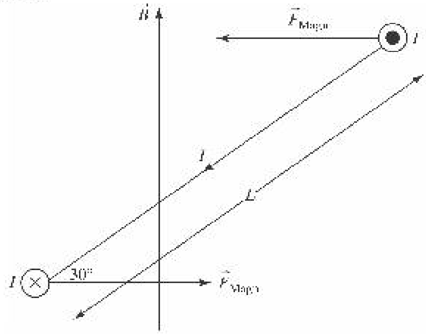
27.43: a) The torque is maximum when the plane of loop is parallel to B.

$\tau = NIBA \sin\phi \Rightarrow \tau_{max} = (15)(2.7\ \text{A})(0.56\ \text{T})\pi(0.08866\ \text{m}/2)^2 \sin 90° = 0.132\ \text{N} \cdot \text{m}.$

b) The torque on the loop is 71% of the maximum when $\sin\phi = 0.71 \Rightarrow \phi = 45°.$

27.44: (a) The force on each segment of the coil is toward the center of the coil, as the net force and net torque are both zero.

(b) As viewed from above:



As in (a), the forces cancel.

$$\begin{aligned}\sum\tau &= 2F_{\text{magn}}\frac{L}{2}\sin\theta \\ &= IlBL\sin\theta \\ &= (1.40\ \text{A})(0.220\ \text{m})(1.50\ \text{T})(0.350\ \text{m})\sin 30° \\ &= 8.09\times10^{-2}\ \text{N}\cdot\text{m}\ \textit{counterclockwise}\end{aligned}$$

27.45: a) $T = 2\pi r/v = 1.5\times10^{-16}\ \text{s}$

b) $I = Q/t = e/t = 1.1\ \text{mA}$

c) $\mu = IA = I\pi r^2 = 9.3\times10^{-24}\ \text{A}\cdot\text{m}^2$

27.46: a) $\phi = 90°: \tau = NIAB\sin(90°) = NIAB$, direction : $\hat{k}\times\hat{j} = -\hat{i}, U = -N\mu B\cos\phi = 0.$

b) $\phi = 0: \tau = NIAB\sin(0) = 0$, no direction, $U = -N\mu B\cos\phi = -NIAB.$

c) $\phi = 90°: \tau = NIAB\sin(90°) = NIAB$, direction : $-\hat{k}\times\hat{j} = \hat{i}, U = -N\mu B\cos\phi = 0.$

d) $\phi = 180°: \tau = NIAB\sin(180°) = 0$, no direction, $U = -N\mu B\cos(180°) = NIAB.$

27.47: $\Delta U = U_f - U_i = -\mu B\cos 0° + \mu B\cos 180° = -2\mu B$

$= -2(1.45\ \text{A}\cdot\text{m}^2)(0.835\ \text{T}) = -2.42\ \text{J}.$

27.48: a) $V_{ab} = \varepsilon + Ir \Rightarrow I = \dfrac{V_{ab} - \varepsilon}{r} = \dfrac{120\text{ V} - 105\text{ V}}{3.2\ \Omega} = 4.7\text{ A}.$

b) $P_{\text{supplied}} = IV_{ab} = (4.7\text{ A})(120\text{ V}) = 564\text{ W}.$

c) $P_{\text{mech}} = IV_{ab} - I^2r = 564\text{ W} - (4.7\text{ A})^2(3.2\ \Omega) = 493\text{ W}.$

27.49: a) $I_f = \dfrac{120\text{ V}}{106\ \Omega} = 1.13\text{ A}.$

b) $I_r = I_{\text{total}} - I_f = 4.82\ A - 1.13\ A = 3.69\ A.$

c) $V = \varepsilon + I_rR_r \Rightarrow \varepsilon = V - I_rR_r = 120\text{ V} - (3.69\text{ A})(5.9\ \Omega) = 98.2\text{ V}.$

d) $P_{\text{mech}} = \varepsilon I_r = (98.2\text{ V})(3.69\text{ A}) = 362\text{ W}.$

27.50: a) Field current $I_f = \dfrac{120\text{ V}}{218\ \Omega} = 0.550\text{ A}.$

b) Rotor current $I_r = I_{\text{total}} - I_f = 4.82\text{ A} - 0.550\text{ A} = 4.27\text{ A}.$

c) $V = \varepsilon + I_rR_r \Rightarrow \varepsilon = V - I_rR_r = 120\text{ V} - (4.27\text{ A})(5.9\ \Omega) = 94.8\text{ V}.$

d) $P_f = I_f^2R_f = (0.550\text{ A})^2(218\ \Omega) = 65.9\text{ W}.$

e) $P_r = I_r^2R_r = (4.27\text{ A})^2(5.9\ \Omega) = 108\text{ W}.$

f) *Power input* = (120 V) (4.82 A) = 578 W.

g) $\textit{Efficiency} = \dfrac{P_{\text{output}}}{P_{\text{input}}} = \dfrac{(578\text{ W} - 65.9\text{ W} - 108\text{ W} - 45\text{ W})}{578\text{ W}} = \dfrac{359\text{ W}}{578\text{ W}} = 0.621.$

27.51: a) $v_d = \dfrac{J}{n|q|} = \dfrac{I}{An|q|}$

$$= \frac{120\text{ A}}{(0.0118\text{ m})(2.3\times10^{-4}\text{ m})(5.85\times10^{28}\text{ m}^{-3})(1.6\times10^{-19}\text{ C})}$$

$$\Rightarrow v_d = 4.72\times10^{-3}\text{ m/s}.$$

b) $E_z = v_dB_y = (4.72\times10^{-3}\text{ m/s})(0.95\text{ T}) = 4.48\times10^{-3}\text{ N/C}$, in the $+z$-direction (negative charge).

c) $V_{\text{Hall}} = zE_z = (0.0118\text{ m})(4.48\times10^{-3}\text{ N/C}) = 5.29\times10^{-5}\text{ V}.$

27.52: $$n = \frac{J_xB_y}{|q|E_z} = \frac{IB_y}{A|q|E_z} = \frac{IB_yz_1}{A|q|\varepsilon_z} = \frac{IB_y}{y_1|q|\varepsilon}$$

$$= \frac{(78.0\text{ A})(2.29\text{ T})}{(2.3\times10^{-4}\text{ m})(1.6\times10^{-19}\text{ C})(1.31\times10^{-4}\text{ V})}$$

$$\Rightarrow n = 3.7\times10^{28}\text{ electrons per cubic meter.}$$

27.53: a) By inspection, using $\vec{F} = q\vec{v}\times\vec{B}$, $\vec{B} = -B\,\hat{j}$ will provide the correct direction for each force. Using either force, say F_2, $B = \dfrac{F_2}{q|v_2|}$.

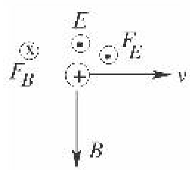
b) $F_1 = q|v_1|B\sin 45° = \dfrac{q|v_2|B}{\sqrt{2}} = \dfrac{F_2}{\sqrt{2}}(\text{since } |v_1| = |v_2|)$.

27.54: a) $\vec{F} = q\vec{v}\times\vec{B} = -qV[B_x(\hat{j}\times\hat{i}) + B_z(\hat{j}\times\hat{k})] = qVB_x\hat{k} - qVB_z\hat{i}$

b) $B_x > 0, B_z < 0$, sign of B_y doesn't matter.

c) $\vec{F} = |q|VB_x\hat{i} - |q|VB_x\hat{k}, \left|\vec{F}\right| = \sqrt{2}|q|vB_x$.

27.55: The direction of $\vec{E}$ is horizontal and perpendicular to $\vec{v}$, as shown in the sketch:



$F_B = qvB,\ F_E = qE$

$F_B = F_E$ for no deflection, so $qvB = qE$

$E = vB = (14.0\text{ m/s})(0.500\text{ T}) = 7.00\text{ V/m}$

We ignored the gravity force. If the target is 5.0 m from the rifle, it takes the bullet 0.36 s to reach the target and during this time the bullet moves downward $y - y_0 = \frac{1}{2}a_y t^2 = 0.62$ m. The magnetic and electric forces we considered are horizontal. A vertical electric field of $E = mg/q = 0.038\text{ V/m}$ would be required to cancel the gravity force. Air resistance has also been neglected.

27.56: a) Motion is circular:

$x^2 + y^2 = R^2 \Rightarrow x = D \Rightarrow y_1 = \sqrt{R^2 - D^2}$ (path of deflected particle)

$y_2 = R$ (equation for tangent to the circle, path of undeflected particle)

$$d = y_2 - y_1 = R - \sqrt{R^2 - D^2} = R - R\sqrt{1 - \frac{D^2}{R^2}} = R\left[1 - \sqrt{1 - \frac{D^2}{R^2}}\right]$$

If $R >> D \Rightarrow d \approx R\left[1 - \left(1 - \frac{1}{2}\frac{D^2}{R^2}\right)\right] = \frac{D^2}{2R}.$

For a particle moving in a magnetic field, $R = \frac{mv}{qB}.$

But $\frac{1}{2}mv^2 = qV$, so $R = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$

Thus, the deflection $d \approx \frac{D^2 B}{2}\sqrt{\frac{q}{2mV}} = \frac{D^2 B}{2}\sqrt{\frac{e}{2mV}}.$

b) $d = \frac{(0.50 \text{ m})^2(5.0\times10^{-5} \text{ T})}{2}\sqrt{\frac{(1.6\times10^{-19} \text{ C})}{2(9.11\times10^{-31} \text{ kg})(750 \text{ V})}} = 0.067 \text{ m} = 6.7 \text{ cm}.$

$d \approx 13\%$ of D, which is fairly significant.

27.57: a) $v_{\max} = \frac{qBR}{m} = \frac{(1.6\times10^{-19} \text{ C})\,(0.85 \text{ T})\,(0.40 \text{ m})}{1.67\times10^{-27} \text{ kg}} = 3.3\times10^7 \text{ m/s}.$

$\Rightarrow \text{E}_{\max} = \frac{1}{2}\,m\,v^2{}_{\max} = \frac{(1.67\times10^{-27} \text{ kg})\,(3.3\times10^7 \text{ m/s}^2)}{2} = 8.9\times10^{-13} \text{ J} = 5.5 \text{ MeV}.$

b) $T = \frac{2\pi R}{v} = \frac{2\pi(0.4 \text{ m})}{3.3\times10^7 \text{ m/s}} = 7.6\times10^{-8} \text{ s}.$

c) If the energy was to be doubled, then the speed would have to be increased by $\sqrt{2}$, as would the magnetic field. Therefore the new magnetic field would be $B_{\text{new}} = \sqrt{2}B_0 = 1.2 \text{ T}.$

d)For alpha particles,

$$E_{\max}(\alpha) = E_{\max}(p)\frac{m_p}{m_\alpha}\frac{q_\alpha{}^2}{q_p{}^2} = E_{\max}(p)\frac{m_p}{(4m_p)}\frac{(2q_p)^2}{q_p{}^2} = E_{\max}(p).$$

27.58: a) $\vec{F} = q\vec{v}\times\vec{B} = q\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ v_x & v_y & v_z\\ B_x & B_y & B_z\end{vmatrix} = q\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 0 & v\\ B_x & B_y & B_z\end{vmatrix} = -qvB_y\hat{i} + qvB_x\hat{j}.$

But $\vec{F} = 3F_0\hat{i} + 4F_0\hat{j}$, so $3F_0 = -qvB_y$ and $4F_0 = qvB_x$

$\Rightarrow B_y = -\frac{3F_0}{qv},\;\; B_x = \frac{4F_0}{qv},\;\; B_z$ is arbitrary.

b) $B = \dfrac{6F_0}{qv} = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2} = \dfrac{F_0}{qv}\sqrt{9 + 16 + B_z{}^2} = \dfrac{F_0}{qv}\sqrt{25 + B_z{}^2}$

$\Rightarrow B_z = \pm\dfrac{11F_0}{qv}$.

27.59: $f = \dfrac{\omega}{2\pi} = \dfrac{qB}{2\pi m} \Rightarrow \dfrac{f_e}{f_{Li}} = \dfrac{q_e B/2\pi m_e}{q_{Li} B/2\pi m_{Li}} = \dfrac{em_{Li}}{3em_e} = \left(\dfrac{1}{3}\right)\dfrac{1.16 \times 10^{-26}\ \text{kg}}{9.11 \times 10^{-31}\ \text{kg}} = 4244.$

27.60: a) $K = 2.7\ \text{MeV}\ (2.7 \times 10^6 \text{eV})\ (1.6 \times 10^{-19}\ \text{J/eV}) = 4.32 \times 10^{-13}\ \text{J}.$

$$\Rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4.32 \times 10^{-13}\ \text{J})}{1.67 \times 10^{-27}\ \text{kg}}} = 2.27 \times 10^7\ \text{m/s}.$$

$$\Rightarrow R = \frac{mv}{qB} = \frac{(1.67 \times 10^{-27}\ \text{kg})\ (2.27 \times 10^7\ \text{m/s})}{(1.6 \times 10^{-19}\ \text{C})\ (3.5\ \text{T})} = 0.068\ \text{m}.$$

Also, $\omega = \dfrac{v}{R} = \dfrac{2.27 \times 10^7\ \text{m/s}}{0.068\ \text{m}} = 3.34 \times 10^8\ \text{rad/s}.$

b) If the energy reaches the final value of 5.4 MeV, the velocity increases by $\sqrt{2}$, as does the radius, to 0.096 m. The angular frequency is unchanged from part (a) at $3.34 \times 10^8\ \text{rad / s}$.

27.61: a) $\vec{F} = q\vec{v} \times \vec{B} = q\left[(v_y B_z)\hat{i} - (v_x B_z)\hat{j}\right] \Rightarrow F^2 = q^2\left[(v_y B_z)^2 - (v_x B_z)^2\right]$

$$\Rightarrow q^2 = \frac{F^2}{B_z{}^2}\frac{1}{(v_y)^2 + (v_x)^2}$$

$$\Rightarrow q = \frac{-1.25\ \text{N}}{0.120\ \text{T}}\sqrt{\frac{1}{\left[4(1.05 \times 10^6\ \text{m/s})\right]^2 + \left[-3(1.05 \times 10^6\ \text{m/s})\right]^2}}$$

$$= -1.98 \times 10^{-6}\ \text{C}.$$

b) $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{q\vec{v} \times \vec{B}}{m} = \dfrac{q}{m}\left[(v_y B_z)\hat{i} - (v_x B_z)\hat{j}\right]$

$$\Rightarrow \vec{a} = \frac{-1.98 \times 10^{-6}\ \text{C}}{2.58 \times 10^{-15}\ \text{kg}}(1.05 \times 10^6\ \text{m/s})\ (-0.120\ \text{T})\left[4\hat{i} + 3\hat{j}\right]$$

$$\Rightarrow \vec{a} = 9.67 \times 10^{13} \text{m/s}^2\ \left[4\hat{i} + 3\hat{j}\right].$$

c) The motion is helical since the force is in the *xy*-plane but the velocity has a *z*-component. The radius of the circular part of the motion is:

$$R = \frac{mv}{qB} = \frac{(2.58 \times 10^{-15}\ \text{kg})\,(5)\,(1.05 \times 10^{6}\ \text{m/s})}{(1.98 \times 10^{-6}\ \text{C})\,(0.120\ \text{T})} = 0.057\ \text{m}.$$

d) $$f = \frac{\omega}{2\pi} = \frac{qB}{2\pi m} = \frac{(1.98 \times 10^{-6}\ \text{C})\,(0.120\ \text{T})}{2\pi(2.58 \times 10^{-15}\ \text{kg})} = 14.7\ \text{MHz}.$$

e) After two complete cycles, the x and y values are back to their original values, $x = R$ and $y = 0$, but z has changed.

$$z = 2Tv_z = \frac{2v_z}{f} = \frac{2(+12)\,(1.05 \times 10^{6}\ \text{m/s})}{1.47 \times 10^{7}\ \text{Hz}} = 1.71\ \text{m}.$$

27.62: a) $$\frac{mv^2}{R} = qE \Rightarrow v\sqrt{\frac{qER}{m}} = \sqrt{\frac{qV_{ab}}{m\ln(b/a)}} = \sqrt{\frac{(1.6 \times 10^{-19}\ \text{C})\,(120\ \text{V})}{(9.11 \times 10^{-31}\ \text{kg})\ln(5.00/0.100)}}$$

$$\Rightarrow v = 2.32 \times 10^{6}\ \text{m/s}.$$

b) $$\frac{mv^2}{R} = q(E + vB) \Rightarrow \left(\frac{m}{R}\right)v^2 - (qB)v - qE = 0$$

$$\Rightarrow (2.28 \times 10^{-29})\,v^2 - (2.08 \times 10^{-23})v - (1.23 \times 10^{-16}) = 0$$

$$\Rightarrow v = 2.82 \times 10^{6}\ \text{m/s or} -1.91 \times 10^{6}\ \text{m/s},$$

but we need the positive velocity to get the correct force, so $v = 2.82 \times 10^{6}\ \text{m/s}$.

c) If the direction of the magnetic field is reversed, then there is a smaller net force and a smaller velocity, and the value is the second root found in part (b),
$\Rightarrow v = 3.19 \times 10^{6}\ \text{m/s}$.

27.63: $$v = \frac{E}{B} = \frac{1.88 \times 10^{4}\ \text{N/C}}{0.701\ \text{T}} = 2.68 \times 10^{4}\ \text{m/s, and } R = \frac{mv}{qB}\text{, so:}$$

$$R_{82} = \frac{82(1.66 \times 10^{-27}\ \text{kg})\,(2.68 \times 10^{4}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})\,(0.701\ \text{T})} = 0.0325\ \text{m}.$$

$$R_{84} = \frac{84(1.66 \times 10^{-27}\ \text{kg})(2.68 \times 10^{4}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})\,(0.701\ \text{T})} = 0.0333\ \text{m}.$$

$$R_{86} = \frac{86(1.66 \times 10^{-27}\ \text{kg})\,(2.68 \times 10^{4}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})\,(0.701\ \text{T})} = 0.0341\ \text{m}.$$

So the distance between two adjacent lines is $2R = 1.6$ mm.

27.64: $F_x = q(v_yB_z - v_zB_y) = 0.$

$$F_y = q(v_z B_x - v_x B_z) = (9.45 \times 10^{-8}\ \text{C})\,(5.85 \times 10^4\ \text{m/s})\,(0.450\ \text{T})$$
$$= 2.49 \times 10^{-3}\ \text{N}.$$
$$F_z = q(v_x B_y - v_y B_x) = -\,(9.45 \times 10^{-8}\ \text{C})\,(-3.11 \times 10^4\,\text{m/s})\,(0.450\ \text{T})$$
$$= 1.32 \times 10^{-3}\ \text{N}.$$

27.65: a) $l_{ab} : \vec{F} = I\,\vec{l}_{ab} \times \vec{B} = I(l_{ab}B)\hat{\boldsymbol{j}} \times \hat{\boldsymbol{i}} = -\,(6.58\ \text{A})(0.750\ \text{m})(0.860\ \text{T})\hat{\boldsymbol{k}}$

$$= (-4.24\ \text{N})\hat{\boldsymbol{k}}.$$

$$l_{bc} : \vec{F} = I\,\vec{l}_{bc} \times \vec{B} = I(l_{bc}B)\left[\frac{(\hat{\boldsymbol{i}} - \hat{\boldsymbol{k}})}{\sqrt{2}} \times \hat{\boldsymbol{i}}\right] = -\,(6.58\ \text{A})(0.750\ \text{m})(0.860\ \text{T})\hat{\boldsymbol{j}}$$

$$= (-4.24\ \text{N})\hat{\boldsymbol{j}}.$$

$$l_{cd} : \vec{F} = I\,\vec{l}_{cd} \times \vec{B} = I(l_{cd}B)\left[\frac{(\hat{\boldsymbol{k}} - \hat{\boldsymbol{j}})}{\sqrt{2}} \times \hat{\boldsymbol{i}}\right] = -\,(6.58\ \text{A})(0.750\ \text{m})(0.860\ \text{T})\left[\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right]$$

$$\Rightarrow \vec{F} = (4.24\ \text{N})\left[\hat{\boldsymbol{j}} + \hat{\boldsymbol{k}}\right].$$

$$l_{de} : \vec{F} = I\,\vec{l}_{de} \times \vec{B} = Il_{de}B\left[-\hat{\boldsymbol{k}} \times \hat{\boldsymbol{i}}\right] = -\,(6.58\ \text{A})(0.750\ \text{m})(0.860\ \text{T})\hat{\boldsymbol{j}} = (-4.24\ \text{N})\hat{\boldsymbol{j}}$$

$$l_{ef} : \vec{F} = I\,\vec{l}_{ef} \times \vec{B} = I(l_{ef}B)(-\hat{\boldsymbol{i}}) \times \hat{\boldsymbol{i}} = 0.$$

b) Summing all the forces in part (a) we have $\vec{F}_{\text{total}} = (-4.24\ \text{N})\hat{\boldsymbol{j}}$.

27.66: a) $F = ILB$, to the right.

b) $v^2 = 2ad \Rightarrow d = \dfrac{v^2}{2a} = \dfrac{v^2 m}{2ILB}$.

c) $d = \dfrac{(1.12 \times 10^4 \text{ m/s})^2 (25 \text{ kg})}{2(2000 \text{ A})(0.50 \text{ m})(0.50 \text{ T})} = 3.14 \times 10^6 \text{ m} = 3140 \text{ km!}$

27.67: The current is to the left, so the force is into the plane.

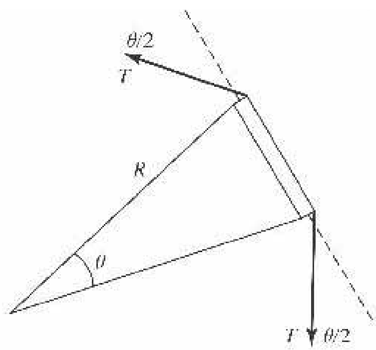
$$\sum F_y = N\cos\theta - \text{Mg} = 0 \text{ and } \sum F_x = N\sin\theta - F_B = 0.$$

$$\Rightarrow F_B = \text{Mg} \tan\theta = ILB \Rightarrow I = \frac{\text{Mg} \tan\theta}{LB}$$

27.68: a) By examining a small piece of the wire (shown below) we find:

$$F_B = ILB = 2T \sin(\theta/2)$$

$$\Rightarrow ILB \approx \frac{2T\theta}{2} = \frac{2TL/R}{2} \Rightarrow \frac{T}{IB} = R.$$



b) For a particle:

$$qvB = \frac{mv^2}{R} \Rightarrow B = \frac{mv}{Rq} = \frac{mvIB}{Tq} \Rightarrow v = \frac{Tq}{mI}.$$

27.69: a) $\dfrac{1}{2}mv_x{}^2 = qV \Rightarrow v_x = \sqrt{\dfrac{2qV}{m}}$. Also $a = \dfrac{qv_x B}{m}$, and $t = \dfrac{x}{v_x}$.

$$\Rightarrow y = \frac{1}{2}at^2 = \frac{1}{2}a\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qv_x B}{m}\right)\left(\frac{x}{v_x}\right)^2 = \frac{1}{2}\left(\frac{qBx^2}{m}\right)\left(\frac{m}{2qV}\right)^{1/2}$$

$$\Rightarrow y = Bx^2\left(\frac{q}{8mV}\right)^{\frac{1}{2}}.$$

b) This can be used for isotope separation since the mass in the denominator leads to different locations for different isotopes.

27.70: (a) During acceleration of the ions:

$$qV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2qV}{m}}$$

In the magnetic field:

$$R = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB}$$

$$m = \frac{qB^2R^2}{2V}$$

(b) $$V\frac{qB^2R^2}{2m} = \frac{(1.60\times10^{-19}\ \text{C})(0.150\ \text{T})^2(0.500\ \text{m})^2}{2(12)(1.66\times10^{-27}\ \text{kg})}$$

$$V = 2.26\times10^4\ \text{volts}$$

(c) The ions are separated by the differences in their diameters.

$$D = 2R = 2\sqrt{\frac{2Vm}{qB^2}}$$

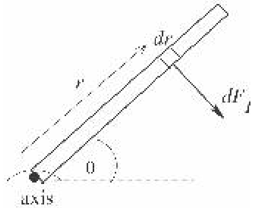
$$\Delta D = D_{14} - D_{12} = 2\left.\sqrt{\frac{2Vm}{qB^2}}\right|_{14} - 2\left.\sqrt{\frac{2Vm}{qB^2}}\right|_{12}$$

$$= 2\sqrt{\frac{2V(1\ \text{amu})}{qB^2}}\left(\sqrt{14}-\sqrt{12}\right)$$

$$= 2\sqrt{\frac{2(2.26\times10^4\ \text{V})(1.66\times10^{-27}\ \text{kg})}{(1.6\times10^{-19}\ \text{C})(0.150\ \text{T})^2}}\left(\sqrt{14}-\sqrt{12}\right)$$

$$= 8.01\times10^{-2}\ \text{m} \approx 8\ \text{cm} - \text{easily distinguishible.}$$

27.71: a)



Divide the rod into infinitesimal sections of length dr.

The magnetic force on this section is $dF_I = IB\,dr$ and is perpendicular to the rod. The torque $d\tau$ due to the force on this section is $d\tau = r dF_I = IBr\,dr$. The total torque is $\int d\tau = IB\int_0^l r\,dr = \frac{1}{2}Il^2B = 0.0442$ N/m, clockwise. This is the same torque calculated from a force diagram in which the total magnetic force $F_I = IlB$ acts at the center of the rod.

b) F_I produces a clockwise torque so the spring force must produce a counterclockwise torque. The spring force must be to the left, the spring is stretched.

Find x, the amount the spring is stretched:

$\sum \tau = 0$, axis at hinge, counterclockwise torques positive

$(kx)l\sin 53° - \frac{1}{2}Il^2B = 0$

$$x = \frac{IlB}{2k\sin 53.0°} = \frac{(6.50\text{ A})(0.200\text{ m})(0.340\text{ T})}{2(4.80\text{ N/m})\sin 53.0°} = 0.05765\text{ m}$$

$U = \frac{1}{2}kx^2 = 7.98\times10^{-3}$ J

27.72: a) $\vec{F} = I\vec{l}\times\vec{B} \Rightarrow F_{PQ} = (5.00\text{ A})(0.600\text{ m})(3.00\text{ T})\sin(0°) = 0\text{ N}$, $F_{RP} = (5.00\text{ A})(0.800\text{ m})(3.00\text{ T})\sin(90°) = 12.0$ N(into the page), $F_{QR} = (5.00\text{ A})(1.00\text{ m})(3.00\text{ T})\left(\frac{0.800}{1.00}\right) = 12.0$ N (out of the page).

b) The net force on the triangular loop of wire is zero.

c) For calculating torque on a uniform wire we can assume that the force on a wire is applied at the wire's center. Also, note that we are finding the torque with respect to the PR-axis (not about a point), and consequently the lever arm will be the distance from the wire's center to the x-axis.

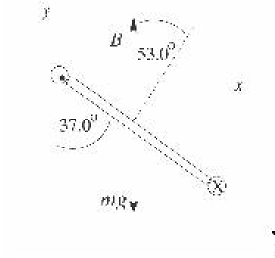
$$|\vec{\tau}| = \left|\vec{r}\times\vec{F}\right| = rF\sin(\theta) \Rightarrow \tau_{PQ} = r(0\text{ N}) = 0,\ \tau_{RP} = (0\text{ m})F\sin\theta = 0,\ \tau_{QR} =$$

$(0.300\text{ m})(12.0\text{ N})\sin(90°) = 3.60\text{ N}\cdot\text{m}$ (pointing to the right and parallel to PR)

d) According to Eqn. 27.28, $\tau = NIAB\sin\phi = (1)(5.00\text{ A})\left(\frac{1}{2}\right)(0.600\text{ m})(0.800\text{ m})(3.00\text{ T})\sin(90°) = 3.60\text{ N}\cdot\text{m}$, which agrees with part (c).

e) The point Q will be rotated out of the plane of the figure.

27.73:



$$\sum \tau = 0,$$

counterclockwise torques positive

$mg(l/2)\sin 37.0° - IAB \sin 53.0°$, with $A = l^2$

$$I = \frac{mg \sin 37°}{2lB \sin 53°} = \frac{mg \tan 37°}{2lB} = 10.0 \text{ A}$$

27.74: a) $\vec{F} = I\vec{l} \times \vec{B} = I(l\hat{k}) \times \vec{B} = Il\left[(-B_y)\hat{i} + (B_x)\hat{j}\right]$

$$\Rightarrow F_x = -IlB_y = -(9.00 \text{ A})(0.250 \text{ m})(-0.985 \text{ T}) = 2.22 \text{ N}.$$

$$F_y = IlB_x = (9.00 \text{ A})(0.250 \text{ m})(-0.242 \text{ T}) = -0.545 \text{ N}$$

$$\Rightarrow F_z = 0, \text{ since the wire is in the z - direction.}$$

b) $F = \sqrt{F_x{}^2 + F_y{}^2} = \sqrt{(2.22 \text{ N})^2 + (0.545 \text{ N})^2} = 2.29 \text{ N}.$

27.75: Summing the torques on the wire from gravity and the magnetic field will enable us to find the magnetic field value.

$$\tau_B = IAB \sin 60° = B(8.2 \text{ A})(0.060 \text{ m})(0.080 \text{ m}) \sin 60° = (0.0341 \text{ N} \cdot \text{m/T})B.$$

There are three sides to consider for the gravitational torque, leading to:

$$\tau_g = m_6 g l_6 \sin\phi + 2m_8 g l_8 \sin\phi,$$

where l_6 is the moment arm from the pivot to the far 6 cm leg and l_8 is the moment arm from the pivot to the centers of mass of the 8 cm legs.

$$\Rightarrow \tau_g = (9.8 \text{ m/s}^2)\sin 30°[(0.00015 \text{ kg/cm})(6 \text{ cm})(0.080 \text{ m})$$
$$+ 2(0.00015 \text{ kg/cm})(8 \text{ cm})(0.040 \text{ m})]$$

$$\Rightarrow \tau_g = 8.23 \times 10^{-4} \text{ N} \cdot \text{m} \Rightarrow B = \frac{8.23 \times 10^{-4} \text{ N} \cdot \text{m}}{0.0341 \text{ N} \cdot \text{m/T}} = 0.024 \text{ T, in the } y\text{-direction.}$$

27.76: a) $\tau = IAB\sin 60° = (15.0\ \text{A})(0.060\ \text{m})(0.080\ \text{m})(0.48\ \text{T})\sin 60° = 0.030\ \text{N}\cdot\text{m}$ in the $-\hat{j}$ direction. To keep the loop in place, you must provide a torque in the $+\hat{j}$ direction.

b) $\tau = IAB\sin 30° = (15.0\ \text{A})(0.60\ \text{m})(0.080\ \text{m})(0.48\ \text{T})\sin 30° = 0.017\ \text{N}\cdot\text{m}$, in the $+\hat{j}$ direction you must provide a torque in the $-\hat{j}$ direction to keep the loop in place.

c) If the loop was pivoted through its center, then there would be a torque on both sides of the loop parallel to the rotation axis. However, the lever arm is only half as large, so the total torque in each case is identical to the values found in parts (a) and (b).

27.77: $$|\vec{\tau}| = I_s|\vec{\alpha}| = I_s\frac{d|\vec{\omega}|}{dt} = -I_s\frac{d^2\phi}{dt^2}$$

but $|\vec{\tau}| = \mu B \sin\phi \cong NIAB\,\phi$ ($\sin\phi \approx \phi$) if ϕ is small)

$$\Rightarrow \frac{d^2\phi}{dt^2} = -\frac{NIAB}{I_s}\phi.$$

This describes simple harmonic motion with

$$\omega^2 = \frac{NIAB}{I_s} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I_s}{NIAB}}.$$

27.78: $$|\vec{\tau}| = \mu B\sin\phi = IAB\sin\phi.$$

$$\phi = 90°, I = qf = \frac{q\omega}{2\pi}, A = \pi r^2 = \pi\left(\frac{L}{2\pi}\right)^2 \Rightarrow \tau = \left(\frac{q\omega}{2\pi}\right)\left(\frac{\pi L^2}{4\pi^2}\right)B = \frac{q\omega\omega^2 B}{8\pi^2}.$$

27.79: The y-components of the magnetic field provide forces which cancel as you go around the loop. The x-components of the magnetic field, however, provide a net force in the $-y$- direction.

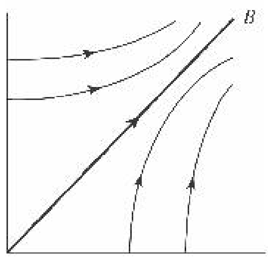
$$F = \int NIB\,dl\,\sin 60° = NIB\sin 60°\int_0^{2\pi R} dl = 2\pi\,RNIB\sin 60°$$

$$\Rightarrow F = 2\pi(0.0156/2\text{ m})\,(50)\,(0.950\text{ A})\,(0.220\text{ T})\sin 60° = 0.444\text{ N}.$$

27.80: $\sum\vec{\tau}_i = \sum\vec{r}_i \times \vec{F}_i = \sum\vec{r}_i \times \vec{F}_i - \vec{r}_p \times \sum\vec{F}_i = \sum(\vec{r}_i - \vec{r}_p) \times \vec{F}_i = \sum\vec{\tau}_i(P).$

Note that we added a term after the second equals sign that was zero because the body is in translational equilibrium.

27.81: a)



b) Side 1: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 y\, dy}{L} \hat{k} = \frac{1}{2} B_0 L I \hat{i}.$

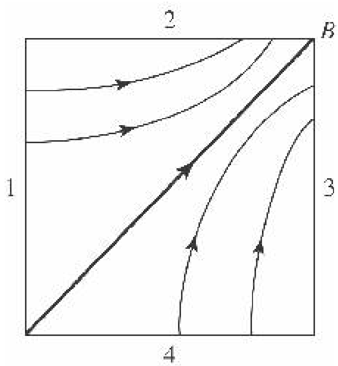
Side 2: $\vec{F} = \int_{0,y=L}^{L} I d\vec{l} \times \vec{B} = I \int_{0,y=L}^{L} \frac{B_0 y\, dx}{L} \hat{j} = -IB_0 L \hat{j}.$

Side 3: $\vec{F} = \int_{L,x=L}^{L} I d\vec{l} \times \vec{B} = I \int_{L,x=L}^{0} \frac{B_0 y\, dy}{L} (-\hat{i}) = -\frac{1}{2} IB_0 L \hat{i}.$

Side 4: $\vec{F} = \int_{L,y=0}^{0} I d\vec{l} \times \vec{B} = I \int_{L,y=0}^{0} \frac{B_0 y\, dx}{L} \hat{j} = 0.$

c) The sum of all forces is $\vec{F}_{\text{total}} = -IB_0 L \hat{j}.$

27.82: a)



b) Side 1: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 y\, dy}{L} (-\hat{k}) = -\frac{1}{2} B_0 L I \hat{k}.$

Side 2: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 x\, dx}{L} \hat{k} = \frac{1}{2} IB_0 L \hat{k}.$

Side 3: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 y dy}{L} \hat{k} = +\frac{1}{2} IB_0 L \hat{k}.$

Side 4: $\vec{F} = \int_0^L I d\vec{l} \times \vec{B} = I \int_0^L \frac{B_0 x dx}{L} (-\hat{k}) = -\frac{1}{2} IB_0 L \hat{k}.$

c) If free to rotate about the x-axis $\Rightarrow \vec{\tau} = \vec{L} \times \vec{F} = \frac{IB_0 L^2}{2} \hat{i} = \frac{1}{2} IAB_0 \hat{i}.$

d) If free to rotate about the y-axis $\Rightarrow \vec{\tau} = \vec{L} \times \vec{F} = \frac{IB_0 L^2}{2} \hat{j} = -\frac{1}{2} IAB_0 \hat{j}.$

e) The form of the torque $\vec{\tau} = \vec{\mu} \times \vec{B}$ is not appropriate, since the magnetic field is not constant.

27.83: a) $\Delta y = 0.350\text{ m} - 0.025\text{ m} = 0.325\text{ m}$, we must subtract off the amount immersed since the bar is accelerating until it leaves the pools and thus hasn't reached v_0 yet.

$$v^2 = 0 = v_0{}^2 - 2g\Delta y \Rightarrow v_0 = \sqrt{2g\Delta y.}$$
$$\Rightarrow v_0 = \sqrt{2(9.8\text{ m/s}^2)(0.325)} = 2.52\text{ m/s}.$$

b) In a distance of 0.025 m the wire's speed increases from zero to 2.52 m/s.

$\Rightarrow a = \dfrac{v^2}{2\Delta y} = \dfrac{(2.52\text{ m/s})^2}{2(0.025\text{ m})} = 127\text{ m/s}^2$. But

$$F = ILB - mg = ma \Rightarrow I = \frac{m(g+a)}{LB} = \frac{(5.40\times10^{-5}\text{ kg})\,((127+9.8)\text{ m/s}^2)}{(0.15\text{ m})\,(0.00650\text{ T})} = 7.58\text{ A}.$$

c) $V = IR \Rightarrow R = \dfrac{V}{I} = \dfrac{1.50\ V}{7.58\ A} = 0.20\ \Omega.$

27.84: a) $I_u = \dfrac{dq}{dt} = \dfrac{\Delta q}{\Delta t} = \dfrac{q_u v}{2\pi r} \Rightarrow I_u = \dfrac{ev}{3\pi r}.$

b) $\mu_u = I_u A = \dfrac{ev}{3\pi r}\pi r^2 = \dfrac{evr}{3}.$

c) Since there are two down quarks, each of half the charge of the up quark,

$$\mu_d = \mu_u = \frac{evr}{3} \Rightarrow \mu_{\text{total}} = \frac{2evr}{3}.$$

d) $v = \dfrac{3\mu}{2er} = \dfrac{3(9.66\times10^{-27}\text{ A}\cdot\text{m}^2}{2(1.60\times10^{-19}\text{ C})(1.20\times10^{-15}\text{m})} = 7.55\times10^7\text{ m/s}.$

27.85: a) $\vec{\mu} = IA\,\hat{n} = -IA\hat{k}$ using the right-hand rule.

b) $\vec{\tau} = \vec{\mu}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -IA \\ B_x & B_y & B_z \end{vmatrix} = \hat{i}(IAB_y) - \hat{j}(IAB_x).$

But $\vec{\tau} = 4D\hat{i} - 3D\hat{j}$, so $IAB_y = 4D, -IAB_x = -3D$

$$\Rightarrow B_x = \frac{3D}{IA},\quad B_y = \frac{4D}{IA}.$$

But $B_0 = \sqrt{B_x{}^2 + B_y{}^2 + B_z{}^2} = \dfrac{13D}{IA}$, so $\sqrt{\dfrac{25D^2}{I^2A^2} + B_z{}^2} = \dfrac{13D}{IA}$

$$\Rightarrow B_z = \pm\frac{12D}{IA}, \text{ but } U = -\vec{\boldsymbol{\mu}}\cdot\vec{\boldsymbol{B}} < 0, \text{ so take } B_z = -\frac{12D}{IA}.$$

27.86: a) $d\vec{l} = dl\hat{t} = Rd\theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right]$. Note that this implies that when $\theta = 0$, the line element points in the + y-direction, and when the angle is $90°$, the line element points in the – x-direction. This is in agreement with the diagram.

$$d\vec{F} = Id\vec{l} \times \vec{B} = IRd\theta\left[-\sin\theta\hat{i} + \cos\theta\hat{j}\right] \times (B_x\hat{i}) \Rightarrow d\vec{F} = IB_xRd\theta\left[-\cos\theta\hat{k}\right]$$

b) $\vec{F} = \int_0^{2\pi} -\cos\theta IB_xR\, d\theta\hat{k} = -IB_xR\int_0^{2\pi}\cos\theta d\theta\hat{k} = 0.$

c) $d\vec{\tau} = \vec{r} \times d\vec{F} = R(\cos\theta\hat{i} + \sin\theta\hat{j}) \times (IB_xR\, d\theta\left[-\cos\theta\hat{k}\right])$

$\Rightarrow d\vec{\tau} = -R^2IB_xd\theta\,(\sin\theta\cos\theta\hat{i} - \cos^2\theta\hat{j}).$

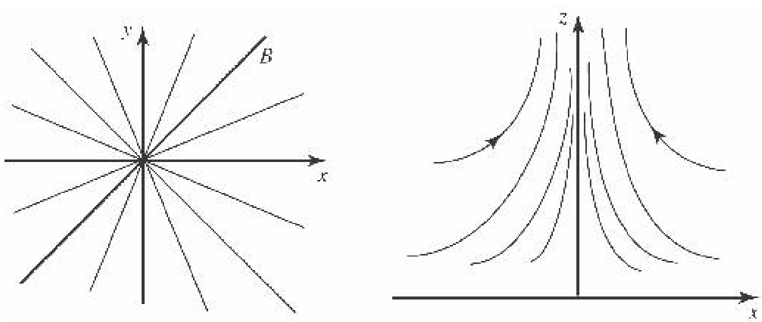
d) $\vec{\tau} = \int d\vec{\tau} = -R^2IB_x\left(\int_0^{2\pi}\sin\theta\cos\theta d\theta\hat{i} - \int_0^{2\pi}\cos^2\theta d\theta\hat{j}\right) = IR^2B_x\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)_0^{2\pi}\hat{j}$

$\Rightarrow \vec{\tau} = IR^2B_x\pi\hat{j} = I\pi R^2B_x\hat{j} = IA\hat{k} \times B_x\hat{i} \Rightarrow \vec{\tau} = \vec{\mu} \times \vec{B}$

27.87: a) $\oint \vec{B}\cdot d\vec{A} = \int_{\text{top}} B_z dA + \int_{\text{barrel}} B_r dA = \int_{\text{top}} (\beta L)dA + \int_{\text{barrel}} B_r dA = 0.$

$\Rightarrow 0 = \beta L\pi r^2 + B_r 2\pi rL \Rightarrow B_r(r) = -\dfrac{\beta r}{2}.$

b) The two diagrams show views of the field lines from the top and side:



27.88: a) $\Delta U = -(\vec{\mu}_f\cdot\vec{B} - \vec{\mu}_i\cdot\vec{B})$

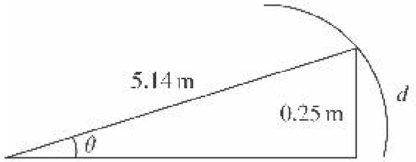
$$= -(\vec{\mu}_f - \vec{\mu}_i)\cdot\vec{B} = \left[-\mu(-\hat{k} - (-0.8\hat{i} + 0.6\hat{j}))\right]\cdot\left[B_0(12\hat{i} + 3\hat{j} - 4\hat{k})\right]$$

$\Rightarrow \Delta U = IAB_0[(-0.8)(+12) + (0.6)(+3) + (+1)(-4)]$

$\Rightarrow \Delta U = (12.5\text{ A})(4.45\times10^{-4}\text{ m}^2)(0.0115\text{ T})(-11.8) = -7.55\times10^{-4}\text{ J}.$

b) $\Delta K = \dfrac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\dfrac{2\Delta K}{I}} = \sqrt{\dfrac{2(7.55\times10^{-4}\text{ J})}{8.50\times10^{-7}\text{ kg}\cdot\text{m}^2}} = 42.1\text{ rad/s}.$

27.89: a) $R = \dfrac{mv}{qB} = \dfrac{(3.20\times10^{-11}\ \text{kg})(1.45\times10^{5}\ \text{m/s})}{(2.15\times10^{-6}\ \text{C})(0.420\ \text{T})} = 5.14\ \text{m}.$



b) The distance along the curve, d, is given by

$$d = R\theta = (5.14\ \text{m})\sin^{-1}(0.25/5.14) = 0.25\ \text{m}.$$

And $t = \dfrac{d}{v} = \dfrac{0.25\ \text{m}}{1.45\times10^{5}\ \text{m/s}} = 1.72\times10^{-6}\,\text{s}.$

c) $\Delta x_1 = d\tan(\theta/2) = (0.25\ \text{m})\tan(2.79°/2) = 6.08\times10^{-3}\ \text{m}.$

d) $\Delta x = \Delta x_1 + \Delta x_2 = 6.08\times10^{-3}\ \text{m} + (0.50\ \text{m})\tan(2.79°) = 0.0304\ \text{m}.$

27.90: a) $\Delta p = FA = IlBA = JlB.$

b) $J = \dfrac{\Delta p}{lB} = \dfrac{(1.00\ \text{atm})(1.013\times10^{5}\ \text{Pa/atm})}{(0.0350\ \text{m})(2.20\ \text{T})} = 1.32\times10^{6}\,\text{A/m}^2.$

27.91: a) The maximum speed occurs at the top of the cycloidal path, and hence the radius of curvature is greatest there. Once the motion is beyond the top, the particle is being slowed by the electric field. As it returns to $y = 0$, the speed decreases, leading to a smaller magnetic force, until the particle stops completely. Then the electric field again provides the acceleration in the y-direction of the particle, leading to the repeated motion.

b) $W = Fd = qEd = qEy = \dfrac{1}{2}mv^2 \Rightarrow v = \sqrt{\dfrac{2qEy}{m}}.$

c) At the top,

$$F_y = qE - qvB = -\frac{mv^2}{R} = -\frac{m}{2y}\frac{2qEy}{m}$$

$$= -qE \Rightarrow 2qE = qvB \Rightarrow v = \frac{2E}{B}.$$

28.1: For a charge with velocity $\vec{v} = (8.00\times10^{6}\ \text{m/s})\hat{j}$, the magnetic field produced at a position r away from the particle is $\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{q\vec{v}\times\hat{r}}{r^2}$. So for the cases below:

a) $\vec{r} = (+0.500\ \text{m})\hat{i} \Rightarrow \hat{v}\times\hat{r} = -\hat{k},\ r_0^2 = \frac{1}{4}$

$$\Rightarrow \vec{B} = -\frac{\mu_0}{4\pi}\frac{qv}{r_0^2}\hat{k} = -\frac{\mu_0}{4\pi}\frac{(6.0\times10^{-6}\ \text{C})(8.0\times10^6\,\text{m/s})}{(0.50\ \text{m})^2}\hat{k} = -(1.92\times10^{-5}\ \text{T})\hat{k} \equiv -B_0\hat{k}.$$

b) $\vec{r} = (-0.500\ \text{m})\hat{j} \Rightarrow \hat{v}\times\hat{r} = 0 \Rightarrow \vec{B} = 0.$

c) $\vec{r} = (0.500\ \text{m})\hat{k} \Rightarrow \hat{v}\times\hat{r} = +\hat{i},\ r_0^2 = \frac{1}{4}.$

$$\Rightarrow \vec{B} = +\frac{\mu_0}{4\pi}\frac{qv}{r_0^2}\hat{i} = B_0\hat{i}.$$

d) $\vec{r} = -(0.500\ \text{m})\hat{j} + (0.500\ \text{m})\hat{k} \Rightarrow \hat{v}\times\hat{r} = -\hat{i},\ r^2 = \frac{1}{2} = 2r_0$

$$\Rightarrow \vec{B} = +\frac{\mu_0}{4\pi}\frac{qv}{r^2}\frac{\hat{i}}{\sqrt{2}} = +\frac{B_0}{2}\frac{\hat{i}}{\sqrt{2}} = +\frac{B_0\hat{i}}{2\sqrt{2}}$$

28.2: $B_{\text{total}} = B + B' = \frac{\mu_0}{4\pi}\left(\frac{qv}{d^2} + \frac{q'v'}{d^2}\right)$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\left(\frac{(8.0\times10^{-6}\ \text{C})(4.5\times10^6\,\text{m/s})}{(0.120\ \text{m})^2} + \frac{(3.0\times10^{-6}\ \text{C})(9.0\times10^6\,\text{m/s})}{(0.120\ \text{m})^2}\right)$$

$\Rightarrow B = 4.38\times10^{-4}\ \text{T}$, into the page.

28.3: $\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\vec{r}}{r^3}$

a) $\vec{v} = v\hat{i},\ \vec{r} = r\hat{i};\ \ \vec{v}\times\vec{r} = \mathbf{0},\ B = 0$

b) $\vec{v} = v\hat{i},\ \vec{r} = r\hat{j};\ \ \vec{v}\times\vec{r} = vr\hat{k},\ r = 0.500\ \text{m}$

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{1\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^5\,\text{m/s})}{(0.500\ \text{m})^2} = 1.31\times10^{-6}\ \text{T}$$

q is negative, so $\vec{B} = -(1.31\times0^{-6}\ \text{T})\hat{k}$

c) $\vec{v} = v\hat{i},\ \vec{r} = (0.500\ \text{m})(\hat{i}+\hat{j});\ \ \vec{v}\times\vec{r} = (0.500\ \text{m})v\hat{k},\ r = 0.7071\,\text{m}$

$$B = \left(\frac{\mu_0}{4\pi}\right)\left(|q||\vec{v}\times\vec{r}|/r^3\right) = \frac{1\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(0.500\ \text{m})(6.80\times10^5\,\text{m/s})}{(0.7071\ \text{m})^3}$$

$B = 4.62\times10^{-7}\ \text{T};\quad \vec{B} = -(4.62\times10^{-7}\ \text{T})\hat{k}$

d) $\vec{v} = v\hat{i},\ \vec{r} = r\hat{k};\ \vec{v}\times\vec{r} = -vr\hat{j},\ r = 0.500\ \text{m}$

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{|q|v}{r^2} = \frac{1\times10^{-7}\ \text{N}\cdot\text{s}^2/\text{C}^2)(4.80\times10^{-6}\ \text{C})(6.80\times10^5\ \text{m/s})}{(0.500\ \text{m})^2}$$

$B = 1.31\times10^{-6}\ \text{T};\quad \vec{B} = (1.31\times10^{-6}\ \text{T})\hat{j}$

28.4: a) Following Example 28.1 we can find the magnetic force between the charges:

$$F_B = \frac{\mu_0}{4\pi}\frac{qq'vv'}{r^2} = (10^{-7}\ \text{T}\cdot\text{m/A})\frac{(8.00\times10^{-6}\ \text{C})(3.00\times10^{-6}\ \text{C})(4.50\times10^{6}\,\text{m/s})(9.00\times10^{6}\,\text{m/s})}{(0.240\ \text{m})^2}$$

$= 1.69\times10^{-3}$ N (the force on the upper charge points up and the force on the lower charge points down).

The Coulomb force between the charges is

$F = k\frac{q_1q_2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(8.00)(3.00)\times10^{-12}\ \text{C}^2}{(0.240\ \text{m})^2} = 3.75\ \text{N}$ (the force on the upper charge points up and the force on the lower charge points down).

The ratio of the Coulomb force to the magnetic force is $\frac{3.75\ \text{N}}{1.69\times10^{-3}\ \text{N}} = 2.22\times10^3 = \frac{c^2}{v_1v_2}$.

b) The magnetic forces are reversed when the direction of only one velocity is reversed but the magnitude of the force is unchanged.

28.5: The magnetic field is into the page at the origin, and the magnitude is

$$B = B + B' = \frac{\mu_0}{4\pi}\left(\frac{qv}{r^2} + \frac{q'v'}{r'^2}\right)$$

$$\Rightarrow B = \frac{\mu_0}{4\pi}\left(\frac{(4.0\times10^{-6}\ \text{C})(2.0\times10^{5}\text{m/s})}{(0.300\ \text{m})^2} + \frac{(1.5\times10^{-6}\ \text{C})(8.0\times10^{5}\text{m/s})}{(0.400\ \text{m})^2}\right)$$

$\Rightarrow B = 1.64\times10^{-6}$ T, into the page.

28.6: a) $q' = -q;\ B_q = \frac{\mu_0 qv}{4\pi d^2}$ into the page; $B_{q'} = \frac{\mu_0 qv'}{4\pi d^2}$ out of the page.

(i) $v' = \frac{v}{2} \Rightarrow B = \frac{\mu_0 qv}{4\pi(2d^2)}$ into the page.

(ii) $v' = v \Rightarrow B = 0.$

(iii) $v' = 2v \Rightarrow B = \frac{\mu_0 qv}{4\pi d^2}$ out of the page.

b) $\vec{F} = q'\vec{v}'\times\vec{B}_q \Rightarrow \frac{\mu_0 q^2 v'v}{4\pi(2d)^2}$ and is attractive.

c) $F_B = \frac{\mu_0 q^2 vv'}{4\pi(2d)^2},\ F_C = \frac{q^2}{4\pi\varepsilon_0(2d)^2} \Rightarrow \frac{F_B}{F_C} = \mu_0\varepsilon_0 vv' = \mu_0\varepsilon_0(3.00\times10^5\,\text{m/s})^2$

$= 1.00\times10^{-6}.$

28.7: a) $\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j} = \cos(150°)\hat{i} + \sin(150°)\hat{j} = -(0.866)\hat{i} + (0.500)\hat{j}.$

b) $d\vec{l}\times\hat{r} = (-dl\ \hat{i})\times(-(0.866)\hat{i} + (0.500)\hat{j}) = -dl(0.500)\hat{k} = -(5.00\times10^{-3}\text{m})\hat{k}$

c) $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\, d\vec{l} \times \hat{r}}{r^2} = -\frac{\mu_0}{4\pi}\frac{I\, dl\,(0.500\text{ m})}{r^2}\hat{k} = -\frac{\mu_0}{4\pi}\frac{(125\text{ A})(0.010\text{ m})(0.500\text{ m})}{(1.20\text{ m})^2}\hat{k}$

$\Rightarrow d\vec{B} = -(4.3\times 10^{-8}\text{ T})\hat{k}.$

28.8: The magnetic field at the given points is:

$$dB_a = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\text{ A})(0.000100\text{ m})}{(0.100\text{ m})^2} = 2.00\times 10^{-6}\text{ T.}$$

$$dB_b = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\text{ A})(0.000100\text{ m})\sin 45^\circ}{2(0.100\text{ m})^2} = 0.705\times 10^{-6}\text{ T.}$$

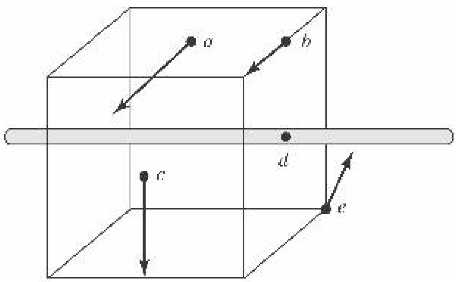
$$dB_c = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{(200\text{ A})(0.000100\text{ m})}{(0.100\text{ m})^2} = 2.00\times 10^{-6}\text{ T.}$$

$$dB_d = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin(0^\circ)}{r^2} = 0.$$

$$dB_e = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

$$\Rightarrow dB_e = \frac{\mu_0}{4\pi}\frac{(200\text{ A})(0.00100\text{ m})}{3(0.100\text{ m})^2}\frac{\sqrt{2}}{\sqrt{3}}$$

$$\Rightarrow dB_e = 0.545\times 10^{-6}\text{ T.}$$



28.9: The wire carries current in the z-direction. The magnetic field of a small piece of wire $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$ at different locations is therefore:

a) $\hat{r} = (2.00\text{ m})\hat{i} \Rightarrow \hat{l}\times\hat{r} = \hat{j}$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}\hat{j} = \frac{\mu_0}{4\pi}\frac{(4.00\text{ A})(5\times 10^{-4}\text{ m})\sin 90^\circ}{(2.00\text{ m})^2} = 5.00\times 10^{-11}\,T\hat{j}.$$

b) $\hat{r} = (2.00\text{ m})\hat{j} \Rightarrow \hat{l}\times\hat{r} = -\hat{i}.$

$$\Rightarrow d\vec{B} = \frac{-\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}\hat{i} = \frac{-\mu_0}{4\pi}\frac{(4.00\ \text{A})(5\times10^{-4}\ \text{m})\sin(90°)}{(2.00\ \text{m})^2}\hat{i}$$
$$= -5.00\times10^{-11}\ \text{T}\hat{i}.$$

c) $\vec{r} = (2.00\ \text{m})\hat{i} + (2.00\ \text{m})\hat{j} \Rightarrow \hat{i}\times\hat{r} = \frac{1}{\sqrt{2}}(\hat{j}-\hat{i})$

$$\Rightarrow d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}\frac{1}{\sqrt{2}}(\hat{j}-\hat{i}) = \frac{\mu_0}{4\pi}\frac{(4.00\ \text{A})(5.0\times10^{-4}\ \text{m})}{(2.00\ \text{m})^2+(2.00\ \text{m})^2}\frac{1}{\sqrt{2}}(\hat{j}-\hat{i})$$
$$= 1.77\times10^{-11}\ \text{T}(\hat{j}-\hat{i})$$

d) $\vec{r} = (2.00\ \text{m})\hat{k} \Rightarrow \hat{i}\times\hat{r} = 0$

28.10: a) At $x = \frac{d}{2} : B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{d/2} + \frac{1}{3d/2}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{8}{3d}\right) = \frac{4\mu_0 I}{3\pi d}$, in the $\hat{j}$ direction.

b) The position $x = -\frac{d}{2}$ is symmetrical with that of part (a), so the magnetic field there is $B = \frac{4\mu_0 I}{3\pi d}$, in the $\hat{j}$ direction.

28.11: a) At the point exactly midway between the wires, the two magnetic fields are in opposite directions and cancel.

b) At a distance a above the top wire, the magnetic fields are in the same direction and add up: $\vec{B} = \frac{\mu_0 I}{2\pi r_1}\hat{k} + \frac{\mu_0 I}{2\pi r_2}\hat{k} = \frac{\mu_0 I}{2\pi a}\hat{k} + \frac{\mu_0 I}{2\pi(3a)}\hat{k} = \frac{2\mu_0 I}{3\pi a}\hat{k}$.

c) At the same distance as part (b), but below the lower wire, yields the same magnitude magnetic field but in the opposite direction: $\vec{B} = -\frac{2\mu_0 I}{3\pi a}\hat{k}$.

28.12: The total magnetic field is the vector sum of the constant magnetic field and the wire's magnetic field. So:

a) At (0, 0, 1 m):

$$\vec{B} = \vec{B}_0 - \frac{\mu_0 I}{2\pi r}\hat{i} = (1.50\times10^{-6}\text{T})\hat{i} - \frac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{i} = -(1.0\times10^{-7}\ \text{T})\hat{i}.$$

b) At (1 m, 0, 0):

$$\vec{B} = \vec{B}_0 + \frac{\mu_0 I}{2\pi r}\hat{k} = (1.50\times10^{-6}\text{T})\hat{i} + \frac{\mu_0(8.00\ \text{A})}{2\pi(1.00\ \text{m})}\hat{k}$$

$$\Rightarrow \vec{B} = (1.50\times10^{-6}\ \text{T})\hat{i} + (1.6\times10^{-6}\ \text{T})\hat{k} = 2.19\times10^{-6}\ \text{T},\ \text{at}\ \theta = 46.8°$$

from x to z.

c) At (0, 0, – 0.25 m): $\vec{B} = \vec{B}_0 + \dfrac{\mu_0 I}{2\pi r}\hat{i} = (1.50\times10^{-6}\,\text{T})\hat{i} + \dfrac{\mu_0(8.00\ \text{A})}{2\pi(0.25\ \text{m})}\hat{i}$

$$= (7.9\times10^{-6}\ \text{T})\hat{i}.$$

28.13: $B = \dfrac{\mu_0 I}{4\pi}\displaystyle\int_{-a}^{a}\dfrac{x\,dy}{(x^2+y^2)^{3/2}} = \dfrac{\mu_0 Ix}{4\pi}\dfrac{y}{x^2(x^2+y^2)^{1/2}}\Bigg|_{-a}^{a} = \dfrac{\mu_0 I}{4\pi}\dfrac{2a}{x(x^2+a^2)^{1/2}}.$

28.14: a) $B_0 = \dfrac{\mu_0 I}{2\pi r} \Rightarrow I = \dfrac{2\pi r B_0}{\mu_0} = \dfrac{2\pi(0.040\ \text{m})\,(5.50\times10^{-4}\ \text{T})}{\mu_0} = 110\ \text{A}.$

b) $B = \dfrac{\mu_0 I}{2\pi r}$, so $B(r = 0.080\ \text{m}) = \dfrac{B_0}{2} = 2.75\times10^{-4}\ \text{T},$

$B(r = 0.160\ \text{m}) = \dfrac{B_0}{4} = 1.375\times10^{-4}\ \text{T}.$

28.15: a) $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\mu_0(800\ \text{A})}{2\pi(5.50\ \text{m})} = 2.90\times10^{-5}\ \text{T},$ to the east.

b) Since the magnitude of the earth's magnetic filed is 5.00×10^{-5} T, to the north, the total magnetic field is now 30° east of north with a magnitude of 5.78×10^{-5} T. This could be a problem!

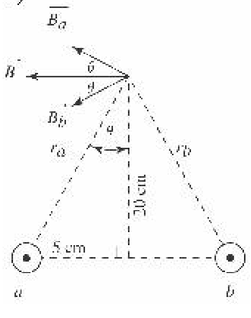
28.16: a) $B = 0$ since the fields are in opposite directions.

b) $B = B_a + B_b = \dfrac{\mu_0 I}{2\pi r_a} + \dfrac{\mu_0 I}{2\pi r_b} = \dfrac{\mu_0 I}{2\pi}\left(\dfrac{1}{r_a} + \dfrac{1}{r_b}\right)$

$= \dfrac{(4\pi\times10^{-7}\,\text{Tm/A})\,(4.0\ \text{A})}{2\pi}\left(\dfrac{1}{0.3\ \text{m}} + \dfrac{1}{0.2\ \text{m}}\right)$

$= 6.67\times10^{-6}\ \text{T} = 6.67\ \mu\text{T}$

c)



Note that $\vec{B}_a \perp r_a$ and $\vec{B}_b \perp r_b$

$$B = B_a \cos\theta + B_b \cos\theta$$
$$= 2B_a \cos\theta$$

$$\tan\ \theta = \frac{5}{20} \rightarrow \theta = 14.04° : r_a = \sqrt{(0.20\text{ m})^2 + (0.05\text{ m})^2}$$

$$B = 2\frac{\mu_0 I}{2\pi r_a}\cos\theta$$
$$= 2\frac{(4\pi + 10^{-7}\text{ Tm/A})\,(4.0\text{ A})}{2\pi\,\sqrt{(0.20\text{ m})^2 + (0.05\text{ m})^2}}\cos 14.04°$$

$= 7.53 \times 10^{-6}$ T $= 7.53\ \mu$T, to the left.

28.17: The only place where the magnetic fields of the two wires are in opposite directions is between the wires, in the plane of the wires.

Consider a point a distance x from the wire carrying I_2 = 75.0 A. B_{tot} will be zero where $B_1 = B_2$.

$$\frac{\mu_0 I_1}{2\pi(0.400\text{ m} - x)} = \frac{\mu_0 I_2}{2\pi x}$$
$$I_2(0.400\text{ m} - x) = I_1 x;\ \ I_1 = 25.0\text{ A}, I_2 = 75.0\text{ A}$$

$x = 0.300$ m; $B_{tot} = 0$ along a line 0.300 m from the wire carrying 75.0 A amd 0.100 m from the wire carrying current 25.0 A.

b) Let the wire with $I_1 = 25.0$ A be 0.400 m above the wire with $I_2 = 75.0$ A. The magnetic fields of the two wires are in opposite directions in the plane of the wires and at points above both wires or below both wires. But to have $B_1 = B_2$ must be closer to wire #1 since $I_1 < I_2$, so can have $B_{tot} = 0$ only at points above both wires.

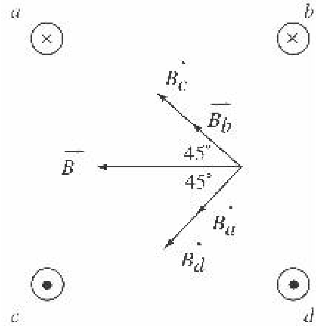
Consider a point a distance x from the wire carrying $I_1 = 25.0$ A. B_{tot} will be zero where $B_1 = B_2$.

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi\ (0.400\text{ m} + x)}$$
$$I_2 x = I_1(0.400\text{ m} + x);\ \ x = 0.200\text{ m}$$

$B_{tot} = 0$ along a line 0.200 m from the wire carrying 25.0 A and 0.600 m from the wire carrying current $I_2 = 75.0$ A.

28.18: (a) and (b) $B = 0$ since the magnetic fields due to currents at opposite corners of the square cancel.

(c)



$$B = B_a \cos 45° + B_b \cos 45° + B_c \cos 45° + B_d \cos 45°$$

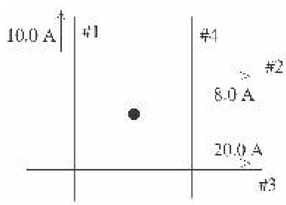
$$= 4B_a \cos 45° = 4\left(\frac{\mu_0 I}{2\pi r}\right)\cos 45°$$

$$r = \sqrt{(10\,\text{cm})^2 + (10\,\text{cm})^2} = 10\sqrt{2}\ \text{cm} = 0.10\sqrt{2}\ \text{m}$$

$$B = 4\frac{(4\pi \times 10^{-7}\ \text{Tm/A})(100\ \text{A})}{2\pi(0.10\sqrt{2}\ \text{m})}\cos 45°$$

$$= 4.0 \times 10^{-4}\ \text{T, to the left.}$$

28.19:



$\vec{B}_1 \otimes, \vec{B}_2 \otimes, \vec{B}_3 \odot$

$$B = \frac{\mu_0 I}{2\pi r};\ r = 0.200\ \text{m for each wire}$$

$B_1 = 1.00 \times 10^{-5}$ T, $B_2 = 0.80 \times 10^{-5}$ T , $B_3 = 2.00 \times 10^{-5}$ T

Let $\odot$ be the positive z-direction. $I_1 = 10.0$ A, $I_2 = 8.0$ A, $I_3 = 20.0$ A

$B_{1z} = -1.00 \times 10^{-5}$ T, $B_{2z} = -0.80 \times 10^{-5}$ T, $B_{3z} = +2.00 \times 10^{-5}$ T

$B_{1z} + B_{2z} + B_{3z} + B_{4z} = 0$

$B_{4z} = -(B_{1z} + B_{2z} + B_{3z}) = -2.0 \times 10^{-6}$ T

To give $\vec{B}_4$ in the $\otimes$ direction the current in wire 4 must be toward the bottom of the page.

$$B_4 = \frac{\mu_0 I}{2\pi r} \text{ so } I_4 = \frac{rB_4}{(\mu_0/2\pi)} = \frac{(0.200\ \text{m})(2.0 \times 10^{-6}\ \text{T})}{(2 \times 10^{-7}\ \text{T}\cdot\text{m/A})} = 2.0\ \text{A}$$

28.20: On the top wire: $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi}\left(\frac{1}{d} - \frac{1}{2d}\right) = \frac{\mu_0 I^2}{4\pi d}$, upward.

On the middle wire, the magnetic fields cancel so the force is zero.

On the bottom wire: $\frac{F}{L} = \frac{\mu_0 I^2}{2\pi}\left(-\frac{1}{d} + \frac{1}{2d}\right) = \frac{\mu_0 I^2}{4\pi d}$, downward.

28.21: We need the magnetic and gravitational forces to cancel:

$$\Rightarrow \lambda L g = \frac{\mu_0 I^2 L}{2\pi h} \Rightarrow h = \frac{\mu_0 I^2}{2\pi \lambda g}$$

28.22: a) $F = \frac{\mu_0 I_1 I_2 L}{2\pi r} = \frac{\mu_0 (5.00\text{ A})(2.00\text{ A})(1.20\text{ m})}{2\pi(0.400\text{ m})} = 6.00 \times 10^{-6}$ N, and the force is repulsive since the currents are in opposite directions.

b) Doubling the currents makes the force increase by a factor of four to $F = 2.40 \times 10^{-5}$ N.

28.23: $\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r} \Rightarrow I_2 = \frac{F}{L}\frac{2\pi r}{\mu_0 I_1} = (4.0 \times 10^{-5}\text{ N/m})\frac{2\pi(0.0250\text{ m})}{\mu_0(0.60\text{ A})} = 8.33$ A.

b) The two wires repel so the currents are in opposite directions.

28.24: There is no magnetic field at the center of the loop from the straight sections. The magnetic field from the semicircle is just half that of a complete loop:

$$B = \frac{1}{2}B_{\text{loop}} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right) = \frac{\mu_0 I}{4R},$$

into the page.

28.25: As in Exercise 28.24, there is no contribution from the straight wires, and now we have two oppositely oriented contributions from the two semicircles:

$$B = (B_1 - B_2) = \frac{1}{2}\left(\frac{\mu_0}{2R}\right)\left|I_1 - I_2\right|,$$

into the page. Note that if the two currents are equal, the magnetic field goes to zero at the center of the loop.

28.26: a) The field still points along the positive x-axis, and thus points into the loop from this location.

b) If the current is reversed, the magnetic field is reversed. At point P the field would then point into the loop.

c) Point the thumb of your right hand in the direction of the magnetic moment, under the given circumstances, the current would appear to flow in the direction that your fingers curl (*i.e.*, clockwise).

28.27: a) $B_x = \mu_0 NI/2a$, so $I = \dfrac{2aB_x}{\mu_0 N} = \dfrac{2(0.024\text{ m})(0.0580\text{ T})}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(800)} = 2.77\text{ A}$

b) At the center, $B_c = \mu_0 NI/2a$. At a distance x from the center,

$$B_x = \frac{\mu_0 NIa^2}{2(x^2 + a^2)^{3/2}} = \left(\frac{\mu_0 NI}{2a}\right)\left(\frac{a^3}{(x^2 + a^2)^{3/2}}\right) = B_c \left(\frac{a^3}{(x^2 + a^2)^{3/2}}\right)$$

$B_x = \frac{1}{2} B_c$ means $\dfrac{a^3}{(x^2 + a^2)^{3/2}} = \dfrac{1}{2}$

$(x^2 + a^2)^3 = 4a^6$, with $a = 0.024$ m, so $x = 0.0184$ m

28.28: a) From Eq. (29-17), $B_{\text{center}} = \dfrac{\mu_0 NI}{2a} = \dfrac{\mu_0(600)\,(0.500\text{ A})}{2(0.020\text{ m})} = 9.42\times10^{-3}\text{ T}.$

b) From Eq. (29-16),

$$B(x) = \frac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}} \Rightarrow B(0.08\text{ m}) = \frac{\mu_0(600)\,(0.500\text{ A})\,(0.020\text{ m})^2}{2((0.080\text{ m})^2+(0.020\text{ m})^2)^{3/2}} = 1.34\times10^{-4}\text{ T}.$$

28.29: $B(x) = \dfrac{\mu_0 NIa^2}{2(x^2+a^2)^{3/2}} \Rightarrow N = \dfrac{2B(x)\,(x^2+a^2)^{3/2}}{\mu_0 Ia^2}$

$$= \frac{2(6.39\times10^{-4}\text{ T})\left[(0.06\text{ m})^2+(0.06\text{ m})^2\right]^{3/2}}{\mu_0(2.50\text{ A})\,(0.06\text{ m})^2} = 69$$

28.30: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} = 3.83\times10^{-4}\text{ T}\cdot\text{m} \Rightarrow I_{\text{encl}} = 305\text{ A}.$

b) -3.83×10^{-4} since $d\vec{l}$ points opposite to $\vec{B}$ everywhere.

28.31: We will travel around the loops in the counterclockwise direction.

a) $I_{\text{encl}} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{l} = 0.$

b) $I_{\text{encl}} = -I_1 = -4.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = -\mu_0(4.0\text{ A}) = -5.03\times10^{-6}\text{T}\cdot\text{m}.$

c) $I_{\text{encl}} = -I_1 + I_2 = -4.0\text{ A} + 6.0\text{ A} = 2.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0(2.0\text{ A})$

$= 2.51\times10^{-6}\text{ T}\cdot\text{m}.$

d) $I_{\text{encl}} = -I_1 + I_2 + I_3 = 4.0\text{ A} \Rightarrow \oint \vec{B}\cdot d\vec{l} = +\mu_0(4.0\text{ A}) = 5.03\times10^{-6}\text{ T}\cdot\text{m}.$

Using Ampere's Law in each case, the sign of the line integral was determined by using the right-hand rule. This determines the sign of the integral for a counterclockwise path.

28.32: Consider a coaxial cable where the currents run in OPPOSITE directions.

a) For $a < r < b, I_{encl} = I \Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}.$

b) For $r > c$, the enclosed current is zero, so the magnetic field is also zero.

28.33: Consider a coaxial cable where the currents run in the SAME direction.

a) For $a < r < b, I_{\text{encl}} = I_1 \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I_1 \Rightarrow B2\pi r = \mu_0 I_1 \Rightarrow B = \frac{\mu_0 I_1}{2\pi r}$.

b) For $r > c, I_{\text{encl}} = I_1 + I_2 \Rightarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 (I_1 + I_2) \Rightarrow B2\pi r = \mu_0 (I_1 + I_2)$

$$\Rightarrow B = \frac{\mu_0 (I_1 + I_2)}{2\pi r}.$$

28.34: Using the formula for the magnetic field of a solenoid:

$$B = \mu_0 nI = \frac{\mu_0 NI}{L} = \frac{\mu_0 (600)\,(8.00\text{ A})}{(0.150\text{ m})} = 0.0402\text{ T}.$$

28.35: a) $B = \frac{\mu_0 NI}{L} \Rightarrow N = \frac{BL}{\mu_0 I} = \frac{(0.0270\text{ T})\,(0.400\text{ m})}{\mu_0 (12.0\text{ A})} = 716\text{ turns}$

$$\Rightarrow n = \frac{N}{L} = \frac{716\text{ turns}}{0.400\text{ m}} = 1790\text{ turns/m}.$$

b) The length of wire required is $2\pi rN = 2\pi (0.0140\text{ m})(116) = 63\text{ m}$.

28.36:

$$B = \mu_0 I \frac{N}{L}$$

$$I = \frac{BL}{\mu_0 N}$$

$$= \frac{(0.150\text{ T})\,(1.40\text{ m})}{(4\pi \times 10^{-7}\text{ Tm/A})(4000)}$$

$$= 41.8\text{ A}$$

28.37:

a) $B = \frac{\mu_0 I}{2\pi r}$, so $I = \frac{Br}{(\mu_0 / 2\pi)} = 3.72 \times 10^6\text{ A}$

b) $B_x = \frac{\mu_0 NI}{2a}$, so $I = \frac{2aB_x}{\mu_0 N} = 2.49 \times 10^5\text{ A}$

c) $B = \mu_0 nI = \mu_0 (N/L) I$, so $I = BL/\mu_0 N = 237\text{ A}$

28.38: Outside a toroidal solenoid there is no magnetic field and inside it the magnetic field is given by $B = \frac{\mu_0 NI}{2\pi r}$.

a) $r = 0.12$ m, which is outside the toroid, so $B = 0$.

b) $r = 0.16\ \text{m} \Rightarrow B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0 (250)\,(8.50\ \text{A})}{2\pi (0.160\ \text{m})} = 2.66 \times 10^{-3}\ \text{T}.$

c) $r = 0.20$ m, which is outside the toroid, so $B = 0$

28.39: $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{\mu_0 (600)\,(0.650\ \text{A})}{2\pi\ (0.070\ \text{m})} = 1.11 \times 10^{-3}\ \text{T}.$

28.40: a) $B = \dfrac{\mu\, NI}{2\pi r} = \dfrac{K_m \mu_0 NI}{2\pi r} = \dfrac{\mu_0 (80)\,(400)\,(0.25\ \text{A})}{2\pi\ (0.060\ \text{m})} = 0.0267\ \text{T}.$

b) The fraction due to atomic currents is $B' = \frac{79}{80} B = \frac{79}{80}(0.0267\ \text{T}) = 0.0263\ \text{T}.$

28.41: a) If $K_m = 1400 \Rightarrow B = \dfrac{K_m \mu_0 NI}{2\pi r} \Rightarrow I = \dfrac{2\pi r\, B}{K_m \mu_0 N} = \dfrac{2\pi(0.0290\text{ m})\,(0.350\text{ T})}{\mu_0(1400)(500)} =$ 0.0725 A.

b) If $K_m = 5200 \Rightarrow I = \dfrac{1400}{5200} I_{\text{part(a)}} = 0.0195\text{ A}.$

28.42: a) $B = \dfrac{K_m \mu_0 NI}{2\pi r} \Rightarrow K_m = \dfrac{2\pi r B}{\mu_0 NI} = \dfrac{2\pi(0.2500\text{ m})\,(1.940\text{ T})}{\mu_0(500)(\,2.400\text{ A})} = 2021.$

b) $X_m = K_m - 1 = 2020.$

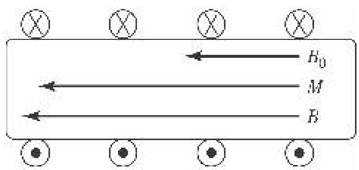
28.43: a) The magnetic field from the solenoid alone is:

(i) $B_0 = \mu_0 nI = \mu_0(6000\text{ m}^{-1})\,(0.15\text{ A}) \Rightarrow B_0 = 1.13 \times 10^{-3}\text{ T}.$

(ii) But $M = \dfrac{K_m - 1}{\mu_0} B_0 = \dfrac{5199}{\mu_0}(1.13 \times 10^{-3}\text{ T}) \Rightarrow M = 4.68 \times 10^6\text{ A/m}.$

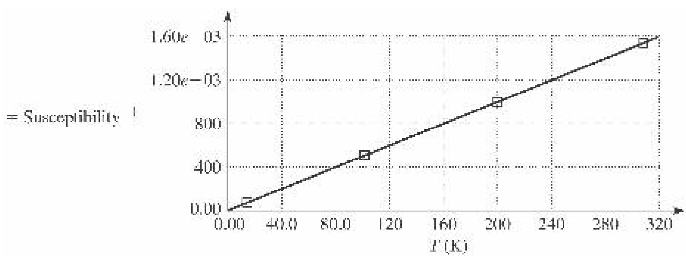
(iii) $B = K_m B_0 = (5200)(1.13 \times 10^{-3}\text{ T}) = 5.88\text{ T}.$

b)



28.44: $\left[\dfrac{\text{J}}{\text{T}}\right] = \left[\dfrac{\text{N}\cdot\text{m}}{\text{N}\cdot\text{s/C}\cdot\text{m}}\right] = \left[\dfrac{\text{C}\cdot\text{m}^2}{\text{s}}\right] = \left[\text{A}\cdot\text{m}^2\right]$

28.45:



The material does obey Curie's Law because we have a straight line for temperature against one over the magnetic susceptibility. The Curie constant from the graph is $C = \dfrac{1}{\mu_0(\text{slope})} = \dfrac{1}{\mu_0(5.13)} = 1.55 \times 10^5\ \text{K} \cdot \text{A/T} \cdot \text{m}.$

28.46: The magnetic field of charge q' at the location of charge q is into the page.

$$\vec{F} = q\vec{v} \times \vec{B}' = (qv)\hat{i} \times \frac{\mu_0}{4\pi}\frac{q\vec{v}' \times \hat{r}}{r^2} = (qv)\hat{i} \times \left(\frac{\mu_0}{4\pi}\frac{qv'\sin\theta}{r^2}\right)(-\hat{k}) = \left(\frac{\mu_0}{4\pi}\frac{qq'\sin\theta}{r^2}\right)\hat{j}$$

where θ is the angle between v' and $\hat{r}'$.

$$\Rightarrow \vec{F} = \left(\frac{\mu_0}{4\pi}\frac{(8.00\times10^{-6}\ \text{C})(5.00\times10^{-6}\ \text{C})(9.00\times10^{4}\ \text{m/s})(6.50\times10^{4}\ \text{m/s})}{(0.500\ \text{m})^2}\left(\frac{0.4}{0.5}\right)\right)\hat{j}$$

$$\Rightarrow \vec{F} = (7.49\times10^{-8}\ \text{N})\hat{j}.$$

28.47: $$F = qvB = qv\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0}{2\pi}\frac{(1.60\times10^{-19}\text{C})(6.00\times10^{4}\ \text{m/s})(2.50\ \text{A})}{(0.045\ \text{m})}$$
$$= 1.07\times10^{-19}\ \text{N}.$$

Let the current run left to right, the electron moves in the opposite direction, below the wire, then the magnetic field at the electron is into the page, and the electron feels a force upward, toward the wire, by the right-hand rule (remember the electron is negative).

28.48: (a) $$a = \frac{F}{m} = \frac{qvB\sin\theta}{m} = \frac{ev}{m}\left(\frac{\mu_0 I}{2\pi r}\right)$$

$$a = \frac{(1.6\times10^{-17}\text{C})(250{,}000\text{m/s})(4\pi\times10^{-7}\,\text{Tm/A})(25\ \text{A})}{(9.11\times10^{-31}\,\text{kg})(2\pi)(0.020\ \text{m})}$$
$$= 1.1\times10^{13}\ \text{m/s}^2,\ \text{away from the wire.}$$

b) The electric force must balance the magnetic force.

$$eE = eVB$$

$$E = vB = v\frac{\mu_0 i}{2\pi r}$$

$$= \frac{(250,000\ \text{m/s})(4\pi\times10^{-7}\ \text{Tm/A})(25\ \text{A})}{2\pi(0.020\ \text{m})}$$

$$= 62.5\ \text{N/C, away from the wire.}$$

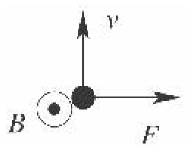
(c) $$mg = (9.11\times10^{-31}\ \text{kg})(9.8\ \text{m/s}^2) \approx 10^{-29}\ \text{N}$$
$$F_{el} = eE = (1.6\times10^{-19}\ \text{C})(62.5\ \text{N/C}) \approx 10^{-17}\ \text{N}$$
$$F_{el} \approx 10^{12}\ F_{grav},$$ so we can neglect gravity.

28.49: Let the wire connected to the 25.0 Ω resistor be #2 and the wire connected to the 10.0 Ω resistor be #1. Both I_1 and I_2 are directed toward the right in the figure, so at the location of the proton I_2 *is* ⊗ and $I_1 = \odot$

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \text{ and } B_2 = \frac{\mu_0 I_2}{2\pi r}, \text{ with r} = 0.0250 \text{ m.}$$

$$B_1 = 8.00 \times 10^{-5} \text{ T}, B_2 = 3.20 \times 10^{-5} \text{ T and } B = B_1 - B_2 = 4.80 \times 10^{-5} \text{ T}$$

and in the direction $\odot$.



Force is to the right.

$$F = qvB = (1.602 \times 10^{-19}\text{C})(650 \times 10^3 \text{ m/s})(4.80 \times 10^{-5} \text{ T}) = 5.00 \times 10^{-18} \text{ N}$$

28.50: The fields add

$$B = B_1 + B_2 = 2B_1 = 2\left[\frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}\right]$$

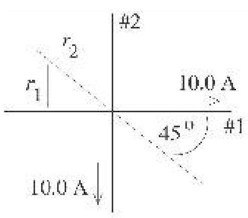
$$= \frac{(4\pi \times 10^{-7}\text{Tm/A})(1.50 \text{ A})(0.20 \text{ m})^2}{[(0.20 \text{ m})^2 + (0.125 \text{ m})^2]^{3/2}} = 5.75 \times 10^{-6} \text{ T}$$

$$F = qv\, B \sin\theta$$

$$= (1.6 \times 10^{-19} \text{ C})(2400 \text{ m/s})(5.75 \times 10^{-6} \text{ T}) \sin 90°$$

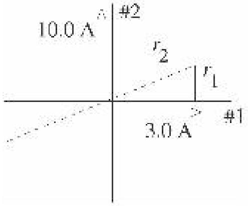
$= 2.21 \times 10^{-21}$ N, perpendicular to the line ab and to the velocity.

28.51: a)



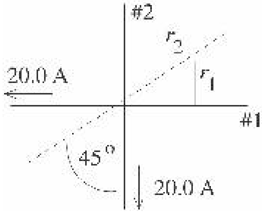
Along the dashed line, $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions. If the line has slope -1.00 then $r_1 = r_2$ and $B_1 = B_2$, so $B_{\text{tot}} = 0$.

b)



Along the dashed line, $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions.
If the line has slope $1/3$ then $r_2 = (10.0/3.0)r_1$
and $B_1 = B_2$, so $B_{\text{tot}} = 0$.

c)



Along the dashed line, $\vec{B}_1$ and $\vec{B}_2$ are in opposite directions.
If the line has slope $+\,1.00$ then $r_1 = r_2$ and $B_1 = B_2$, so $B_{\text{tot}} = 0$.

28.52: a) $$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}_0 \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_{0x} & v_{0y} & v_{0z} \\ 1 & 0 & 0 \end{vmatrix}$$

$$= \frac{\mu_0}{4\pi}\frac{q}{r^2}\left(v_{0z}\hat{j} - v_{0y}\hat{k}\right) = (6.00 \times 10^{-6}\ \text{T})\hat{j}$$

$$\Rightarrow \frac{\mu_0}{4\pi}\frac{q}{r^2}v_{0y} = 0 \Rightarrow v_{0y} = 0 \text{ and } -\frac{\mu_0}{4\pi}\frac{|q|}{r^2}v_{0z} = 6.00 \times 10^{-6}\ \text{T}$$

$$\Rightarrow v_{0z} = -\frac{4\pi(6.00 \times 10^{-6}\ \text{T})(0.25\ \text{m})^2}{\mu_0(-7.20 \times 10^{-3}\ \text{C})} = -521\ \text{m/s}.$$

And $v_{0x} = \pm\sqrt{v_0^2 - {v_{0y}}^2 - {v_{0z}}^2} = \pm\sqrt{(800\ \text{m/s})^2 - (-521\ \text{m/s})^2} = \pm 607\ \text{m/s}.$

b) $$\vec{B}(0, 0.250\ \text{m}, 0) = \frac{\mu_0}{4\pi}\frac{q\vec{v}_0 \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ v_{0x} & v_{0y} & v_{0z} \\ 0 & 1 & 0 \end{vmatrix} = +\frac{\mu_0}{4\pi}\frac{q}{r^2}\left(v_{0x}\hat{k} - v_{0z}\hat{i}\right)$$

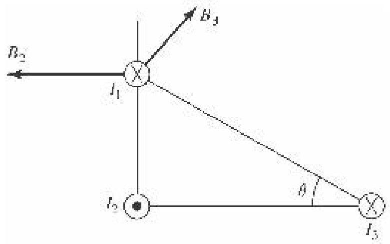
$$\Rightarrow B(0,\, 0.250\text{ m},\, 0) = \frac{\mu_0}{4\pi}\frac{|q|}{r^2}v_0 = \frac{\mu_0}{4\pi}\frac{(-7.20\times 10^{-3}\text{ C})}{(0.250\text{ m})^2}800\text{ m/s} = 9.2\times 10^{-6}\text{ T}.$$

28.53: Choose a cube of edge length L, with one face on the y-z plane. Then:

$$0=\oint \vec{B}\cdot d\vec{A}=\iint_{x=L} \vec{B}\cdot d\vec{A}=\iint_{x=L} \frac{B_0 x}{a}\hat{i}\cdot d\vec{A}=\frac{B_0 L}{a}\iint_{x=L} dA=\frac{B_0 L^3}{a}\Rightarrow B_0=0,$$

so the only possible field is a zero field.

28.54: a)



b) $\vec{B}_2=-\left(\frac{\mu_0 I_2}{2\pi r^2}\right)\hat{i}$ $\qquad$ $\vec{B}_3=\left(\frac{\mu_0 I_3}{2\pi r_3}\right)(\sin\theta\hat{i}+\cos\theta\hat{j})$

And so

$$\vec{B}=\left(\frac{\mu_0}{2\pi}\right)\left(\left(-\frac{I_2}{r_2}+\frac{I_3}{r_3}\sin\theta\right)\hat{i}+\frac{I_3}{r_3}\cos\theta\,\hat{j}\right)$$

$$\Rightarrow \vec{B}=\left(\frac{\mu_0}{2\pi}\right)\left(\left(-\frac{I_2}{(0.030\text{ m})}+\frac{I_3}{(0.050\text{ m})}(0.6)\right)\hat{i}+\frac{I_3}{(0.050\text{ m})}(0.8)\hat{j}\right)$$

$$\Rightarrow \vec{B}=\left(\frac{\mu_0}{2\pi}\right)\left((12I_3-33.3I_2)\,\hat{i}+(16I_3)\,\hat{j}\right)$$

$$=\frac{\mu_0}{2\pi}\left((12)(4.00\text{ A})-(33.3)(2.00\text{ A}))\,\hat{i}+(16)(4.00\text{ A})\hat{j}\right)$$

$$=-3.72\times10^{-6}\text{ T}\hat{i}+1.28\times10^{-5}\text{ T}\hat{j}.$$

c) $\vec{F}=I_1\vec{l}\times\vec{B}=I_1 lB_x\hat{j}+I_1 lB_y\hat{i}$

$$\Rightarrow (1.00\text{ A})(0.010\text{ m})[(3.72\times10^{-6}\text{ T})\hat{j}+(1.28\times10^{-5}\text{ T})\hat{i}]$$

$=3.72\times10^{-8}\text{ T}\hat{j}+1.28\times10^{-7}\text{ T}\hat{i}$; $F=1.33\times10^{-7}$ N, $16.2°$ counterclockwise from $+x$-axis.

28.55: a) If the magnetic field at point P is zero, then from Figure (28.46) the current I_2 must be out of the page, in order to cancel the field from I_1. Also:

$$B_1=B_2\Rightarrow\frac{\mu_0 I_1}{2\pi r_1}=\frac{\mu_0 I_2}{2\pi r_2}\Rightarrow I_2=I_1\frac{r_2}{r_1}=(6.00\text{ A})\frac{(0.500\text{ m})}{(1.50\text{ m})}=2.00\text{ A}.$$

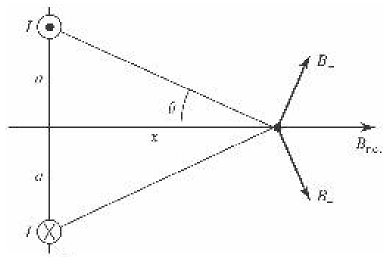
b) Given the currents, the field at Q points to the right and has magnitude

$$B_Q = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = \frac{\mu_0}{2\pi}\left(\frac{6.00\text{ A}}{0.500\text{ m}} - \frac{2.00\text{ A}}{1.50\text{ m}}\right) = 2.13\times10^{-6}\text{ T.}$$

c) The magnitude of the field at S is given by the sum of the squares of the two fields because they are at right angles. So:

$$B_S = \sqrt{B_1^{\,2} + B_2^{\,2}} = \frac{\mu_0}{2\pi}\sqrt{\left(\frac{I_1}{r_1}\right)^2 + \left(\frac{I_2}{r_2}\right)^2} = \frac{\mu_0}{2\pi}\sqrt{\left(\frac{6.00\text{ A}}{0.60\text{ m}}\right)^2 + \left(\frac{2.00\text{ A}}{0.80\text{ m}}\right)^2} = 2.1\times10^{-6}\text{ T.}$$

28.56: a)



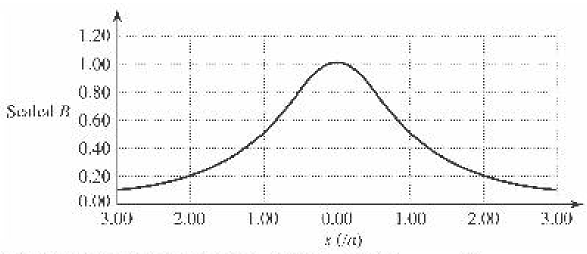
b) At a position on the x-axis:

$$B_{\text{net}} = 2\frac{\mu_0 I}{2\pi r}\sin\theta = \frac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\frac{a}{\sqrt{x^2+a^2}}$$

$$\Rightarrow B_{\text{net}} = \frac{\mu_0 I a}{\pi(x^2+a^2)},$$

in the positive x-direction , as shown at left.

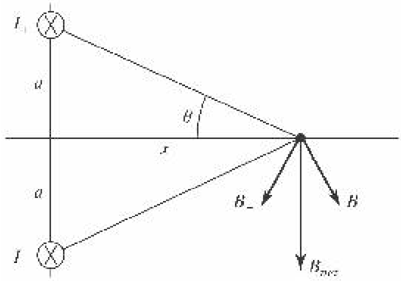
c)



d) The magnetic field is a maximum at the origin, $x = 0$.

e) When $x >> a$, $B \approx \frac{\mu_0 I a}{\pi x^2}$.

28.57: a)



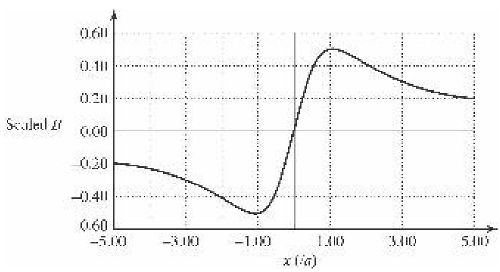
b) At a position on the x-axis:

$$B_{\text{net}} = 2\frac{\mu_0 I}{2\pi r}\cos\theta = \frac{\mu_0 I}{\pi\sqrt{x^2+a^2}}\frac{x}{\sqrt{x^2+a^2}}$$

$$\Rightarrow B_{\text{net}} = \frac{\mu_0 I x}{\pi\left(x^2+a^2\right)},$$

in the negative y-direction, as shown at left.

c)



d) The magnetic field is a maximum when:

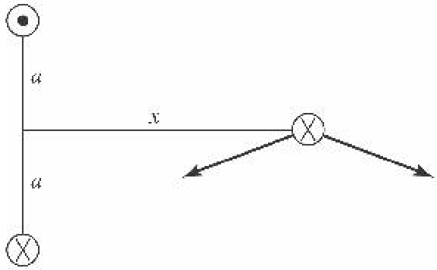
$$\frac{dB}{dx} = 0 = \frac{C}{x^2+a^2} - \frac{2Cx^2}{\left(x^2+a^2\right)^2} \Rightarrow \left(x^2+a^2\right) = 2x^2 \Rightarrow x = \pm a$$

e) When $x >> a$, $B \approx \frac{\mu_0 I}{\pi x}$, which is just like a wire carrying current 2I.

28.58: a) Wire carrying current into the page, so it feels a force downward from the other wires, as shown at right.

$$\frac{F}{L} = IB = I\left(\frac{\mu_0 I a}{\pi\left(x^2+a^2\right)}\right)$$

$$\Rightarrow \frac{F}{L} = \frac{\mu_0 (6.00\ A)^2 (0.400\ m)}{\pi\left((0.600\ m)^2 + (0.400\ m)^2\right)} = 1.11\times10^{-5}\ \text{N/m}.$$



b) If the wire carries current out of the page then the forces felt will be the opposite of part (a) . Thus the force will be 1.11×10^{-5} N/m, upward.

28.59: The current in the wires is $I = \varepsilon/\mathrm{R} = (45.0\ \mathrm{V})/(0.500\ \Omega) = 90.0$ A. The currents in the wires are in opposite directions, so the wires repel. The force each wire exerts on the other is

$$F = \frac{\mu_0 II'L}{2\pi r} = \frac{(2\times 10^{-7}\ \mathrm{N/A^2})(90.0\ \mathrm{A})^2(3.50\ \mathrm{m})}{(0.0150\ \mathrm{m})} = 0.378\ \mathrm{N}$$

To hold the wires at rest, each spring exerts a force of 0.189 N on each wire.

$$F = kx \ \text{ so } \ k = F/x = (0.189\ \mathrm{N})/(0.0050\ \mathrm{m}) = 37.8\ \mathrm{N/m}$$

28.60: a) Note that the Earth's magnetic field exerts no force on wire B, since the current in wire B is parallel to the Earth's magnetic field. Thus, for equilibrium, the remaining two forces that act on wire B must cancel. Assuming that the length of wire B is L and that wire A carries a current I we obtain

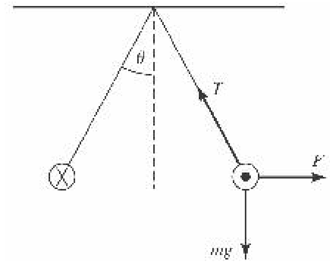
$$-\frac{\mu_0 I(1.0\ A)L}{2\pi(0.050\ m)} + \frac{\mu_0(1.0\ \mathrm{A})(3.0\ \mathrm{A})L}{2\pi(0.100\ \mathrm{m})} = 0$$

So

$$I = (3.0\ \mathrm{A})\cdot\frac{0.050\ \mathrm{m}}{0.100\ \mathrm{m}} = 1.5\ \mathrm{A}$$

b) Note that the force on wire B that is generated by wire C is to the right. Thus, if the current in wire C is increased, wire B will slide to the right.

28.61:



The wires are in equilibrium, so:

$$x: F = T\sin\theta \text{ and } y: T\cos\theta = mg$$

$$\Rightarrow F = IlB = T\sin\theta = mg\tan\theta \Rightarrow I = \frac{mg\tan\theta}{lB}$$

But $B = \dfrac{\mu_0 I}{2\pi r} \Rightarrow I = \dfrac{2\pi r\, mg\tan\theta}{l\mu_0 I} \Rightarrow I = \sqrt{\dfrac{2\pi r\, mg\tan\theta}{l\mu_0}}$

And $r = [2(0.0400\text{ m})\sin(6.00°)] = 8.36\times10^{-3}\text{ m}$.

$$\Rightarrow I = \sqrt{\frac{2\pi(8.36\times10^{-3}\text{ m})(0.0125\text{ kg/m})(9.80\text{ m/s}^2)\tan(6.00°)}{\mu_0}} = 23.2\text{ A}.$$

28.62: The forces on the top and bottom segments cancel, leaving the left and right sides:

$$\vec{F} = \vec{F}_l + \vec{F}_r = -(IlB_l)\hat{i} + (IlB_r)\hat{i} = Il\left(-\frac{\mu_0 I_{\text{wire}}}{2\pi r_l} + \frac{\mu_0 I_{\text{wire}}}{2\pi r_r}\right)\hat{i} = \frac{\mu_0 IlI_{\text{wire}}}{2\pi}\left(\frac{1}{r_r} - \frac{1}{r_l}\right)\hat{i}$$

$$\Rightarrow \vec{F} = \frac{\mu_0(5.00\text{ A})(0.200\text{ m})(14.0\text{ A})}{2\pi}\left(\frac{1}{0.100\text{ m}} - \frac{1}{0.026\text{ m}}\right)\hat{i} = -(7.97\times10^{-5}\text{ N})\hat{i}.$$

28.63: a) $x >> a \Rightarrow B = \dfrac{N\mu_0 Ia^2}{2(x^2+a^2)^{3/2}} \approx \dfrac{N\mu_0 Ia^2}{2x^3}$ and $|\vec{\tau}| = |\vec{\mu}\times\vec{B}| = \mu B\sin\theta$

$$\Rightarrow \tau = (N'I'A')\left(\frac{N\mu_0 Ia^2}{2x^3}\right)\sin\theta = \frac{NN'\mu_0\pi I\, I'a^2a'^2\sin\theta}{2x^3}$$

b) $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\theta = -(N'I'\pi a'^2)\left(\dfrac{N\mu_0 Ia^2}{2x^3}\right)\cos\theta = -\dfrac{NN'\mu_0\pi II'a^2\,a'^2\cos\theta}{2x^3}.$

c) Having $x >> a$ allows us to simplify the form of the magnetic field, whereas assuming $x >> a'$ means we can assume that the magnetic field from the first loop is constant over the second loop.

28.64: $B = B_a - B_b = \dfrac{1}{2}\left(\dfrac{\mu_0 I}{2}\right)\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = \dfrac{\mu_0 I}{4a}\left(1 - \dfrac{a}{b}\right)$, out of the page.

28.65: a) Recall for a single loop: $B = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$. Here we have two loops, each of N turns, and measuring the field along the x-axis from between them means that the "x" in the formula is different for each case:

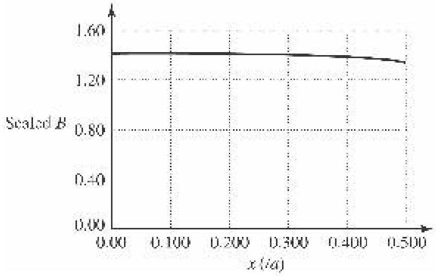
Left coil: $\quad x \to x + \dfrac{a}{2} \Rightarrow B_l = \dfrac{\mu_0 NIa^2}{2((x+a/2)^2+a^2)^{3/2}}.$

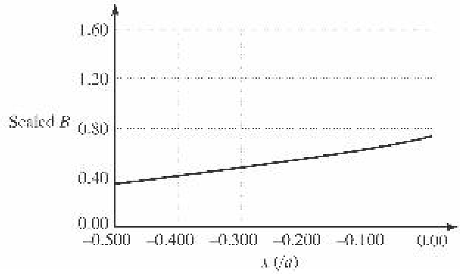
Right coil: $\quad x \to x - \dfrac{a}{2} \Rightarrow B_r = \dfrac{\mu_0 NIa^2}{2((x-a/2)^2+a^2)^{3/2}}.$

So the total field at a point x from the point between them is:

$$B = \frac{\mu_0 NIa^2}{2}\left(\frac{1}{((x+a/2)^2+a^2)^{3/2}} + \frac{1}{((x-a/2)^2+a^2)^{3/2}}\right).$$

b) Below left: Total magnetic field. Below right: Magnetic field from right coil.





c) At point P, $x = 0 \Rightarrow B = \dfrac{\mu_0 NIa^2}{2}\left(\dfrac{1}{((a/2)^2 + a^2)^{3/2}} + \dfrac{1}{((-a/2)^2 + a^2)^{3/2}}\right)$

$$\Rightarrow B = \frac{\mu_0 NIa^2}{(5a^2/4)^{3/2}} = \left(\frac{4}{5}\right)^{3/2} \frac{\mu_0 NI}{a}.$$

d) $B = \left(\dfrac{4}{5}\right)^{3/2} \dfrac{\mu_0 NI}{a} = \left(\dfrac{4}{5}\right)^{3/2} \dfrac{\mu_0 (300)(6.00\text{ A})}{(0.080\text{ m})} = 0.0202\text{ T}.$

e) $\dfrac{dB}{dx} = \dfrac{\mu_0 NIa^2}{2}\left(\dfrac{-3(x+a/2)}{((x+a/2)^2 + a^2)^{5/2}} + \dfrac{-3(x-a/2)}{((x-a/2)^2 + a^2)^{5/2}}\right)$

$$\Rightarrow \left.\frac{dB}{dx}\right|_{x=0} = \frac{\mu_0 NIa^2}{2}\left(\frac{-3(a/2)}{((a/2)^2 + a^2)^{5/2}} + \frac{-3(-a/2)}{((-a/2)^2 + a^2)^{5/2}}\right) = 0.$$

$$\frac{d^2B}{dx^2} = \frac{\mu_0 NIa^2}{2}\left(\frac{-3}{((x+a/2)^2 + a^2)^{5/2}} + \frac{6(x+a/2)^2(5/2)}{((x+a/2)^2 + a^2)^{7/2}}\right.$$

$$\left. + \frac{-3}{((x-a/2)^2 + a^2)^{5/2}} + \frac{6(x-a/2)^2(5/2)}{((x-a/2)^2 + a^2)^{7/2}}\right)$$

$$\Rightarrow \left.\frac{d^2B}{dx^2}\right|_{x=0} = \frac{\mu_0 NIa^2}{2}\left(\frac{-3}{((a/2)^2 + a^2)^{5/2}} + \frac{6(a/2)^2(5/2)}{((a/2)^2 + a^2)^{7/2}} + \frac{-3}{((a/2)^2 + a^2)^{5/2}} + \frac{6(-a/2)^2(5/2)}{((a/2)^2 + a^2)^{7/2}}\right) =$$

Since both first and second derivatives are zero, the field can only be changing very slowly.

28.66: A wire of length l produces a field $B = \frac{\mu_0 I}{4\pi}\frac{l}{x\sqrt{x^2 + (l/2)^2}}$. Here all edges produce a field into the page so we can just add them up:

$$x = a/2 \text{ and } l = b \Rightarrow B_{\text{left}} = \frac{\mu_0 I}{4\pi}\frac{b}{(a/2)\sqrt{(a/2)^2 + (b/2)^2}} = \frac{\mu_0 I}{\pi}\left(\frac{b}{a}\right)\frac{1}{\sqrt{a^2 + b^2}}.$$

$$x = b/2 \text{ and } l = a \Rightarrow B_{\text{top}} = \frac{\mu_0 I}{4\pi}\frac{a}{(b/2)\sqrt{(b/2)^2 + (a/2)^2}} = \frac{\mu_0 I}{\pi}\left(\frac{a}{b}\right)\frac{1}{\sqrt{a^2 + b^2}}.$$

And the right and bottom edges just produce the same contribution as the left and top, respectively. Thus the total magnetic field is:

$$B=\frac{2\mu_0 I}{\pi}\left(\frac{b}{a}+\frac{a}{b}\right)\frac{1}{\sqrt{a^2+b^2}}=\frac{2\mu_0 I}{\pi ab}\sqrt{a^2+b^2}.$$

28.67: The contributions from the straight segments is zero since $d\vec{l}\times\vec{r}=0$. The magnetic field from the curved wire is just one quarter of a full loop:

$$\Rightarrow B=\frac{1}{4}\left(\frac{\mu_0 I}{2R}\right),$$

and is out of the page.

28.68: The horizontal wire yields zero magnetic field since $d\vec{l}\times\vec{r}=0$. The vertical current provides the magnetic field of HALF of an infinite wire. (The contributions from all infinitesimal pieces of the wire point in the same direction, so there is no vector addition or components to worry about.)

$$\Rightarrow B=\frac{1}{2}\left(\frac{\mu_0 I}{2\pi R}\right),$$

and is out of the page.

28.69: a) $I=\int_s J dA=\int_s \alpha r r dr d\theta=\alpha 2\pi\int_0^R r^2 dr=\frac{2\pi\alpha R^3}{3}\Rightarrow\alpha=\frac{3I}{2\pi R^3}.$

b) $(i)\ r\le R\Rightarrow I_{\text{encl}}=\frac{3I}{2\pi R^3}\int_s r^2 dr d\theta=\frac{3I}{2\pi R^3}2\pi\int_0^r r^2 dr=I\frac{r^3}{R^3}.$

$$\Rightarrow\oint\vec{B}\cdot d\vec{l}=B2\pi r=\mu_0 I_{\text{encl}}=\mu_0\left(I\frac{r^3}{R^3}\right)\Rightarrow B=\frac{\mu_0 I r^2}{2\pi R^3}.$$

$$(ii)\ r\ge R\Rightarrow I_{\text{encl}}=I\Rightarrow\oint\vec{B}\cdot d\vec{l}=B2\pi r=\mu_0 I_{\text{encl}}=\mu_0 I\Rightarrow B=\frac{\mu_0 I}{2\pi r}.$$

28.70: $a) r<a\Rightarrow I_{\text{encl}}=I\left(\frac{A_r}{A_a}\right)=I\left(\frac{r^2}{a^2}\right)\Rightarrow\oint\vec{B}\cdot d\vec{l}=B2\pi r=\mu_0 I_{\text{encl}}=\mu_0 I\left(\frac{r^2}{a^2}\right)$

$$\Rightarrow B=\frac{\mu_0 I r}{2\pi a^2}.$$

When $r=a$, $B=\frac{\mu_0 I}{2\pi a}$ which is just what was found from Exercise 28.32, part (a).

$$b)\; b < r < c \Rightarrow I_{\text{encl}} = I - I\left(\frac{A_{b\to r}}{A_{b\to c}}\right) = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right)$$

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = \mu_0 I\left(\frac{c^2 - r^2}{c^2 - b^2}\right) \Rightarrow B = \frac{\mu_0 I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right).$$

When $r = b$, $B = \dfrac{\mu_0 I}{2\pi b}$, just as in Ex. Exercise 28.32, part (a), and at $r = c$, $B = 0$, just as in Ex. Exercise 28.32, part (b).

28.71: If there is a magnetic field component in the z-direction, it must be constant because of the symmetry of the wire. Therefore the contribution to a surface integral over a closed cylinder, encompassing a long straight wire will be zero: no flux through the barrel of the cylinder, and equal but opposite flux through the ends. The radial field will have no contribution through the ends, but through the barrel:

$$0 = \oint_s \vec{B} \cdot d\vec{A} = \oint_s \vec{B}_r \cdot d\vec{A} = \int_{\text{barrel}} \vec{B}_r \cdot d\vec{A} = \int_{\text{barrel}} B_r dA = B_r A_{\text{barrel}} = 0 \Rightarrow B_r = 0.$$

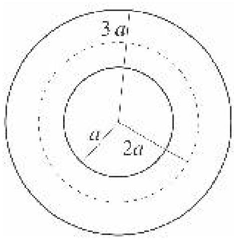
28.72: a) $r < a \Rightarrow I_{\text{encl}} = 0 \Rightarrow B = 0.$

b) $a < r < b \Rightarrow I_{\text{encl}} = I\left(\dfrac{A_{a\to r}}{A_{a\to b}}\right) = I\left(\dfrac{\pi(r^2 - a^2)}{\pi(b^2 - a^2)}\right) = I\dfrac{(r^2 - a^2)}{(b^2 - a^2)}$

$$\Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I \frac{(r^2 - a^2)}{(b^2 - a^2)} \Rightarrow B = \frac{\mu_0 I}{2\pi r}\frac{(r^2 - a^2)}{(b^2 - a^2)}.$$

c) $r > b \Rightarrow I_{\text{encl}} = I \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}.$

28.73:



Apply Ampere's law to a circular path of radius $2a$.

$B(2\pi r) = \mu_0 I_{\text{encl}}$

$$I_{\text{encl}} = I\left(\frac{(2a)^2 - a^2}{(3a)^2 - a^2}\right) = 3I/8$$

$B = \dfrac{3}{16}\dfrac{\mu_0 I}{2\pi a}$; this is the magnetic field inside the metal at a distance of $2a$ from the cylinder axis.

Outside the cylinder, $B = \dfrac{\mu_0 I}{2\pi r}$. The value of r where these two fields are equal is given by $1/r = 3/(16a)$ and $r = 16a/3$.

28.74: At the center of the circular loop the current I_2 generates a magnetic field that is into the page–so the current I_1 must point to the right. For complete cancellation the two fields must have the same magnitude

$$\frac{\mu_0 I_1}{2\pi D} = \frac{\mu_0 I_2}{2R}$$

Thus, $I_1 = \frac{\pi D}{R} I_2$

28.75: a) $I = \int_s \vec{J} \cdot d\vec{A} = \frac{2I_0}{\pi a^2} \int_s \left(1 - \frac{r^2}{a^2}\right) r dr d\theta = \frac{2I_0}{\pi a^2} 2\pi \int_0^a \left(r - \frac{r^3}{a^2}\right) dr =$

$\frac{4I_0}{a^2}\left(\frac{r^2}{2} - \frac{r^4}{4a^2}\right)\Bigg|_0^a \Rightarrow I = \frac{4I_0}{a^2}\left(\frac{a^2}{2} - \frac{a^4}{4a^2}\right) = I_0.$

b) For $r \geq a \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} \Rightarrow \mu_0 I_0 \Rightarrow B = \frac{\mu_0 I_0}{2\pi r}.$

c) For $r \leq a \Rightarrow I_{\text{encl}} = \oint_s \vec{J} \cdot d\vec{A} = \frac{2I_0}{\pi a^2} \int_s \left(1 - \frac{r'^2}{a^2}\right) r' dr' d\theta = \frac{2I_0}{\pi a^2} 2\pi \int_0^r \left(r' - \frac{r'^3}{a^2}\right) dr'$

$$\Rightarrow I_{\text{encl}} = \frac{4I_0}{a^2}\left(\frac{r'^2}{2} - \frac{r'^4}{4a^2}\right)\Bigg|_0^r = 2I_0 \frac{r^2}{a^2}\left(1 - \frac{r^2}{2a^2}\right).$$

d) For $r \leq a \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = 2\mu_0 I_0 \frac{r^2}{a^2}\left(1 - \frac{r^2}{2a^2}\right)$

$$\Rightarrow B = \frac{\mu_0 I_0 r}{\pi a^2}\left(1 - \frac{r^2}{2a^2}\right).$$

At $r = a$, $B = \frac{\mu_0 I_0}{2\pi a}$ for both parts (b) and (d).

28.76: a) $I_0 = \int_s \vec{J} \cdot d\vec{A} = \int_s \left(\frac{b}{r} e^{(r-a)/\delta}\right) r dr d\theta = 2\pi b \int_0^a e^{(r-a)/\delta} dr = 2\pi b \delta\, e^{(r-a)/\delta}\Big|_0^a =$

$2\pi b\delta(1 - e^{-a/\delta}) \Rightarrow I_0 = 2\pi(600 \text{ A/m})\,(0.025 \text{ m})\,(1 - e^{(0.050/0.025)}) = 81.5 \text{ A}.$

b) For $r \geq a \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \Rightarrow B = \frac{\mu_0 I_0}{2\pi r}.$

c) $r \leq a \Rightarrow I(r) = \int_s \vec{J} \cdot d\vec{A} = \int_s \left(\frac{b}{r'} e^{(r'-a)/\delta}\right) r' dr' d\theta$

$$= 2\pi b \int_0^r e^{(r-a)/\delta} dr = 2\pi b \delta e^{(r'-a)/\delta}\Big|_0^r$$

$$\Rightarrow I(r) = 2\pi b\delta(e^{(r'-a)/\delta} - e^{-a/\delta}) = 2\pi b\delta e^{-a/\delta}(e^{r/\delta} - 1) \Rightarrow I(r) = I_0 \frac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)}.$$

d) For $r \leq a \Rightarrow \oint \vec{B} \cdot d\vec{l} = B(r)2\pi r = \mu_0 I_{\text{encl}} = \mu_0 I_0 \frac{(e^{r/\delta} - 1)}{(e^{a/\delta} - 1)} \Rightarrow B = \frac{\mu_0 I_0 (e^{r/\delta} - 1)}{2\pi r(e^{a/\delta} - 1)}.$

e) At $r = \delta = 0.025 \text{ m} \Rightarrow B = \dfrac{\mu_0 I_0 (e-1)}{2\pi\delta(e^{a/\delta}-1)} = \dfrac{\mu_0 (81.5 \text{ A})}{2\pi(0.025 \text{ m})} \dfrac{(e-1)}{(e^{0.050/0.025}-1)}$

$= 1.75 \times 10^{-4}$ T.

At $r = a = 0.050 \text{ m} \Rightarrow B = \dfrac{\mu_0 I_0}{2\pi a} \dfrac{(e^{a/\delta}-1)}{(e^{a/\delta}-1)} = \dfrac{\mu_0 (81.5 \text{ A})}{2\pi(0.050 \text{ m})} = 3.26 \times 10^{-4}$ T.

At $r = 2a = 0.100 \text{ m} \Rightarrow B = \dfrac{\mu_0 I_0}{2\pi r} = \dfrac{\mu_0 (81.5 \text{ A})}{2\pi(0.100 \text{ m})} = 1.63 \times 10^{-4}$ T.

28.77: $\int_{-\infty}^{\infty} B_x dx = \int_{-\infty}^{\infty} \dfrac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}} dx = \dfrac{\mu_0 I}{2} \int_{-\infty}^{\infty} \dfrac{1}{((x/a)^2+1)^{3/2}} d(x/a)$

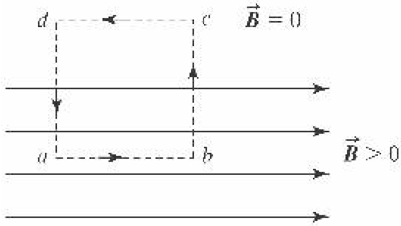
$$= \frac{\mu_0 I}{2} \int_{-\infty}^{\infty} \frac{dz}{(z^2+1)^{3/2}} \Rightarrow \int_{-\infty}^{\infty} B_x dx = \frac{\mu_0 I}{2} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \frac{\mu_0 I}{2} (\sin\theta)\Big|_{-\pi/2}^{\pi/2} = \mu_0 I,$$

where we used the substitution $z = \tan\theta$ to go from the first to second line. This is just what Ampere's Law tells us to expect if we imagine the loop runs along the x-axis closing on itself at infinity: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I.$

28.78: $\oint \vec{B} \cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $B = B_0 \hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$.

$$\int_{abcde} \vec{B} \cdot d\vec{l} = B_{ab}L - B_{cd}L = 0,$$

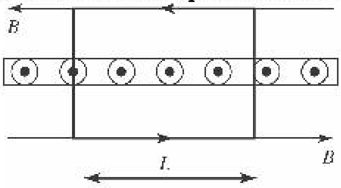
but $B_{cd} = 0$. $B_{ab}L = 0$, but $B_{ab} \neq 0$. This is a contradiction and violates Ampere's Law. See the figure on the next page.



28.79: a) Below the sheet, all the magnetic field contributions from different wires add up to produce a magnetic field that points in the positive x-direction. (Components in the z-direction cancel.) Using Ampere's Law, where we use the fact that the field is anti-symmetrical above and below the current sheet, and that the legs of the path perpendicular provide nothing to the integral: So, at a distance a beneath the sheet the magnetic field is:

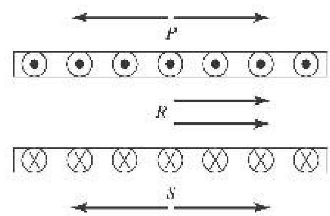
$$I_{\text{encl}} = nLI \Rightarrow \oint \vec{B} \cdot d\vec{l} = B2L = \mu_0 nLI \Rightarrow B = \frac{\mu_0 nI}{2},$$

in the positive x-direction. (Note there is no dependence on a.)



b) The field has the same magnitude above the sheet, but points in the negative x-direction.

28.80: Two infinite sheets, as in Problem 28.79, are placed one above the other, with their currents opposite.



a) Above the two sheets, the fields cancel (since there is no dependence upon the distance from the sheets).

b) In between the sheets the two fields add up to yield $B = \mu_0 nI$, to the right.

c) Below the two sheets, their fields again cancel (since there is no dependence upon the distance from the sheets).

28.81: $M_{Fe} = (\mu_{\text{atom of } Fe})(\#\, Fe \text{ atoms/m}^3) = (\mu_{\text{atom of } Fe}) N_A (\#\, Fe \text{ moles/m}^3)$

$$\Rightarrow M_{Fe} = (\mu_{\text{atom of } Fe}) N_A \frac{\rho_{Fe}}{m_{\text{mol}}(\text{Fe})} \Rightarrow \mu_{\text{atom of } Fe} = \frac{M_{Fe} m_{\text{mol}}(\text{Fe})}{N_A \rho_{Fe}}$$

$$\Rightarrow \mu_{\text{atom of } Fe} = \frac{(6.50 \times 10^4 \text{ A/m})(0.0558 \text{ kg/mol})}{(6.02 \times 10^{23} \text{ atoms/mol})(7.8 \times 10^3 \text{ kg/m}^3)}$$

$$= 7.72 \times 10^{-25} \text{ A} \cdot \text{m}^2.$$

$$\Rightarrow \mu_{\text{atom of } Fe} = \frac{7.72 \times 10^{-25} \text{ A} \cdot \text{m}^2}{9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2} \mu_B = 0.0833 \mu_B.$$

28.82: The microscopic magnetic moments of an initially unmagnetized ferromagnetic material experience torques from a magnet that aligns the magnetic domains with the external field, so they are attracted to the magnet. For a paramagnetic material, the same

attraction occurs because the magnetic moments align themselves parallel to the external field.

For a diamagnetic material, the magnetic moments align anti-parallel to the external field so it is like two magnets repelling each other.

b) The magnet can just pick up the iron cube so the force it exerts is:

$$F_{Fe} = m_{Fe} g = \rho_{Fe} a^3 g = (7.8 \times 10^3 \text{ kg/m}^3)(0.020 \text{ m})^3 (9.8 \text{ m/s}^2) = 0.612 \text{ N}.$$

But $F_{Fe} = IaB = \dfrac{\mu_{Fe} B}{a} = 0.612 \text{ N} \Rightarrow \dfrac{B}{a} = \dfrac{0.612 \text{ N}}{\mu_{Fe}}$.

So if the magnet tries to lift the aluminum cube of the same dimensions as the iron block, then the upward force felt by the cube is:

$$F_{Al} = \frac{\mu_{Al} B}{a} = \frac{\mu_{Al}}{\mu_{Fe}} 0.612 \text{ N} = \frac{K_{Al}}{H_{Fe}} 0.612 \text{ N} = \frac{1.000022}{1400} 0.612 \text{ N} = 4.37 \times 10^{-4} \text{ N}.$$

But the weight of the aluminum cube is:

$$W = m_{Al} g = \rho_{Al} a^3 g = (2.7 \times 10^3 \text{ kg/m}^3)(0.020 \text{ m})^3 (9.8 \text{ m/s}^2) = 0.212 \text{ N}.$$

So the ratio of the magnetic force on the aluminum cube to the weight of the cube is $\frac{4.37 \times 10^{-4} \text{ N}}{0.212 \text{ N}} = 2.1 \times 10^{-3}$, and the magnet cannot lift it.

c) If the magnet tries to lift a silver cube of the same dimensions as the iron block, then the DOWNWARD force felt by the cube is:

$$F_{Al} = \frac{\mu_{Ag} B}{a} = \frac{\mu_{Ag}}{\mu_{Fe}} 0.612 \text{ N} = \frac{K_{Ag}}{K_{Fe}} 0.612 \text{ N} = \frac{(1.00 - 2.6 \times 10^{-5})}{1400} 0.612 \text{ N}$$
$$= 4.37 \times 10^{-4} \text{ N}.$$

But the weight of the silver cube is:

$$W = m_{Ag} g = \rho_{Ag} a^3 g = (10.5 \times 10^3 \text{ kg/m}^3)(0.020 \text{ m})^3 (9.8 \text{ m/s}^2) = 0.823 \text{ N}.$$

So the ratio of the magnetic force on the silver cube to the weight of the cube is $\frac{4.37 \times 10^{-4} \text{ N}}{0.823 \text{ N}} = 5.3 \times 10^{-4}$, and the magnet's effect would not be noticeable.

28.83: a) The magnetic force per unit length between two parallel, long wires is:

$$\frac{F}{L} = IB = \frac{\mu_0}{2\pi d} I^2 = \frac{\mu_0}{2\pi d}\left(\frac{I_0}{\sqrt{2}}\right)^2 = \frac{\mu_0}{4\pi d}\left(\frac{V}{R}\right)^2 = \frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2,$$

where $\frac{I_0}{\sqrt{2}}$ is the rms current over the short discharge time.

$$\frac{F}{L}=\frac{m}{L}a=\lambda a=\frac{\mu_0}{4\pi d}\left(\frac{Q_0}{RC}\right)^2 \Rightarrow a=\frac{\mu_0 Q_0{}^2}{4\pi\lambda dR^2C^2} \Rightarrow v_0=at=aRC=\frac{\mu_0 Q_0{}^2}{4\pi\lambda dRC}.$$

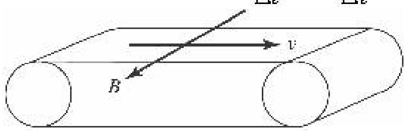
b) $v_0=\dfrac{\mu_0(CV)^2}{4\pi\lambda dRC}=\dfrac{\mu_0 CV^2}{4\pi\lambda dR}=\dfrac{\mu_0(2.50\times10^{-6}\text{ F})(3000\text{ V})^2}{4\pi(4.50\times10^{-3}\text{ kg/m})(0.03\text{ m})(0.048\ \Omega)}=0.347\text{ m/s}.$

c) Height that the wire reaches above the original height:

$$\frac{1}{2}mv_0{}^2=mgh \Rightarrow h=\frac{v_0{}^2}{2g}=\frac{(0.347\text{ m/s})^2}{2(9.80\text{ m/s}^2)}=6.14\times10^{-3}\text{ m}.$$

28.84: The amount of charge on a length Δx of the belt is:

$$\Delta Q=L\Delta x\sigma \Rightarrow I=\frac{\Delta Q}{\Delta t}=L\frac{\Delta x}{\Delta t}\sigma=Lv\sigma.$$



Approximating the belt as an infinite sheet:

$$B=\frac{\mu_0 I}{2L}=\frac{\mu_0 v\sigma}{2},$$

out of the page, as shown at left.

28.85: The charge on a ring of radius r is $q=\sigma A=\sigma 2\pi r dr=\dfrac{2Qrdr}{a^2}$. If the disk rotates at n turns per second, then the current from that ring is:

$$I=\frac{\Delta q}{\Delta t}=nq=\frac{2Qnrdr}{a^2} \Rightarrow dB=\frac{\mu_0 I}{2r}=\frac{\mu_0}{2r}\frac{2Qnrdr}{a^2}=\frac{\mu_0 nQdr}{a^2}.$$

So we integrate out from the center to the edge of the disk to find:

$$B=\int_0^a dB=\int_0^a\frac{\mu_0 nQdr}{a^2}=\frac{\mu_0 nQ}{a}.$$

28.86: There are two parts to the magnetic field: that from the half loop and that from the straight wire segment running from $-a$ to a.

$$B_x(ring)=\frac{1}{2}B_{\text{loop}}=-\frac{\mu_0 Ia^2}{4(x^2+a^2)^{3/2}}$$

$$dB_y(ring)=dB\sin\theta\sin\phi=\frac{\mu_0 I}{4\pi}\frac{dl}{(x^2+a^2)}\frac{x}{(x^2+a^2)^{1/2}}\sin\phi=\frac{\mu_0 Iax\sin\phi\, d\phi}{4\pi(x^2+a^2)^{3/2}}$$

$$\Rightarrow B_y(ring) = \int_0^{\pi} dB_y(ring) = \int_0^{\pi} \frac{\mu_0 Iax \sin\phi d\phi}{4\pi(x^2 + a^2)^{3/2}} = \frac{\mu_0 Iax}{4\pi(x^2 + a^2)^{3/2}} \cos\phi \Big|_0^{\pi}$$

$$= -\frac{\mu_0 Iax}{2\pi(x^2 + a^2)^{3/2}}.$$

$B_y(rod) = \frac{\mu_0 Ia}{2\pi x(x^2+a^2)^{1/2}}$, using Eq. (28.8). So the total field components are:

$$B_x = -\frac{\mu_0 Ia^2}{4(x^2 + a^2)^{3/2}}$$

and

$$B_y = \frac{\mu_0 Ia}{2\pi x(x^2 + a^2)^{1/2}} \left(1 - \frac{x^2}{x^2 + a^2}\right) = \frac{\mu_0 Ia^3}{2\pi x(x^2 + a^2)^{3/2}}.$$

29.1: $\Phi_{B_f} = NBA$, and $\Phi_{B_i} = NBA\cos 37.0° \Rightarrow \Delta\Phi_B = NBA(1-\cos 37.0°)$

$$\Rightarrow \varepsilon = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{NBA(1-\cos 37.0°)}{\Delta t}$$

$$= -\frac{(80)(1.10\text{ T})(0.400\text{ m})(0.25\text{ m})(1-\cos 37.0°)}{0.0600\text{ s}}$$

$$\Rightarrow |\varepsilon| = 29.5\text{ V}.$$

29.2: a) Before: $\Phi_B = NBA = (200)(6.0\times10^{-5}\text{ T})(12\times10^{-4}\text{ m}^2)$

$$= 1.44\times10^{-5}\text{ T}\cdot\text{m}^2;\text{ after}: 0$$

b) $$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = \frac{NBA}{\Delta t} = \frac{(200)(6.0\times10^{-5}\text{ T})(1.2\times10^{-3}\text{ m}^2)}{0.040\text{ s}} = 3.6\times10^{-4}\text{ V}.$$

29.3: a) $$\varepsilon = \frac{\Delta\Phi_B}{\Delta t} = \frac{NBA}{\Delta t} = IR = \left(\frac{Q}{\Delta t}\right)R \Rightarrow QR = NBA \Rightarrow Q = \frac{NBA}{R}.$$

b) A credit card reader *is* a search coil.

c) Data is stored in the charge measured so it is independent of time.

29.4: From Exercise (29.3),

$$Q = \frac{NBA}{R} = \frac{(90)(2.05\text{ T})(2.20\times10^{-4}\text{ m}^2)}{6.80\,\Omega+12.0\,\Omega} = 2.16\times10^{-3}\text{ C}.$$

29.5: From Exercise (29.3),

$$Q = \frac{NBA}{R} \Rightarrow B = \frac{QR}{NA} = \frac{(3.56\times10^{-5}\text{ C})(60.0\,\Omega+45.0\,\Omega)}{(120)(3.20\times10^{-4}\text{ m}^2)} = 0.0973\text{ T}.$$

29.6: a) $$\varepsilon = \frac{Nd\Phi_B}{dt} = NA\frac{d}{dt}(B) = NA\frac{d}{dt}\left((0.012\text{ T/s})t + (3.00\times10^{-5}\text{ T/s}^4)t^4\right)$$

$$\Rightarrow \varepsilon = NA\left((0.012\text{ T/s}) + (1.2\times10^{-4}\text{ T/s}^4)t^3\right)$$

$$= 0.0302\text{ V} + (3.02\times10^{-4}\text{ V/s}^3)t^3.$$

b) At $t = 5.00\text{ s} \Rightarrow \varepsilon = 0.0302\text{ V} + (3.02\times10^{-4}\text{ V/s}^2)(5.00\text{ s})^3 = +0.0680\text{ V}$

.

$$\Rightarrow I = \frac{\varepsilon}{R} = \frac{0.0680\text{ V}}{600\,\Omega} = 1.13\times10^{-4}\text{ A}$$

29.7: a) $\varepsilon = -\dfrac{d\Phi_B}{dt} = -\dfrac{d}{dt}\left[NAB_0\left(1-\cos\left(\dfrac{2\pi t}{T}\right)\right)\right] = -\dfrac{2\pi NAB_0}{T}\sin\left(\dfrac{2\pi t}{T}\right)$ for

$0 < t < T$; zero otherwise..

b) $\varepsilon = 0$ at $t = \dfrac{T}{2}$

c) $\varepsilon_{\max} = \dfrac{2\pi NAB_0}{T}$ occurs at $t = \dfrac{T}{4}$ and $t = \dfrac{3T}{4}$.

d) From $0 < t < \frac{T}{2}$, B is getting larger and points in the $+z$ direction. This gives a clockwise current looking down the $-z$ axis. From $\frac{T}{2} t < T$, B is getting smaller but still points in the $+z$ direction. This gives a counterclockwise current.

29.8: a) $\left|\varepsilon_{\text{ind}}\right| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}(B_1 A)$

$$\left|\varepsilon_{\text{ind}}\right| = A \sin 60° \frac{dB}{dt} = A \sin 60° \frac{d}{dt}\left((1.4\ \text{T})e^{-0.057s^{-1}t}\right)$$

$$= (\pi r^2)(\sin 60°)(1.4\ \text{T})(0.057\text{s}^{-1})e^{-0.057s^{-1}t}$$

$$= \pi(0.75\ \text{m})^2(\sin 60°)(1.4\ \text{T})(0.057\ \text{s}^{-1})e^{-0.057s^{-1}t}$$

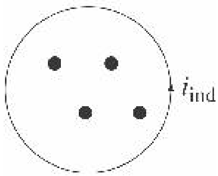
$$= 0.12\ \text{V}\ e^{-0.057\ s^{-1}t}$$

b) $\varepsilon = \dfrac{1}{10}\varepsilon_0 = \dfrac{1}{10}(0.12\ \text{V})$

$$\frac{1}{10}(0.12\ \text{V}) = 0.12\ \text{V}\ e^{-0.057\ s^{-1}t}$$

$$\ln(1/10) = -0.057\ \text{s}^{-1}t \rightarrow t = 40.4\text{s}$$

c) B is getting weaker, so the flux is decreasing. By Lenz's law, the induced current must cause an upward magnetic field to oppose the loss of flux. Therefore the induced current must flow *counterclockwise* as viewed from above.



29.9: a) $c = 2\pi r$ and $A = \pi r^2$ so $A = c^2/4\pi$

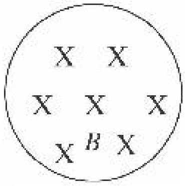
$\Phi_B = BA = (B/4\pi)c^2$

$|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right| = \left(\dfrac{B}{2\pi}\right)c\left|\dfrac{dc}{dt}\right|$

At $t = 9.0$ s, $c = 1.650\ \text{m} - (9.0\ \text{s})(0.120\ \text{s}) = 0.570\ \text{m}$

$|\varepsilon| = (0.500\ \text{T})(1/2\pi)(0.570\ \text{m})(0.120\ \text{m/s}) = 5.44\ \text{mV}$

b)



current

Flux ⊗ is decreasing so the flux of the induced

Φ_{ind} is ⊗ and I is clockwise.

29.10: According to Faraday's law (assuming that the area vector points in the positive z-direction)

$$\varepsilon = -\frac{\Delta\Phi}{\Delta t} = -\frac{0-(1.5\ \text{T})\pi(0.120\ \text{m})^2}{2.0\times10^{-3}\ \text{s}} = +34\ \text{V}(\text{counterclockwise})$$

29.11: $\Phi_B = BA\cos\phi;\ \phi$ is the angle between the normal to the loop and $\vec{B}$, so $\phi = 53°$

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = (A\cos\phi)(dB/dt) = (0.100\ \text{m})^2\cos 53°(1.00\times10^{-3}\ \text{T/s}) = 6.02\times10^{-6}\ \text{V}$$

29.12: a)

$$|\varepsilon| = \frac{d\Phi_B}{dt} = \frac{d}{dt}(NBA\cos\omega t) = NBA\,\omega\sin\omega t \text{ and } 1200\ \text{rev/min} = 20\ \text{rev/s, so:}$$

$$\Rightarrow \varepsilon_{\max} = NBA\omega = (150)(0.060\ \text{T})\pi(0.025\ \text{m})^2(440\ \text{rev/min})(1\ \text{min}/60\text{sec})(2\pi\ \text{rad/rev}) = 0.81$$

b) Average $\varepsilon = \frac{2}{\pi}\varepsilon_{\max} = \frac{2}{\pi}0.814\ \text{V} = 0.518\ \text{V}.$

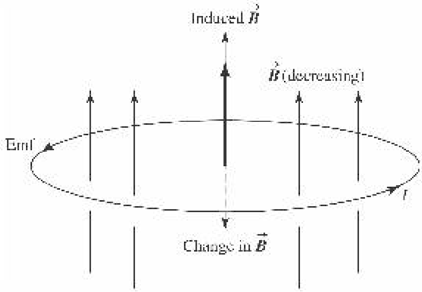
29.13: From Example 29.5,

$$\varepsilon_{av} = \frac{2N\omega BA}{\pi} = \frac{2(500)(56\ \text{rev/s})(2\pi\ \text{rad/rev})(0.20\ \text{T})(0.10\ \text{m})^2}{\pi} = 224\ \text{V}$$

29.14: $\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(NBA\cos\omega t) = NBA\omega\sin\omega t \Rightarrow \varepsilon_{\max} = NBA\omega$

$$\Rightarrow \omega = \frac{\varepsilon_{\max}}{NBA} = \frac{2.40\times10^{-2}\ \text{V}}{(120)(0.0750\ \text{T})(0.016\ \text{m})^2} = 10.4\ \text{rad}/s.$$

29.15:



29.16: a) If the magnetic field is increasing into the page, the induced magnetic field must oppose that change and point opposite the external field's direction, thus requiring a counterclockwise current in the loop.

b) If the magnetic field is decreasing into the page, the induced magnetic field must oppose that change and point in the external field's direction, thus requiring a clockwise current in the loop.

c) If the magnetic field is constant, there is no changing flux, and therefore no induced current in the loop.

29.17: a) When the switch is opened, the magnetic field to the right decreases. Therefore the second coil's induced current produces its own field to the right. That means that the current must pass through the resistor from point a to point b.

b) If coil B is moved closer to coil A, more flux passes through it toward the right. Therefore the induced current must produce its own magnetic field to the left to oppose the increased flux. That means that the current must pass through the resistor from point b to point a.

c) If the variable resistor R is decreased, then more current flows through coil A, and so a stronger magnetic field is produced, leading to more flux to the right through coil B. Therefore the induced current must produce its own magnetic field to the left to oppose the increased flux. That means that the current must pass through the resistor from point b to point a.

29.18: a) With current passing from $a \to b$ and is increasing the magnetic, field becomes stronger to the left, so the induced field points right, and the induced current must flow from right to left through the resistor.

b) If the current passes from $b \to a$, and is decreasing, then there is less magnetic field pointing right, so the induced field points right, and the induced current must flow from right to left through the resistor.

c) If the current passes from $b \to a$, and is increasing, then there is more magnetic field pointing right, so the induced field points left, and the induced current must flow from left to right through the resistor.

29.19: a) Φ_B is $\odot$ and increasing so the flux Φ_{ind} of the induced current is clockwise.

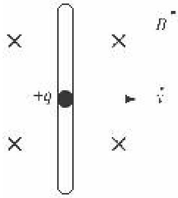
b) The current reaches a constant value so Φ_B is constant. $d\Phi_B / dt = 0$ and there is no induced current.

c) Φ_B is $\odot$ and decreasing, so Φ_{ind} is $\odot$ and current is counterclockwise.

29.20: a) $\varepsilon = vBl = (5.0 \text{ m/s})(0.750 \text{ T})(1.50 \text{ m})$

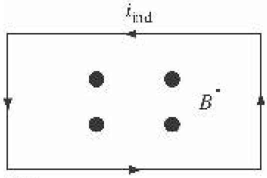
$= 5.6 \text{ V}$

b) (i)



Let q be a positive charge in the moving bar. The magnetic force on this charge $\vec{F} = q\vec{v} \times \vec{B}$, which points *upward*. This force pushes the current in a *counterclockwise* direction through the circuit.

(ii) The flux through the circuit is increasing, so the induced current must cause a magnetic field out of the paper to oppose this increase. Hence this current must flow in a *counterclockwise* sense.



c) $$\varepsilon = Ri$$
$$i = \frac{\varepsilon}{R} = \frac{5.6\ \text{V}}{25\ \Omega} = 0.22\ \text{A}$$

29.21: $[vBL] = \left[\frac{\text{m}}{\text{s}}\text{Tm}\right] = \left[\frac{\text{m}}{\text{s}}\frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\text{m}\right] = \left[\frac{\text{N}\cdot\text{m}}{\text{C}}\right] = \left[\frac{\text{J}}{\text{C}}\right] = [\text{V}].$

29.22: a) $\varepsilon = vBL = (5.00\ \text{m/s})(0.450\ \text{T})(0.300\ \text{m}) = 0.675\ \text{V}.$

b) The potential difference between the ends of the rod is just the motional emf $V = 0.675\ \text{V}.$

c) The positive charges are moved to end b, so b is at the higher potential.

d) $E = \frac{V}{L} = \frac{0.675\ \text{V}}{0.300\ \text{m}} = 2.25\frac{\text{V}}{\text{m}}.$

e) b

29.23: a) $\varepsilon = vBL \Rightarrow v = \frac{\varepsilon}{BL} = \frac{0.620\ \text{V}}{(0.850\ \text{T})(0.850\ \text{m})} = 0.858\ \text{m/s}.$

b) $I = \frac{\varepsilon}{R} = \frac{0.620\ \text{V}}{0.750\ \Omega} = 0.827\ \text{A}.$

c) $F = ILB = (0.827\ \text{A})(0.850\ \text{m})(0.850\ \text{T}) = 0.598\ \text{N}$, to the left, since you must pull it to get the current to flow.

29.24: a) $\varepsilon = vBL = (7.50\ \text{m/s})(0.800\ \text{T})(0.500\ \text{m}) = 3.00\ \text{V}.$

b) The current flows counterclockwise since its magnetic field must oppose the increasing flux through the loop.

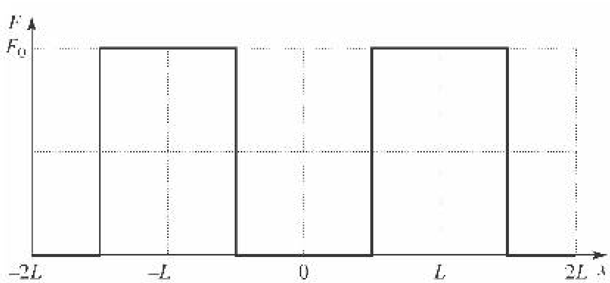
c) $F = ILB = \dfrac{\varepsilon LB}{R} = \dfrac{(3.00\ \text{V})(0.500\ \text{m})(0.800\ \text{T})}{1.50\ \Omega} = 0.800\ \text{N},$ to the right.

d) $P_{\text{mech}} = Fv = (0.800\ \text{N})(7.50\ \text{m/s}) = 6.00\ \text{W}.$

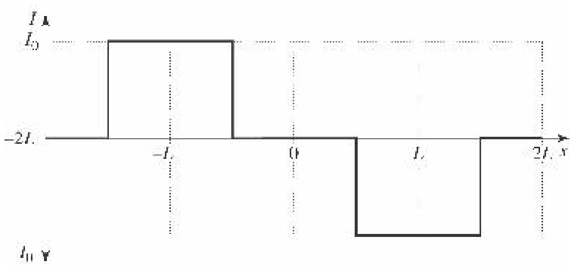
$$P_{\text{elec}} = \frac{\varepsilon^2}{R} = \frac{(3.00\ \text{V})^2}{1.50\ \Omega} = 6.00\ \text{W. So both rates are equal.}$$

29.25: For the loop pulled through the region of magnetic field,

a)



b)



Where $\varepsilon = vBL = IR \Rightarrow I_0 = \dfrac{vBL}{R}$ and $F_0 = ILB = \dfrac{vB^2L^2}{R}$.

29.26: a) Using Equation (29.6): $\varepsilon = vBL \Rightarrow B = \dfrac{\varepsilon}{vL} = \dfrac{0.450\ \text{V}}{(4.50\ \text{m/s})(0.120\ \text{m})} = 0.833\ \text{T}.$

b) Point a is at a higher potential than point b, because there are more positive charges there.

29.27: $\varepsilon = \dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}(BA) = \dfrac{d}{dt}(\mu_0 nIA) = \mu_0 nA\dfrac{dI}{dt}$ and $\oint \vec{E}\cdot d\vec{l} = \varepsilon \Rightarrow$

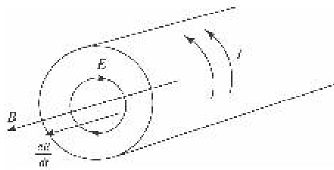
$$E = \frac{\varepsilon}{2\pi r} = \frac{\mu_0 nA}{2\pi r}\frac{dI}{dt} = \frac{\mu_0 nr}{2}\frac{dI}{dt}.$$

a) $r = 0.50\text{ cm} \Rightarrow E = \dfrac{\mu_0 (900\text{ m}^{-1})(0.0050\text{ m})}{2}(60\text{ A/s}) = 1.70 \times 10^{-4}\text{ V/m}.$

b) $r = 1.00\text{cm} \Rightarrow E = 3.39 \times 10^{-4}\text{ V/m}.$

29.28: a) $\dfrac{d\Phi_B}{dt} = A\dfrac{dB}{dt} = \pi r_1^{\,2}\dfrac{dB}{dt}.$

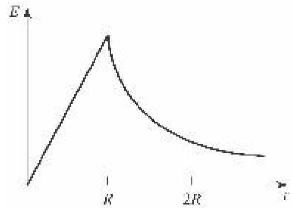
b) $E = \dfrac{1}{2\pi r_1}\dfrac{d\Phi_B}{dt} = \dfrac{\pi r_1^{\,2}}{2\pi r_1}\dfrac{dB}{dt} = \dfrac{r_1}{2}\dfrac{dB}{dt}.$



c) All the flux is within $r < R$, so outside the solenoid

$$E = \frac{1}{2\pi r_2}\frac{d\Phi_B}{dt} = \frac{\pi R^2}{2\pi r_2}\frac{dB}{dt} = \frac{R^2}{2r_2}\frac{dB}{dt}.$$

d)



e) At $r = R/2$:

$$\Rightarrow \varepsilon = \frac{d\Phi_B}{dt} = \pi(R/2)^2 \frac{dB}{dt} = \frac{\pi R^2}{4}\frac{dB}{dt}.$$

f) At $r = R \Rightarrow \varepsilon = \dfrac{d\Phi_B}{dt} = \pi R^2 \dfrac{dB}{dt}$.

g) At $r = 2R \Rightarrow \varepsilon = \dfrac{d\Phi_B}{dt} = \pi R^2 \dfrac{dB}{dt}$. Note that the emf is independent of the distance from the center of the cylinder as long as one is outside it.

29.29: a) The induced electric field lines are concentric circles since they cause the current to flow in circles.

b) $E = \dfrac{1}{2\pi r}\varepsilon = \dfrac{1}{2\pi r}\dfrac{d\Phi_B}{dt} = \dfrac{1}{2\pi r}A\dfrac{dB}{dt} = \dfrac{r}{2}\dfrac{dB}{dt} = \dfrac{0.100\text{ m}}{2}(0.0350\text{ T/s})$

$\Rightarrow E = 1.75\times10^{-3}$ V/m, in the clockwise direction, since the induced magnetic field must reinforce the decreasing external magnetic field.

c) $I = \dfrac{\varepsilon}{R} = \dfrac{\pi r^2}{R}\dfrac{dB}{dt} = \dfrac{\pi(0.100\text{ m})^2}{4.00\,\Omega}(0.0350\text{ T/s}) = 2.75\times10^{-4}$ A.

d) $\varepsilon = IR = IR_{TOT}/2 = \dfrac{(2.75\times10^{-4}\text{ A})(4.00\,\Omega)}{2} = 5.50\times10^{-4}$ V.

e) If the ring was cut and the ends separated slightly, then there would be a potential difference between the ends equal to the induced emf:

$$\varepsilon = \pi r^2\frac{dB}{dt} = \pi(0.100\text{ m})^2(0.0350\text{ T/s}) = 1.10\times10^{-3}\text{ V}.$$

29.30: $\varepsilon = \dfrac{d\Phi_B}{dt} = \dfrac{d}{dt}(BA) = \dfrac{d}{dt}(\mu_0 nIA) = \mu_0 nA\dfrac{dI}{dt} \Rightarrow \dfrac{dI}{dt} = \dfrac{E\cdot 2\pi r}{\mu_0 nA}$

$$\Rightarrow \frac{dI}{dt} = \frac{(8.00\times10^{-6}\text{ V/m})2\pi(0.0350)}{\mu_0(400\text{ m}^{-1})\pi(0.0110\text{ m})^2} = 9.21\text{ A/s}.$$

29.31: a)

$W = \int \vec{F}\cdot d\vec{l} = qE2\pi R = (6.50\times10^{-6}\text{ C})(8.00\times10^{-6}\text{ V/m})2\pi(0.0350\text{ m}) = 1.14\times10^{-11}$ J.

(b) For a conservative field, the work done for a closed path would be zero.

(c) $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt} \Rightarrow EL = BA\dfrac{di}{dt}$. A is the area of the solenoid.

For a circular path:

$E2\pi r = BA\dfrac{di}{dt} =$ constant for all circular paths that enclose the solenoid.

So $W = qE2\pi r =$ constant for all paths outside the solenoid.

$W = 1.14 \times 10^{-11}$ J if $r = 7.00$ cm.

29.32: $\varepsilon = -\dfrac{N\Delta\Phi_B}{\Delta t} = -\dfrac{NA(B_f - B_i)}{\Delta t} = \dfrac{NA\mu_o nI}{\Delta t}$

$= \dfrac{\mu_o (12)(8.00 \times 10^{-4}\ \text{m}^2)(9000\ \text{m}^{-1})(0.350\ \text{A})}{0.0400\ \text{s}}$

$\Rightarrow \varepsilon = 9.50 \times 10^{-4}$ V.

29.33: $i_D = \varepsilon\dfrac{d\Phi_E}{dt} = (3.5 \times 10^{-11}\ \text{F/m})(24.0 \times 10^3\ \text{V} \cdot \text{m/s}^3)t^2$

$i_D = 21 \times 10^{-6}$ A gives $t = 5.0$ s

29.34: According to Eqn.29.14 $\varepsilon = \dfrac{i_D}{\left(\dfrac{d\Phi_E}{dt}\right)} = \dfrac{12.9 \times 10^{-12}\,\text{A}}{4(8.76 \times 10^3\ \text{V} \cdot \text{m/s}^4)(26.1 \times 10^{-3}\ \text{s})^3} =$

2.07×10^{-11} F/m. Thus, the dielectric constant is $K = \frac{\varepsilon}{\varepsilon_0} = 2.34$.

29.35: a) $j_D = \varepsilon_0 \dfrac{dE}{dt} = \varepsilon_0 \dfrac{i_c}{\varepsilon_0 A} = \dfrac{i_c}{A} = \dfrac{0.280\ \text{A}}{\pi(0.0400\ \text{m})^2} = 55.7\ \text{A/m}^2$.

b) $\dfrac{dE}{dt} = \dfrac{j_D}{\varepsilon_0} = \dfrac{55.7\ \text{A/m}^2}{\varepsilon_0} = 6.29 \times 10^{12}\ \text{V/m} \cdot \text{s}$.

c) Using Ampere's Law

$$r < R : B = \frac{\mu_0}{2\pi}\frac{r}{R^2} i_D = \frac{\mu_0}{2\pi}\frac{0.0200\ \text{m}}{(0.0400\ \text{m})^2}(0.280\ \text{A}) = 7.0 \times 10^{-7}\ \text{T}.$$

d) Using Ampere's Law

$$r < R : B = \frac{\mu_0 r}{2\pi R^2} i_D = \frac{\mu_0}{2\pi}\frac{(0.0100\ \text{m})}{(0.0400\ \text{m})^2}(0.280) = 3.5 \times 10^{-7}\ \text{T}.$$

29.36: a) $Q = CV = \left(\frac{\varepsilon A}{d}\right)V = \frac{(4.70)\varepsilon_0 (3.00\times10^{-4}\ \text{m}^2)(120\ \text{V})}{2.50\times10^{-3}\ \text{m}} = 5.99\times10^{-10}\ \text{C}.$

b) $\frac{dQ}{dt} = i_c = 6.00\times10^{-3}\ \text{A}.$

c) $j_D = \varepsilon\frac{dE}{dt} = K\varepsilon_0\frac{i_c}{K\varepsilon_0 A} = \frac{i_c}{A} = j_c \Rightarrow i_D = i_c = 6.00\times10^{-3}\ \text{A}.$

29.37: a) $q = i_c t = (1.80\times10^{-3}\text{ A})(0.500\times10^{-6}\text{ s}) = 0.900\times10^{-9}\text{ C}$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{A\varepsilon_0} = \frac{0.900\times10^{-9}\text{ C}}{(5.00\times10^{-4}\text{ m}^2)\varepsilon_0} = 2.03\times10^{5}\text{ V/m}.$$

$$\Rightarrow V = Ed = (2.03\times10^{5}\text{ V/m})(2.00\times10^{-3}\text{ m}) = 406\text{ V}.$$

b) $\dfrac{dE}{dt} = \dfrac{i_c}{A\varepsilon_0} = \dfrac{1.80\times10^{-3}\text{ A}}{(5.00\times10^{-4}\text{ m}^2)\varepsilon_0} = 4.07\times10^{11}\text{V/m}\cdot\text{s},$ and is constant in time.

c) $j_D = \varepsilon_0 \dfrac{dE}{dt} = \varepsilon_0(4.07\times10^{11}\text{ V/m}\cdot\text{s}) = 3.60\text{ A/m}^2$

$\Rightarrow i_D = j_D A = (3.60\text{ A/m}^2)(5.00\times10^{-4}\text{ m}^2) = 1.80\times10^{-3}\text{A},$ which is the same as i_c.

29.38: a) $E = \rho J = \dfrac{\rho I}{A} = \dfrac{(2.0\times10^{-8}\ \Omega\text{m})(16\text{ A})}{2.1\times10^{-6}\text{ m}^2} = 0.15\text{ V/m}.$

b) $\dfrac{dE}{dt} = \dfrac{d}{dt}\left(\dfrac{\rho I}{A}\right) = \dfrac{\rho}{A}\dfrac{dI}{dt} = \dfrac{2.0\times10^{-8}\ \Omega\text{m}}{2.1\times10^{-6}\text{ m}^2}(4000\ A/s) = 38\ V/\text{m}\cdot\text{s}.$

c) $j_D = \varepsilon_0 \dfrac{dE}{dt} = \varepsilon_0(38\text{ V/s}\cdot\text{s}) = 3.4\times10^{-10}\ A/\text{m}^2.$

d) $i_D = j_D A = (3.4\times10^{-10}\text{ A/m}^2)(2.1\times10^{-6}\text{ m}^2) = 7.14\times10^{-16}\text{ A}$

$\Rightarrow B_D = \dfrac{\mu_0 I_D}{2\pi r} = \dfrac{\mu_0(7.14\times10^{-16}\text{ A})}{2\pi(0.060\text{ m})} = 2.38\times10^{-21}\text{ T},$ and this is a negligible contribution. $B_c = \dfrac{\mu_0 I_c}{2\pi r} = \dfrac{\mu_0}{2\pi}\dfrac{(16\text{ A})}{(0.060\text{ m})} = 5.33\ \times10^{-5}\text{ T}.$

29.39: In a superconductor there is no internal magnetic field, and so there is no changing flux and no induced emf, and no induced electric field.

$$0 = \oint_{\substack{\text{Inside}\\ \text{material}}} \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} = \mu_0(I_c + I_D) = \mu_0 I_c \Rightarrow I_c = 0,$$

and so there is no current inside the material. Therefore, it must all be at the surface of the cylinder.

29.40: Unless some of the regions with resistance completely fill a cross-sectional area of a long type-II superconducting wire, there will still be no total resistance. The regions of no resistance provide the path for the current. Indeed, it will be like two resistors in parellel, where one has zero resistance and the other is non-zero. The equivalent resistance is still zero.

29.41: a) For magnetic fields less than the critical field, there is no internal magnetic field, so:

Inside the superconductor: $\vec{B} = 0, \vec{M} = -\dfrac{\vec{B}_0}{\mu_0} = -\dfrac{(0.130\ T)\hat{i}}{\mu_0} = -(1.03\times10^5\ \text{A/m})\hat{i}$.

Outside the superconductor: $\vec{B} = \vec{B}_0 = (0.130T)\hat{i}, \vec{M} = 0$.

b) For magnetic fields greater than the critical field, $\chi = 0 \Rightarrow \vec{M} = 0$ both inside and outside the superconductor, and $\vec{B} = \vec{B}_0 = (0.260\ \text{T})\hat{i}$, both inside and outside the superconductor.

29.42: a) Just under $\vec{B}_{c1}$ (threshold of superconducting phase), the magnetic field in the material must be zero, and $\vec{M} = -\dfrac{\vec{B}_{c1}}{\mu_0} = -\dfrac{55\times10^{-3}\ \text{T}\hat{i}}{\mu_0} = -(4.38\times10^4\ \text{A/m})\hat{i}$.

b) Just over $\vec{B}_{c2}$ (threshold of normal phase), there is zero magnetization, and $\vec{B} = \vec{B}_{c2} = (15.0\ \text{T})\hat{i}$.

29.43: a) The angle ϕ between the normal to the coil and the direction of $\vec{B}$ is 30.0°.

$|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right| = (N\pi r^2)\, dB/dt$ and $I = |\varepsilon|/R$.

For $t < 0$ and $t > 1.00$ s, $dB/dt = 0, |\varepsilon| = 0$ and $I = 0$

For $0 \le t \le 1.00$ s, $dB/dt = (0.120\ \text{T})\pi \sin \pi t$

$|\varepsilon| = (N\pi r^2)\pi(0.120\ \text{T})\sin \pi t = (0.9475\ \text{V}) \sin \pi t$

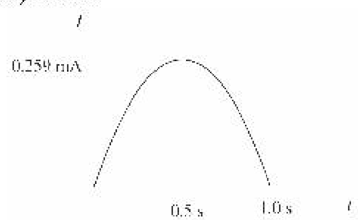
R for wire : $R_w = \dfrac{\rho L}{A} = \dfrac{\rho L}{\pi r^2}$; $\rho = 1.72\times10^{-8}\,\Omega\cdot\text{m}, r = 0.0150\times10^{-3}\text{m}$

$L = Nc = N2\pi r = (500)\,(2\pi)\,(0.0400\ \text{m}) = 125.7\ \text{m}$

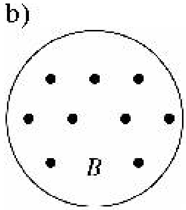
$R_w = 3058\ \Omega$ and the total resistance of the circuit is

$R = 3058\ \Omega + 600\ \Omega = 3658\ \Omega$

$I = |\varepsilon|/R = (0.259\ \text{mA}) \sin \pi t$



b)



B increasing so Φ_B is $\odot$ and increasing

Φ_{ind} is $\otimes$ so I is clockwise

29.44: a) The large circuit is an RC circuit with a time constant of $\tau = RC = (10\ \Omega)\,(20\times10^{-6}\,\text{F}) = 200\mu\text{s}$. Thus, the current as a function of time is

$$i = \left(\frac{100\ \text{V}}{10\ \Omega}\right)e^{-\frac{t}{200\ \mu\text{s}}}$$

At $t = 200\mu\text{s}$, we obtain $i = (10\ \text{A})\,(e^{-1}) = 3.7\ \text{A}$.

b) Assuming that only the long wire nearest the small loop produces an appreciable magnetic flux through the small loop and referring to the solution of Problem 29.54 we obtain

$$\Phi_B = \int_c^{c+a}\frac{\mu_0 ib}{2\pi r}\,dr = \frac{\mu_0 ib}{2\pi}\ln\left(1+\frac{a}{c}\right)$$

So the emf induced in the small loop at $t = 200\mu\text{s}$ is

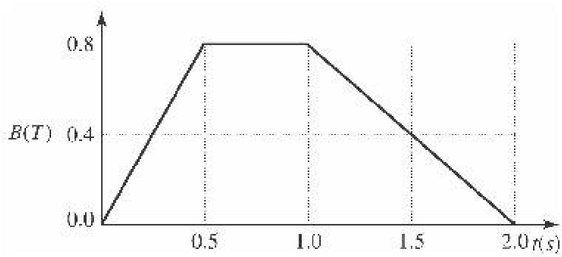
$$\varepsilon = -\frac{d\Phi}{dt} = -\frac{\mu_0 b}{2\pi}\ln\left(1+\frac{a}{c}\right)\frac{di}{dt} = -\frac{(4\pi\times10^{-7}\,\frac{\text{Wb}}{\text{A}\cdot\text{m}})\,(0.200\ \text{m})}{2\pi}\ln(3.0)\left(-\frac{3.7\ \text{A}}{200\times10^{-6}\,\text{s}}\right) = +0.81$$

Thus, the induced current in the small loop is $i' = \frac{\varepsilon}{R} = \frac{0.81\ \text{mV}}{25(0.600\ \text{m})(1.0\ \Omega/\text{m})} = 54\mu\text{A}$.

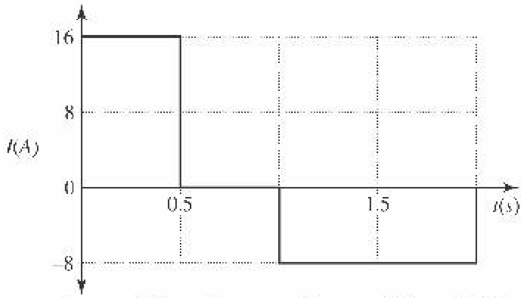
c) The induced current will act to oppose the decrease in flux from the large loop. Thus, the induced current flows counterclockwise.

d) Three of the wires in the large loop are too far away to make a significant contribution to the flux in the small loop–as can be seen by comparing the distance c to the dimensions of the large loop.

29.45: a)



b)



c) $E_{\max} = \dfrac{\varepsilon}{N2\pi r} = \dfrac{1}{2\pi} NA \dfrac{dB}{dt} = \dfrac{1}{2\pi r} \pi r^2 \dfrac{dB}{dt} = \dfrac{r}{2} \dfrac{dB}{dt} = \dfrac{(0.50 \text{ m})}{2} \dfrac{0.80 \text{ T}}{0.50 \text{ s}} = 0.4 \text{ V/m}.$

29.46: a) $I = \dfrac{\varepsilon}{R} = \dfrac{1}{R} \dfrac{d\Phi_B}{dt} = \dfrac{1}{R} \dfrac{d(BA \cos \omega t)}{dt} = \dfrac{BA\, \omega \sin \omega t}{R}.$

b) $P = I^2 R = \dfrac{B^2 A^2 \omega^2 \sin^2 \omega t}{R}.$

c) $\mu = IA = \dfrac{BA^2 \omega \sin \omega t}{\mathrm{R}}.$

d) $\tau = \mu B \sin\phi = \mu \mathrm{B} \sin \omega t = \dfrac{\mathrm{B}^2 A^2 \omega \sin^2 \omega t}{R}.$

e) $P = \tau\omega = \dfrac{B^2 A^2 \omega^2 \sin^2 \omega t}{R}$, which is the same as part (b).

29.47: a) $\Phi_B = BA = \dfrac{\mu_0 i}{2a} \pi a^2 = \dfrac{\mu_0 i \pi a}{2}.$

b) $\varepsilon = -\dfrac{d\Phi_B}{dt} = iR \Rightarrow -\dfrac{d}{dt}\left(\dfrac{\mu_0 i \pi a}{2}\right) = -\dfrac{\mu_0 \pi a}{2} \dfrac{di}{dt} = iR \Rightarrow \dfrac{di}{dt} = -i \dfrac{2R}{\mu_0 \pi a}$

c) Solving $\dfrac{di}{i} = -dt \dfrac{2R}{\mu_0 \pi a}$ for $i(t)$ yields $i(t) = i_0 e^{-t(2R/\mu_0 \pi a)}$.

d) We want $i(t) = i_0 (0.010) = i_0 e^{-t(2R/\mu_0 \pi a)} \Rightarrow \ln(0.010) = -t(2R/\mu_0 \pi a)$

$$\Rightarrow t = -\frac{\mu_0 \pi a}{2R} \ln(0.010) = -\frac{\mu_0 \pi (0.50 \text{ m})}{2(0.10\ \Omega)} \ln(0.010) = 4.55 \times 10^{-5} \text{ s}.$$

e) We can ignore the self-induced currents because it takes only a very short time for them to die out.

29.48: a) Choose the area vector to point out of the page. Since the area and its orientation to the magnetic field are fixed, we can write the induced emf in the 10 cm radius loop as

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A_z \frac{dB_z}{dt} = -\pi(0.10 \text{ m})^2 \frac{dB_z}{dt} = 10^{-4}[(20.0 \text{ V}) - (4.00 \text{ V/s})t]$$

After solving for $\frac{dB_z}{dt}$ and integrating we obtain

$$B_z(t = 2.00\text{s}) - B_z(t = 0\text{ s}) = -\frac{10^{-4}}{\pi(0.10\text{ m})^2}\int_0^2 [(20.0\text{ V}) - (4.00\text{ V/s})t]dt.$$

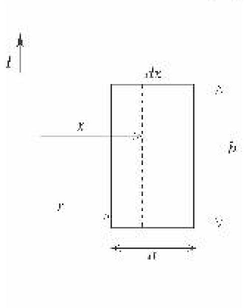
Thus,

$$B_z = (-0.800\text{ T}) - \frac{10^{-2}\text{ m}^{-2}}{\pi}\left[(20.0\text{ V})(2.00\text{ s}) - (2.00\text{ V/s})(2.00\text{ s})^2\right] = -0.902\text{ T}$$

b) Repeat part (a) but set $\varepsilon = -(2.00\times10^{-3}\text{ V}) + (4.00\times10^{-4}\text{ V/s})\,t$ to obtain $B_z = -0.698\text{ T}$

c) In part (a) the flux has decreased (*i.e.*, it has become more negative) and in part (b) the flux has increased. Both results agree with the expectations of Lenz's law.

29.49:a) (i) $|\varepsilon| = \left|\dfrac{d\Phi_B}{dt}\right|$



Consider a narrow strip of width dx and a distance x from the long wire.

The magnetic field of the wire at the strip is $B = \mu_0 I/2\pi x$.

The flux through the strip is

$$d\Phi_B = Bb\,dx = (\mu_0 Ib/2\pi)\,(dx/x)$$

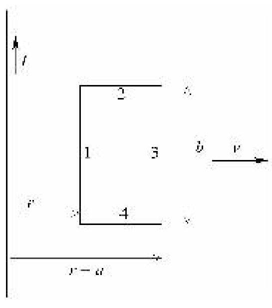
The total flux through the loop is $\Phi_B = \int d\Phi_B = \left(\dfrac{\mu_0 Ib}{2\pi}\right)\displaystyle\int_r^{r+a}\dfrac{dx}{x}$

$$\Phi_B = \left(\frac{\mu_0 Ib}{2\pi}\right)\ln\left(\frac{r+\mathrm{a}}{r}\right)$$

$$\frac{d\Phi_B}{dt} = \frac{d\Phi_B}{dt}\frac{dr}{dt} = \frac{\mu_0 Ib}{2\pi}\left(-\frac{a}{r(r+a)}\right)\ v$$

$$|\varepsilon| = \frac{\mu_0 Iabv}{2\pi r(r+a)}$$

(ii) $\varepsilon = Bvl$ for a bar of length l moving at speed v perpendicular to magnetic field B.



The emf in each side of the loop is

$$\varepsilon_1 = \left(\frac{\mu_0 I}{2\pi r}\right) vb,$$

$$\varepsilon_2 = \left(\frac{\mu_0 I}{2\pi (r+a)}\right) vb,$$

$$\varepsilon_2 = \varepsilon_4 = 0$$

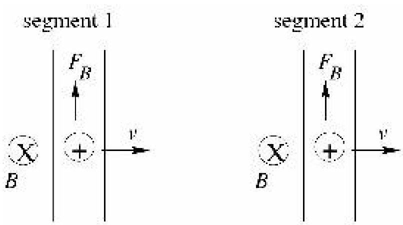
Both emfs ε_1 and ε_2 are directed toward the top of the loop so oppose each other. The net emf is

$$\varepsilon = \varepsilon_1 - \varepsilon_2 = \frac{\mu_0 Ivb}{2\pi}\left(\frac{1}{r} - \frac{1}{r+a}\right) = \frac{\mu_0 Iabv}{2\pi r(r+a)}$$

This expression agrees with what was obtained in (i) using Faraday's law.

b) (i) $\vec{B}$ is $\otimes$. Φ_B is $\otimes$ and decreasing, so the flux Φ_{ind} of the induced current is $\otimes$ and the current is clockwise.

(ii)



B is larger at segment 1 since it is closer to the long wire, so F_B is larger in segment 1 and the induced current in the loop is clockwise. This agrees with the direction deduced in (i) using Lenz's law.

c) when $v = 0$ the induced emf should be zero; the expression in part (a) gives this. When $a \to 0$ the flux goes to zero and the emf should approach zero; the expression in part (a) gives this. When $r \to \infty$ the magnetic field through the loop goes to zero and the emf should go to zero; the expression in part (a) gives this.

29.50: a) Rotating about the $y-$axis :

$$\varepsilon_{max} = \frac{d\Phi_B}{dt} = \omega BA = (35.0 \text{ rad/s})\,(0.450 \text{ T})\,(6.00 \times 10^{-2} \text{ m}) = 0.945 \text{ V}.$$

b) Rotating about the $x-$axis : $\frac{d\Phi_B}{dt} = 0 \Rightarrow \varepsilon = 0.$

c) Rotating about the $z-$axis :

$$\varepsilon_{max} = \frac{d\Phi_B}{dt} = \omega BA = (35.0 \text{ rad/s})\,(0.450 \text{ T})\,(6.00 \times 10^{-2} \text{ m}) = 0.945 \text{ V}.$$

29.51: From Example 29.4, $\varepsilon = \omega BA \sin \omega t$; $\varepsilon_{max} = \omega BA$

For N loops, $\varepsilon_{max} = N\omega BA$

$N = 400$, $B = 1.5$ T, $A = (0.100\text{ m})^2$, $\varepsilon_{max} = 120$ V

$\omega = \varepsilon_{max}/NBA = (20\text{ rad/s})\,(1\text{ rev}/2\pi\text{ rad})\,(60\text{ s}/1\text{min}) = 190\text{ rpm}$

29.52: a) The flux through the coil is given by $NBA\cos(\omega t)$, where N is the number of turns, B is the strength of the Earth's magnetic field, and ω is the angular velocity of the rotating coil. Thus, $\varepsilon = \omega\, NBA\sin(\omega t)$, which has a peak amplitude of $\varepsilon_0 = \omega\, NBA.$ Solving for A we obtain

$$A = \frac{\varepsilon_0}{\omega NB} = \frac{9.0\text{ V}}{(30\text{ rev/min})\,(1\text{ min/60s})\,(2\pi\text{ rad/rev})\,(2000\text{ turns})\,(8.0\times10^{-5}\text{T})} = 18\text{ m}^2$$

b) Assuming a point on the coil at maximum distance from the axis of rotation we have

$$v = r\omega = \sqrt{\frac{A}{\pi}}\omega = \sqrt{\frac{18\text{ m}^2}{\pi}}\,(30\text{ rev/min})\,(1\text{ min}/60\text{ s})\,(2\pi\text{ rad/rev}) = 7.5\text{ m/s}.$$

29.53: a) $\varepsilon = -\dfrac{\Delta\Phi_B}{\Delta t} = -B\dfrac{\Delta A}{\Delta t} = -B\dfrac{-\pi r^2}{\Delta t} = (0.950\text{ T})\dfrac{\pi(0.0650/2\text{ m})^2}{0.250\text{ s}} = 0.0126\text{ V}.$

b) Since the flux through the loop is decreasing, the induced current must produce a field that goes into the page. Therefore the current flows from point a through the resistor to point b.

29.54: a) When $I = i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}$, into the page.

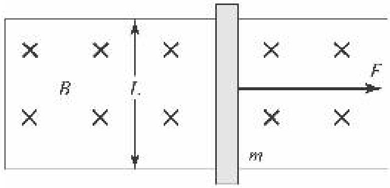
b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r}Ldr.$

c) $\Phi_B = \displaystyle\int_a^b d\Phi_B = \frac{\mu_0 iL}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 iL}{2\pi}\ln(b/a).$

d) $\varepsilon = \dfrac{d\Phi_B}{dt} = \dfrac{\mu_0 L}{2\pi}\ln(b/a)\dfrac{di}{dt}.$

e) $\varepsilon = \dfrac{\mu_0(0.240\text{ m})}{2\pi}\ln(0.360/0.120)\,(9.60\text{ A/s}) = 5.06\times10^{-7}\text{ V}.$

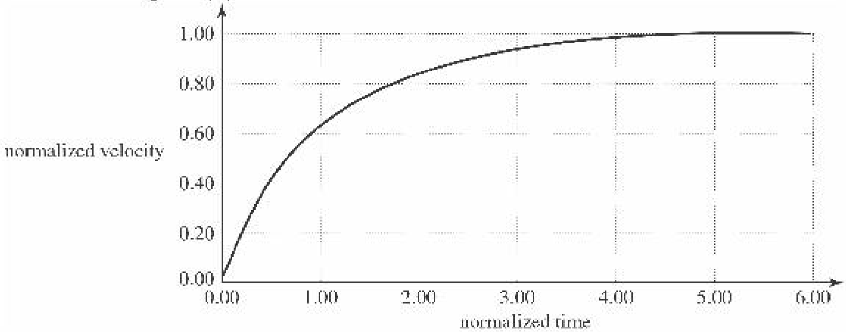
29.55: a)



$\varepsilon = vBL = IR \Rightarrow I = \dfrac{vBL}{R}$, and $F - F_B = F - ILB = ma$

$\Rightarrow a = \left(\dfrac{F - ILB}{m}\right) = \dfrac{F}{m} - \dfrac{vB^2L^2}{mR}.$

$\Rightarrow \dfrac{dv}{dt} = \dfrac{F}{m} - \dfrac{vB^2L^2}{mR} \Rightarrow v(t) = v_t\left(1 - e^{-t(B^2L^2/mR)}\right)$, where v_t is the terminal velocity calculated in part (b).



b) The terminal speed v_t occurs when the pulling force is equaled by the magnetic force: $F_B = ILB = \left(\dfrac{v_t LB}{R}\right)LB = \dfrac{v_t L^2 B^2}{R} = F \Rightarrow v_t = \dfrac{FR}{L^2B^2}.$

29.56: The bar will experience a magnetic force due to the induced current in the loop. According to Example 29.6, the induced voltage in the loop has a magnitude BLv, which opposes the voltage of the battery, ε. Thus, the net current in the loop is $I = \frac{\varepsilon - BLv}{R}$. The acceleration of the bar is $a = \frac{F}{m} = \frac{ILB\sin(90°)}{m} = \frac{(\varepsilon - BLv)\,LB}{mR}$.

a) To find $v(t)$, set $\frac{dv}{dt} = a = \frac{(\varepsilon - BLv)LB}{mR}$ and solve for v using the method of separation of variables:

$$\int_0^v \frac{dv}{(\varepsilon - BLv)} = \int_0^t \frac{LB}{mR}\,dt \rightarrow v = \frac{\varepsilon}{BL}\left(1 - e^{-\frac{B^2L^2}{mR}t}\right) = (10\ \text{m/s})\left(1 - e^{-\frac{t}{3.13}}\right).$$

Note that the graph of this function is similar in appearance to that of a charging capacitor.

b) $I = \varepsilon/R = 2.4\ \text{A}$; $F = ILB = 2.88\ \text{N}$; $a = F/m = 3.2\ \text{m/s}^2$

c) When

$v = 2.0\ \text{m/s}$, $a = \dfrac{[12\ \text{V} - (1.5\ \text{T})\,(0.8\ \text{m})\,(2.0\ \text{m/s})]\,(0.8\ \text{m})\,(1.5\ \text{T})}{(0.90\ \text{kg})\,(5.0\Omega)} = 2.6\ \text{m/s}^2$

d) Note that as the velocity increases, the acceleration decreases. The velocity will asymptotically approach the terminal velocity $\frac{\varepsilon}{BL} = \frac{12\ \text{V}}{(1.5\ \text{T})\,(0.8\ \text{m})} = 10\ \text{m/s}$, which makes the acceleration zero.

29.57: $\varepsilon = Bvl$; $B = 8.0 \times 10^{-5}$ T, $L = 2.0$ m

Use $\sum \vec{F} = m\vec{a}$ applied to the satellite motion to find the speed v of the satellite.

$$G\frac{mm_E}{r^2} = m\frac{v^2}{r};\ r = 400 \times 10^3\ \text{m} + R_E$$

$$v = \sqrt{\frac{Gm_E}{r}} = 7.665 \times 10^3\ \text{m/s}$$

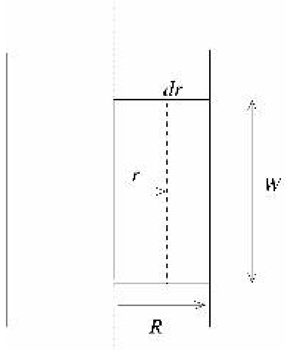
Using this v gives $\varepsilon = 1.2$ V

29.58: a) According to Example 29.6 the induced emf is $\varepsilon = BLv = (8\times10^{-5}\,\text{T})$ $(0.004\ \text{m})(300\ m/s) = 96\mu\ \text{V} \approx 0.1\ \text{mV}$. Note that L is the size of the bar measured in a direction that is perpendicular to both the magnetic field and the velocity of the bar. Since a positive charge moving to the east would be deflected upward, the top of the bullet will be at a higher potential.

b) For a bullet that travels south, the induced emf is zero.

c) In the direction parallel to the velocity the induced emf is zero.

29.59: From Ampere's law (Example 28.9), the magnetic field inside the wire, a distance r from the axis, is $B(r) = \mu_0 Ir/2\pi R^2$.



Consider a small strip of length W and width dr that is a distance r from the axis of the wire.
The flux through the strip is

$$d\Phi_B = B(r)W\ dr = \frac{\mu_0 IW}{2\pi R^2} r\ dr$$

The total flux through the rectangle is

$$\Phi_B = \int d\Phi_B = \left(\frac{\mu_0 IW}{2\pi R^2}\right)\int_0^R r\ dr = \frac{\mu_0 IW}{4\pi}$$

Note that the result is independent of the radius R of the wire.

29.60: a) $\Phi_B = BA = B_0\pi r_0^2(1 - 3(t/t_0)^2 + 2(t/t_0)^3)$.

b) $\varepsilon = -\dfrac{d\Phi_B}{dt} = -B_0\pi r_0^2 \dfrac{d}{dt}(1 - 3(t/t_0)^2 + 2(t/t_0)^3) = -\dfrac{B_0\pi r_0^2}{t_0}(-6(t/t_0) + 6(t/t_0)^2)$

$$\Rightarrow \varepsilon = -\frac{6B_0\pi r_0^2}{t_0} = \left(\left(\frac{t}{t_0}\right)^2 - \left(\frac{t}{t_0}\right)\right) \text{so at } t = 5.0\times10^{-3}\text{s},,$$

$$\varepsilon = -\frac{6B_0\pi(0.0420\text{ m})^2}{0.010\text{ s}}\left(\left(\frac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)^2 - \left(\frac{5.0\times10^{-3}\text{ s}}{0.010\text{ s}}\right)\right) = 0.0665\text{ V, counterclockwise.}$$

c) $i = \dfrac{\varepsilon}{R_{\text{total}}} \Rightarrow R_{\text{total}} = r + R = \dfrac{\varepsilon}{i} \Rightarrow r = \dfrac{0.0655\text{ V}}{3.0\times10^{-3}\text{ A}} - 12\,\Omega = 10..2\,\Omega.$

d) Evaluating the emf at $t = 1.21\times10^{-2}$ s, using the equations of part (b):
$\varepsilon = -0.0676$ V, and the current flows clockwise, from b to a through the resistor.

e) $\varepsilon = 0 \Rightarrow 0 = \left(\left(\dfrac{t}{t_0}\right)^2 - \left(\dfrac{t}{t_0}\right)\right) \Rightarrow 1 = \dfrac{t}{t_0} \Rightarrow t = t_0 = 0.010$ s.

29.61: a) $d\varepsilon = (\vec{v}\times\vec{B})\cdot d\vec{r} = vBdr = \dfrac{\mu_0 Iv}{2\pi r}dr \Rightarrow \varepsilon\dfrac{\mu_0 Iv}{2\pi}\displaystyle\int_d^{d+L}\dfrac{dr}{r} = \dfrac{\mu_0 Iv}{2\pi}\ln\left(\dfrac{d+L}{d}\right).$

b) The magnetic force is strongest at the top end, closest to the current carrying wire. Therefore, the top end, point a, is the higher potential since the force on positive charges is greatest there, leading to more positives gathering at that end.

c) If the single bar was replaced by a rectangular loop, the edges parallel to the wire would have no emf induced, but the edges perpendicular to the wire will have an emf induced, just as in part (b). However, no current will flow because each edge will have its highest potential closest to the current carrying wire. It would be like having two batteries of opposite polarity connected in a loop.

29.62: Wire A : $\vec{v}\times\vec{B} = 0 \Rightarrow \varepsilon = 0.$

Wire C: $\varepsilon = vBL\sin\phi = (0.350\text{ m/s})(0.120\text{ T})(0.500\text{ m})\sin 45° = 0.0148\text{ V}.$

Wire
D: $\varepsilon = vBL\sin\phi = (0.350\text{ m/s})(0.120\text{ T})\sqrt{2}(0.500\text{ m})\sin 45° = 0.0210\text{ V}.$

29.63: a) $d\varepsilon = (\vec{v}\times\vec{B})\cdot d\vec{r} = \omega r\,Bdr \Rightarrow \varepsilon = \displaystyle\int_0^L \omega rBdr = \frac{1}{2}\omega L^2 B$

$$= \frac{(8.80\text{ rad/sec})(0.24\text{ m})^2(0.650\text{ T})}{2} = 0.164\text{ V}.$$

b) The potential difference between its ends is the same as the induced emf.

c) Zero, since the force acting on each end points toward the center.

$$\Delta V_{\text{center}} = \frac{\varepsilon_{\text{part(a)}}}{4} = 0.0410\text{ V}.$$

29.64: a) From Example 29.7, the power required to keep the bar moving at a constant velocity is $P = \frac{(BLv)^2}{R} \Rightarrow R = \frac{(BLv)^2}{P} = \frac{[(0.25\text{ T})(3.00\text{ m/s})]^2}{25\text{ W}} = 0.090\ \Omega.$

b) For a 50 W power dissipation we would require that the resistance be decreased to half the previous value.

c) Using the resistance from part (a) and a bar length of 0.20 m

$$P = \frac{(BLv)^2}{R} = \frac{[(0.25\text{ T})(0.20\text{ m})(2.0\text{ m/s})]^2}{0.090\ \Omega} = 0.11\text{ W}$$

29.65: a) $I = \dfrac{\varepsilon}{R} = \dfrac{vBa}{R} \Rightarrow F = IaB = \dfrac{vB^2a^2}{R}.$

b)

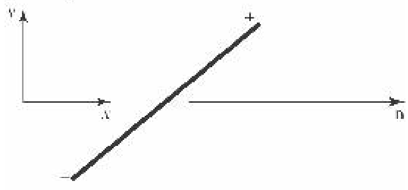
$$F = ma = m\frac{dv}{dt} = \frac{vB^2a^2}{R} \Rightarrow \int_{v_0}^{v} \frac{dv'}{v'} = \frac{B^2a^2}{mR}\int_0^t dt' \Rightarrow v = v_0 e^{-t(B^2a^2/mR)} = \frac{dx}{dt} \Rightarrow$$

$$\int_0^x dx' = v_0 \int_0^\infty e^{-t'(B^2a^2/mR)}dt' \Rightarrow x = -\frac{mRv_0}{B^2a^2} = e^{-t'(B^2a^2/mR)}\Big|_0^\infty = \frac{mRv_0}{B^2a^2}.$$

29.66: a) $\varepsilon = (\vec{v} \times \vec{B}) \cdot \vec{L} = (4.20\text{ m/s})\hat{i} \times ((0.120\text{ T})\hat{i} - (0.220\text{ T})\hat{j} - (0.0900\text{ T})\hat{k}) \cdot \vec{L}$

$\Rightarrow \varepsilon = \left((0.378\text{ V/m})\hat{j} - (0.924\text{ V/m})\hat{k}\right) \cdot \left((0.250\text{ m})(\cos 36.9°\hat{i} + \sin 36.9°\hat{j})\right)$

$\Rightarrow \varepsilon = (0.378)(0.250)\sin 36.9° = 0.0567\text{ V}.$



29.67: At point $a: \varepsilon = \dfrac{d\Phi_B}{dt} = A\dfrac{dB}{dt} = \pi r^2\dfrac{dB}{dt}$ and $F = qE = q\dfrac{\varepsilon}{2\pi r} = \dfrac{qr}{2}\dfrac{dB}{dt}$, to the left. At point b, the field is the same magnitude as at a since they are the same distance from the center. So $F = \dfrac{qr}{2}\dfrac{dB}{dt}$, but upward.

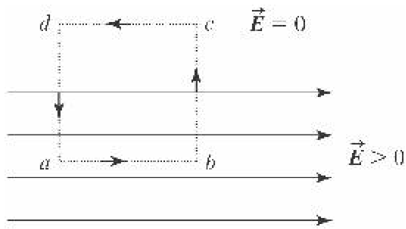
At point c, there is no force by symmetry arguments: one cannot have one direction picked out over any other, so the force must be zero.

29.68: $\oint \vec{E} \cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}.$

If $\vec{B}$ = constant then $\frac{d\Phi_B}{dt} = 0$, so $\oint \vec{E} \cdot d\vec{l} = 0$.

$$\int_{abcda} \vec{E} \cdot d\vec{l} = E_{ab}L - E_{da}L = 0, \text{ but } E_{da} = 0 \text{ so } E_{da}L = 0.$$

But since we assumed $E_{ab} \neq 0$, this contradicts Faraday's law. Thus, we can't have a uniform electric field abruptly drop to zero in a region in which the magnetic field is constant.



29.69: At the terminal speed, the upward force F_B exerted on the loop due to the induced current equals the downward force of gravity: $F_B = mg$

$$\varepsilon = Bvs, I = Bvs/R \text{ and } F_B = IsB = B^2s^2v/R$$

$$\frac{B^2s^2v_T}{R} = mg \text{ and } v_T = \frac{mgR}{B^2s^2}$$

$$m = \rho_m V = \rho_m (4s)\pi(d/2)^2 = \rho_m \pi s d^2$$

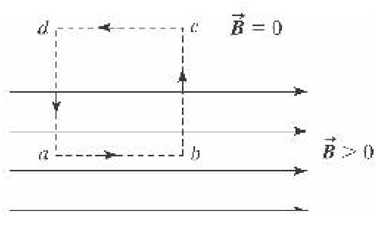
$$R = \frac{\rho L}{A} = \frac{\rho_R 4s}{\frac{1}{4}\pi d^2} = \frac{16\rho_R s}{\pi d^2}$$

Using these expressions for m and R gives $v_T = 16\rho_m \rho_R g/B^2$

29.70: $\oint \vec{B} \cdot d\vec{l} = 0$ (no currents in the region). Using the figure, let $B = B_0\hat{i}$ for $y < 0$ and $B = 0$ for $y > 0$.

$$\int_{abcde} \vec{B} \cdot d\vec{l} = B_{ab}L - B_{cd}L = 0,$$

but $B_{cd} = 0$. $B_{ab}L = 0$, but $B_{ab} \neq 0$. This is a contradiction and violates Ampere's Law. See the figure on the next page.



29.71: a) $j_c = \frac{I}{A} = \frac{V}{AR} = \frac{VA}{Ad\rho} = \frac{\text{V}}{d\rho} = \frac{q}{Cd\rho} = \frac{qd}{K\varepsilon_0 Ad\rho}$

and

$$RC = \frac{\rho d}{A}\frac{K\varepsilon_0 A}{d} = K\varepsilon_0 \rho.$$

$$\Rightarrow j_c(t) = \frac{q}{K\varepsilon_0 A\rho} = \frac{Q_0}{K\varepsilon_0 A\rho}e^{-t/RC} = \frac{Q_0}{K\varepsilon_0 A\rho}e^{-t/K\varepsilon_0\rho}$$

b) $j_D(t) = K\varepsilon_0 \frac{dE}{dt} = K\varepsilon_0 \frac{d(\rho j_c)}{dt} = K\varepsilon_0 \rho \frac{Q_0}{K\varepsilon_0 A\rho}\frac{d(e^{-t/K\varepsilon_0\rho})}{dt}$

$$= -\frac{Q_0}{K\varepsilon_0 A\rho}e^{-t/K\varepsilon_0\rho} = -j_c(t).$$

29.72: a) $j_c(\text{max}) = \frac{E_0}{\rho} = \frac{0.450\ \text{V/m}}{2300\ \Omega\ \text{m}} = 1.96\times10^{-4}\ \text{A/m}^2.$

b) $j_D(\text{max}) = \varepsilon_0\frac{dE}{dt} = \varepsilon_0\omega E_0 = 2\pi\varepsilon_0 fE_0 = 2\pi\varepsilon_0(120\ \text{Hz})(0.450\ \text{V/m})$

$$\Rightarrow j_D(\text{max}) = 3.00\times10^{-9}\ \text{A/m}^2.$$

c) If $j_c = j_D \Rightarrow \frac{E_0}{\rho} = \omega\varepsilon_0 E_0 \Rightarrow \omega = \frac{1}{\rho\varepsilon_0} = 4.91\times10^7\ \text{rad/s}$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{4.91\times10^7\ \text{rad/s}}{2\pi} = 7.82\times10^6\text{Hz}.$$

d) The two current densities are out of phase by $90°$ because one has a sine function and the other has a cosine, so the displacement current leads the conduction current by 90°.

29.73: a) $\vec{\tau}_G = \sum \vec{r}_{cm}\times m\vec{g}$, summed over each leg,

$$= (0)\left(\frac{m}{4}\right)g\ \sin(90-\phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\ \sin(90-\phi) + \left(\frac{L}{2}\right)\left(\frac{m}{4}\right)g\ \sin(90-\phi)$$

$$+ (L)\left(\frac{m}{4}\right)g\sin(90-\phi)$$

$$= \frac{mgL}{2}\cos\phi\ \text{(clockwise)}.$$

$\vec{\tau}_B = \vec{\mu}\times\vec{B} = IAB\sin\phi$ (counterclockwise).

$I = \frac{\varepsilon}{R} = \frac{BA}{R}\frac{d}{dt}\cos\phi = -\frac{BA}{R}\frac{d\phi}{dt}\sin\phi = \frac{BA\omega}{R}\sin\phi.$ The current is going counterclockwise looking to the $-\hat{k}$ direction.

$$\Rightarrow \tau_B = \frac{B^2A^2\omega}{R}\sin^2\phi = \frac{B^2L^4\omega}{R}\sin^2\phi,$$

so $\tau = \frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi,$ opposite to the direction of the rotation.

b) $\tau = I\alpha$ (I being the moment of inertia).

About this axis $I = \frac{5}{12}mL^2$.

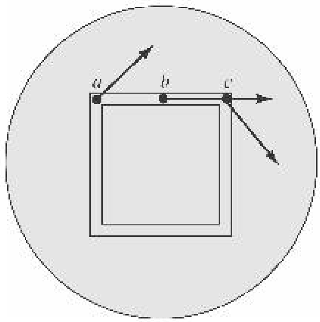
$$\text{So } \alpha = \frac{12}{5}\frac{1}{mL^2}\left[\frac{mgL}{2}\cos\phi - \frac{B^2L^4\omega}{R}\sin^2\phi\right]$$

$$= \frac{6g}{5L}\cos\phi - \frac{12B^2L^2\omega}{5m\ R}\sin^2\phi.$$

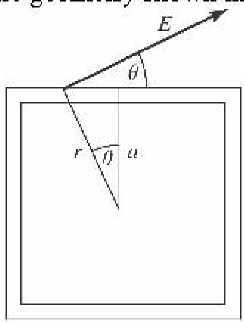
c) The magnetic torque slows down the fall (since it opposes the gravitational torque).

d) Some energy is lost through heat from the resistance of the loop.

29.74: a) For clarity, figure is rotated so B comes out of the page.



b) To work out the amount of the electric field that is in the direction of the loop at a general position, we will use the geometry shown in the diagram below.



$$E_{\text{loop}} = E\cos\theta \text{ but } E = \frac{\varepsilon}{2\pi r} = \frac{\varepsilon}{2\pi(a/\cos\theta)} = \frac{\varepsilon\cos\theta}{2\pi a}$$

$$\Rightarrow E_{\text{loop}} = \frac{\varepsilon\cos^2\theta}{2\pi a} \text{but } \varepsilon = \frac{d\Phi_B}{dt} = A\frac{dB}{dt}\pi r^2\frac{dB}{dt} = \frac{\pi a^2}{\cos^2\theta}\frac{dB}{dt}$$

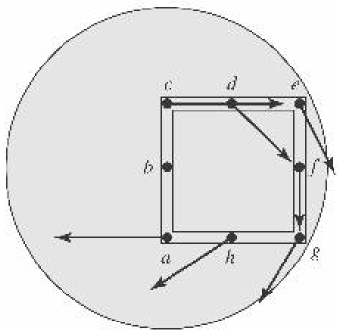
$\Rightarrow E_{\text{loop}} = \dfrac{\pi a^2}{2\pi a}\dfrac{dB}{dt} = \dfrac{a}{2}\dfrac{dB}{dt}$, which is exactly the value for a ring, obtained in Exercise 29.29, and has no dependence on the part of the loop we pick.

c) $I = \dfrac{\varepsilon}{R} = \dfrac{A}{R}\dfrac{dB}{dt} = \dfrac{L^2}{R}\dfrac{dB}{dt} = \dfrac{(0.20\,\text{m})^2(0.0350\ \text{T/s})}{1.90\,\Omega} = 7.37\times10^{-4}\ \text{A}.$

d) $\varepsilon_{ab} = \dfrac{1}{8}\varepsilon = \dfrac{1}{8}L^2\dfrac{dB}{dt} = \dfrac{(0.20\,\text{m})^2(0.0350\ \text{T/s})}{8} = 1.75\times10^{-4}\ \text{V}.$

But there is potential drop $V = IR = -1.75\times10^{-4}$ V, so the potential difference is zero.

29.75: a)



b) The induced emf on the side ac is zero, because the electric field is always perpendicular to the line ac.

c) To calculate the total emf in the loop, $\varepsilon = \dfrac{d\Phi_B}{dt} = A\dfrac{dB}{dt} = L^2\dfrac{dB}{dt}$

$\Rightarrow \varepsilon = (0.20\,\text{m})^2(0.035\,\text{T/s}) = 1.40\times10^{-3}\ \text{V}.$

d) $I = \dfrac{\varepsilon}{R} = \dfrac{1.40\times10^{-3}\ \text{V}}{1.90\,\Omega} = 7.37\times10^{-4}\ \text{A}.$

e) Since the loop is uniform, the resistance in length ac is one quarter of the total resistance. Therefore the potential difference between a and c is:
$V_{ac} = IR_{ac} = (7.37\times10^{-4}\ \text{A})(1.90\,\Omega/4) = 3.50\times10^{-4}\ \text{V}$, and the point a is at a higher potential since the current is flowing from a to c.

29.76: a) As the bar starts to slide, the flux is decreasing, so the current flows to increase the flux, which means it flows from a to b.

b) The magnetic force on the bar must eventually equal that of gravity.

$$F_B = iLB = \frac{LB}{R}\varepsilon = \frac{LB}{R}\frac{d\Phi_B}{dt} = \frac{LB}{R}B\frac{dA}{dt} = \frac{LB^2}{R}(vL\cos\phi) = \frac{vL^2B^2}{R}\cos\phi$$

$$\Rightarrow F_g = mg\tan\phi = \frac{v_t L^2B^2}{R}\cos\phi \Rightarrow v_t = \frac{Rmg\tan\phi}{L^2B^2\cos\phi}.$$

c) $i = \dfrac{\varepsilon}{R} = \dfrac{1}{R}\dfrac{d\Phi_B}{dt} = \dfrac{1}{R}B\dfrac{dA}{dt} = \dfrac{B}{R}(vL\cos\phi) = \dfrac{vL\,B\cos\phi}{R} = \dfrac{mg\tan\phi}{LB}.$

d) $P = i^2R = \dfrac{Rm^2g^2\tan^2\phi}{L^2B^2}.$

e) $P_g = Fv\cos(90° - \phi) = mg\left(\dfrac{Rmg\tan\phi}{L^2B^2\cos\phi}\right)\sin\phi \Rightarrow P_g = \dfrac{Rm^2g^2\tan^2\phi}{L^2B^2}$, which is the same as found in part (d).

29.77: The primary assumption throughout the problem is that the square patch is small enough so that the velocity is constant over its whole areas, that is, $v = \omega r \approx \omega d$.

a) $\omega \rightarrow$ clockwise, $B \rightarrow$ into page :

$\varepsilon = vBL = \omega d\ BL$

$\Rightarrow I = \dfrac{\varepsilon}{R} = \dfrac{\varepsilon A}{\rho L} = \dfrac{\omega dBA}{\rho}$. Since $\vec{v} \times \vec{B}$ points outward, A is just the cross-sectional area tL.

$\Rightarrow I = \dfrac{\omega dBLt}{\rho}$ flowing radially outward since $\vec{v} \times \vec{B}$ points outward.

b) $\vec{\tau} = \vec{d} \times \vec{F}_b$; $\vec{F}_B = I\vec{L} \times \vec{B} = ILB$ pointing counterclockwise.

So $\tau = \dfrac{\omega d^2 B^2 L^2 t}{\rho}$ pointing out of the page (a counterclockwise torque opposing the clockwise rotation).

c) If $\omega \rightarrow$ counterclockwise and $B \rightarrow$ into page,

$\Rightarrow I \rightarrow$ flow inward radially since $\vec{v} \times \vec{B}$ points inward.

$\tau \rightarrow$ clockwise (again opposing the motion);

If $\omega \rightarrow$ counterclockwise and $B \rightarrow$ out of the page

$\Rightarrow I \rightarrow$ radially outward

$\tau \rightarrow$ clockwise (opposing the motion)

The magnitudes of I and τ are the same as in part (a).

30.1: a) $\varepsilon_2 = M(di_1/dt) = (3.25 \times 10^{-4}\ \text{H})\ (830\ \text{A/s}) = 0.270\ \text{V}$, and is constant.

b) If the second coil has the same changing current, then the induced voltage is the same and $\varepsilon_1 = 0.270\ \text{V}$.

30.2: For a toroidal solenoid, $M = N_2 \Phi_{B_2}/i_1$, and $\Phi_{B_2} = \mu_0 N_1 i_1 A/2\pi r$.

So, $M = \mu_0 A N_1 N_2 / 2\pi r$.

30.3: a) $M = N_2 \Phi_{B_2}/i_1 = (400)\ (0.0320\ \text{Wb})/(6.52\ \text{A}) = 1.96\ \text{H}$.

b) When $i_2 = 2.54\ \text{A}$, $\Phi_{B_1} = i_2 M / N_1 = (2.54\ \text{A})\ (1.96\ \text{H})/(700) = 7.11 \times 10^{-3}\ \text{Wb}$.

30.4: a) $M = \varepsilon_2/(di/dt) = 1.65 \times 10^{-3}\ \text{V}/\ (-0.242\ \text{A/s}) = 6.82 \times 10^{-3}\ \text{H}$.

b) $N_2 = 25$, $i_1 = 1.20\ \text{A}$,

$\Rightarrow \Phi_{B_2} = i_1 M / N_2 = (1.20\ \text{A})\ (6.82 \times 10^{-3}\ \text{H})/25$

$= 3.27 \times 10^{-4}\ \text{Wb}$.

c) $di_2/dt = 0.360$ A/s and $\varepsilon_1 = Mdi_2/dt = (6.82\times10^{-3}\text{ H})(0.360\text{ A/s}) = 2.45$ mV.

30.5: $1\text{ H} = 1\text{ Wb/A} = 1\text{ Tm}^2/\text{A} = 1\text{ Nm/A}^2 = 1\text{ J/A}^2 = 1\text{ (J/AC)s} = 1\text{ (V/A)s} = 1\,\Omega\text{s}.$

30.6: For a toroidal solenoid, $L = N\Phi_B/i = \varepsilon/(di/dt)$. So solving for N we have:

$$N = \varepsilon i/\Phi_B(di/dt) = \frac{(12.6\times10^{-3}\text{ V})\,(1.40\text{ A})}{(0.00285\text{ Wb})\,(0.0260\text{ A/s})} = 238\text{ turns.}$$

30.7: a) $|\varepsilon| = L(di_1/dt) = (0.260\text{ H})\,(0.0180\text{ A/s}) = 4.68\times10^{-3}\text{ V}.$

b) Terminal a is at a higher potential since the coil pushes current through from b to a and if replaced by a battery it would have the $+$ terminal at a.

30.8: a) $$L_{K_m} = K_m\mu_0 N^2 A/2\pi r = \frac{(500\mu_0)\,(1800)^2\,(4.80\times10^{-5}\text{ m}^2)}{2\pi(0.120\text{ m})} = 0.130\text{ H.}$$

b) Without the material, $L = \dfrac{1}{K_m}L_{K_m} = \dfrac{1}{500}(0.130\text{ H}) = 2.60\times10^{-4}\text{ H.}$

30.9: For a long, straight solenoid:
$L = N\Phi_B/i$ and $\Phi_B = \mu_0 NiA/l \Rightarrow L = \mu_0 N^2 A/l.$

30.10: a) Note that points a and b are reversed from that of figure 30.6. Thus, according to Equation 30.8, $\frac{di}{dt} = \frac{V_b - V_a}{L} = \frac{-1.04\text{ V}}{0.260\text{ H}} = -4.00$ A/s. Thus, the current is decreasing.

b) From above we have that $di = (-4.00\text{ A/s})dt$. After integrating both sides of this expression with respect to t, we obtain

$$\Delta i = (-4.00\text{ A/s})\Delta t \Rightarrow i = (12.0\text{ A}) - (4.00\text{ A/s})\,(2.00\text{ s}) = 4.00\text{ A.}$$

30.11: a) $L = \varepsilon/(di/dt) = (0.0160\text{ V})/(0.0640\text{ A/s}) = 0.250$ H.

b) $\Phi_B = iL/N = (0.720\text{ A})\,(0.250\text{ H})/(400) = 4.50\times10^{-4}$ Wb.

30.12: a) $U = \dfrac{1}{2}LI^2 = (12.0\text{ H})\,(0.300\text{ A})^2/2 = 0.540\text{ J.}$

b) $P = I^2R = (0.300\text{ A})^2\,(180\,\Omega) = 16.2$ W.

c) No. Magnetic energy and thermal energy are independent. As long as the current is constant, $U =$ constant.

30.13: $U = \frac{1}{2}LI^2 = \frac{\mu_0 N^2 A I^2}{4\pi r}$

$$\Rightarrow N = \sqrt{\frac{4\pi r U}{\mu_0 A I^2}} = \sqrt{\frac{4\pi(0.150\text{ m})(0.390\text{ J})}{\mu_0(5.00\times10^{-4}\text{m}^2)\ (12.0\text{ A})^2}} = 2850\text{ turns.}$$

30.14: a) $U = Pt = (200\text{ W})\,(24\text{ h/day}\times 3600\text{ s/h}) = 1.73\times10^7\text{ J}.$

b) $U = \frac{1}{2}LI^2 \Rightarrow L = \frac{2U}{I^2} = \frac{2(1.73\times10^7\text{ J})}{(80.0\text{ A})^2} = 5406\text{ H}.$

30.15: Starting with Eq. (30.9), follow exactly the same steps as in the text except that the magnetic permeability μ is used in place of μ_0.

30.16: a) free space: $U = u\text{V} = \frac{B^2}{2\mu_0}\text{V} = \frac{(0.560\text{ T})^2}{2\mu_0}(0.0290\text{ m}^3) = 3619\text{ J}.$

b) material with $K_\text{m} = 450 \Rightarrow U = u\text{V} = \frac{B^2}{2K_\text{m}\mu_0}\text{V} = \frac{(0.560\text{ T})^2}{2(450)\mu_0}(0.0290\text{ m}^3) = 8.04\text{ J}.$

30.17: a) $u = \frac{U}{Vol} = \frac{B^2}{2\mu_0} \Rightarrow \text{Volume} = \frac{2\mu_0 U}{B^2} = \frac{2\mu_0(3.60\times10^6\text{ J})}{(0.600\text{ T})^2} = 25.1\text{ m}^3.$

b) $B^2 = \frac{2\mu_0 U}{Vol} = \frac{2\mu_0(3.60\times10^6\text{ J})}{(0.400\text{ m})^3} = 141.4\text{ T}^2 \Rightarrow B = 11.9\text{ T}.$

30.18: a) $B = \frac{\mu_0 NI}{2\pi r} = \frac{\mu_0(600)\,(2.50\text{ A})}{2\pi\,(0.0690\text{ m})} = 4.35\text{ mT}.$

b) From Eq. (30.10), $u = \frac{B^2}{2\mu_0} = \frac{(4.35\times10^{-3}\text{ T})^2}{2\mu_0} = 7.53\text{ J/m}^3.$

c) Volume $\text{V} = 2\pi r A = 2\pi\ (0.0690\text{ m})\ (3.50\times10^{-6}\text{ m}^2) = 1.52\times10^{-6}\text{ m}^3.$

d) $U = uV = (7.53\text{ J/m}^3)\ (1.52\times10^{-6}\text{m}^3) = 1.14\times10^{-5}\text{ J}.$

e) $L = \frac{\mu_0 N^2 A}{2\pi r} = \frac{\mu_0(600)^2\ (3.50\times10^{-6}\text{ m}^2)}{2\pi\ (0.0690\text{ m})} = 3.65\times10^{-6}\text{ H}.$

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(3.65\times10^{-6}\ \text{H})\,(2.50\ \text{A})^2 = 1.14\times10^{-5}\ \text{J same as (d).}$$

30.19: a) $\dfrac{di}{dt} = \dfrac{\varepsilon - iR}{L}$. When $i = 0 \Rightarrow \dfrac{di}{dt} = \dfrac{6.00\ \text{V}}{2.50\ \text{H}} = 2.40\ \text{A/s}$.

b) When $i = 1.00\ \text{A} \Rightarrow \dfrac{di}{dt} = \dfrac{6.00\ \text{V} - (0.500\ \text{A})\,(8.00\ \Omega)}{2.50\ \text{H}} = 0.800\ \text{A/s}$.

c) At $t = 0.200\ \text{s} \Rightarrow i = \dfrac{\varepsilon}{R}(1 - e^{-(R/L)t}) = \dfrac{6.00\ \text{V}}{8.00\ \Omega}(1 - e^{-(8.00\,\Omega/2.50\,\text{H})\,(0.250\,s)}) = 0.413\ \text{A}$.

d) As $t \to \infty \Rightarrow i \to \dfrac{\varepsilon}{R} = \dfrac{6.00\ \text{V}}{8.00\ \Omega} = 0.750\ \text{A}$.

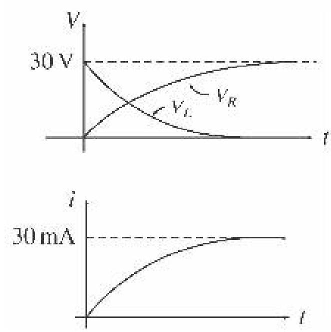
30.20: (a) $i_{\max} = \frac{30\ \text{V}}{1000\ \Omega} = 0.030\ \text{A} = 30\ \text{mA}$, long after closing the switch.

b)

$$i = i_{\max}(1 - \mathrm{e}^{-t/(L/R)}) = 0.030\ \text{A}\left(1 - \mathrm{e}^{-\frac{20\,\mu s}{10\,\mu s}}\right)$$
$$= 0.0259\ \text{A}$$
$$V_{\text{R}} = Ri = (1000\ \Omega)\,(0.0259\ \text{A}) = 26\ \text{V}$$
$$V_{\text{L}} = \varepsilon_{\text{Battery}} - V_{\text{R}} = 30\ \text{V} - 26\ \text{V} = 4.0\ \text{V}$$

(or, could use $V_L = L\frac{di}{dt}$ at $t = 20\ \mu\text{s}$)

c)



30.21: a) $i = \varepsilon/R(1 - e^{-t/\tau})$, $\tau = L/R$

$i_{\max} = \varepsilon/R$ so $i = i_{\max}/2$ when $(1 - e^{-t/\tau}) = \frac{1}{2}$, and $e^{-t/\tau} = \frac{1}{2}$

$$-t/\tau = \ln\left(\tfrac{1}{2}\right) \text{ and } t = \frac{L\ln 2}{\text{R}} = \frac{(\ln 2)\,(1.25\times10^{-3}\text{H})}{50.0\ \Omega} = 17.3\ \mu s$$

b) $U = \frac{1}{2}Li^2$; $U_{max} = \frac{1}{2}Li^2{}_{max}$

$U = \frac{1}{2}U_{max}$ when $i = i_{max}/\sqrt{2}$

$1 - e^{-t/\tau} = 1\sqrt{2}$ so $e^{-t/\tau} = 1 - 1/\sqrt{2} = 0.2929$

$t = -L\ \ln(0.2929)/R = 30.7\ \mu s$

30.22: a) $U = \dfrac{1}{2}LI^2 \Rightarrow I = \sqrt{\dfrac{2U}{L}} = \sqrt{\dfrac{2(0.260\ \text{J})}{0.115\ \text{H}}} = 2.13\ \text{A}$

$$\Rightarrow \varepsilon = IR = (2.13\ \text{A})\,(120\ \Omega) = 256\ \text{V}.$$

b) $i = Ie^{-(R/L)t}$ and $U = \dfrac{1}{2}Li^2 = \dfrac{1}{2}Li^2e^{-2(R/L)t} = \dfrac{1}{2}U_0 = \dfrac{1}{2}\left(\dfrac{1}{2}LI^2\right)$

$$\Rightarrow e^{-2(R/L)t} = \frac{1}{2}$$

$$\Rightarrow t = -\frac{L}{2R}\ln\left(\frac{1}{2}\right) = -\frac{0.115\ \text{H}}{2(120\ \Omega)}\ln\left(\frac{1}{2}\right) = 3.32\times10^{-4}\ \text{s}.$$

30.23: a) $I_0 = \dfrac{\varepsilon}{R} = \dfrac{60\ \text{V}}{240\ \Omega} = 0.250\ \text{A}.$

b) $i = I_0e^{-(R/L)t} = (0.250\ \text{A})\,e^{-(240\ \Omega/0.160\ \text{H})(4.00\times10^{-4}\ s)} = 0.137\ \text{A}.$

c) $V_{cb} = V_{ab} = iR = (0.137\ \text{A})\,(240\ \Omega) = 32.9\ \text{V}$, and c is at the higher potential.

d) $\dfrac{i}{I_0} = \dfrac{1}{2} = e^{-(R/L)t_{1/2}} \Rightarrow t_{1/2} = -\dfrac{L}{R}\ln\left(\dfrac{1}{2}\right) = -\dfrac{(0.160\ \text{H})}{(240\ \Omega)}\ln\left(\dfrac{1}{2}\right) = 4.62\times10^{-4}\ \text{s}.$

30.24: a) At $t = 0 \Rightarrow v_{ab} = 0$ and $v_{bc} = 60\ \text{V}$.

b) As $t \to \infty \Rightarrow v_{ab} \to 60\ \text{V}$ and $v_{bc} \to 0$.

c) When $i = 0.150\ \text{A} \Rightarrow v_{ab} = iR = 36.0\ \text{V}$ and $v_{bc} = 60.0\ \text{V} - 36.0\ \text{V} = 24.0\ \text{V}$.

30.25: a) $P = \varepsilon i = \varepsilon I_0\ (1 - e^{-(R/L)t}) = \dfrac{\varepsilon^2}{R}(1 - e^{-(R/L)t}) = \dfrac{(6.00\ \text{V})^2}{8.00\ \Omega}(1 - e^{-(8.00\ \Omega/2.50\ \text{H})t})$

$$\Rightarrow P = (4.50\ \text{W})\,(1 - e^{-(3.20\ s^{-1})t}).$$

b) $P_R = i^2R = \dfrac{\varepsilon^2}{R}(1 - e^{-(R/L)t})^2 = \dfrac{(6.00\ \text{V})^2}{8.00\ \Omega}(1 - e^{-(8.00\ \Omega/2.50\ \text{H})t})^2$

$$\Rightarrow P_R = (4.50\ \text{W})\,(1 - e^{-(3.20\ s^{-1})t})^2.$$

c) $P_L = iL\dfrac{di}{dt} = \dfrac{\varepsilon}{R}(1-e^{-(R/L)t})\,L\left(\dfrac{\varepsilon}{L}e^{-(R/L)t}\right) = \dfrac{\varepsilon^2}{R}(e^{-(R/L)t} - e^{-2(R/L)t})$

$$\Rightarrow P_L = (4.50\ \text{W})\,(e^{-(3.20\ \text{s}^{-1})t} - e^{-(6.40\ \text{s}^{-1})t}).$$

d) Note that if we expand the exponential in part (b), then parts (b) and (c) add to give part (a), and the total power delivered is dissipated in the resistor and inductor.

30.26: When switch 1 is closed and switch 2 is open:

$$L\frac{di}{dt} + iR = 0 \Rightarrow \frac{di}{dt} = -i\frac{R}{L} \Rightarrow \int_{I_0}^{i}\frac{di'}{i'} = -\frac{R}{L}\int_0^t dt'$$

$$\Rightarrow \ln(i/I_0) = -\frac{R}{L}t \Rightarrow i = I_0 e^{-t(R/L)}.$$

30.27: Units of $L/R = \text{H}/\Omega = (\Omega\ \text{s})/\Omega = \text{s} =$ units of time.

30.28: a) $\omega = \dfrac{1}{\sqrt{LC}} = 2\pi f$

$$\Rightarrow L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2(1.6\times10^6)^2(4.18\times10^{-12}\text{F})} = 2.37\times10^{-3}\ \text{H}.$$

b) $C_{\text{max}} = \dfrac{1}{4\pi^2 f^2{}_{\text{min}} L} = \dfrac{1}{4\pi^2(5.40\times10^5)^2(2.37\times10^{-3}\text{H})} = 3.67\times10^{-11}\ \text{F}.$

30.29: a) $T = \dfrac{2\pi}{\omega} = 2\pi\sqrt{LC} = 2\pi\sqrt{(1.50\ \text{H})(6.00\times10^{-5}\ \text{F})}$

$= 0.0596\ \text{s}, \omega = 105\ \text{rad/s}.$

b) $Q = CV = (6.00\times10^{-5}\ \text{F})(12.0\ \text{V}) = 7.20\times10^{-4}\ \text{C}.$

c) $U_0 = \dfrac{1}{2}CV^2 = \dfrac{1}{2}(6.00\times10^{-5}\ \text{F})(12.0\ \text{V})^2 = 4.32\times10^{-3}\ \text{J}.$

d) At $t = 0$, $q = Q = Q\cos(\omega t + \phi) \Rightarrow \phi = 0.$

$t = 0.0230\ \text{s}, q = Q\cos(\omega t) = (7.20\times10^{-4}\text{C})\ \cos\left(\dfrac{0.0230\ \text{s}}{\sqrt{(1.50\ \text{H})(6.00\times10^{-5}\text{F})}}\right)$

$= -5.43\times10^{-4}$ C. Signs on plates are opposite to those at $t = 0$.

e) $t = 0.0230\ \text{s}, i = \dfrac{dq}{dt} = -\omega Q\ \sin(\omega t)$

$$\Rightarrow i = -\frac{7.20\times10^{-4}\,\text{C}}{\sqrt{(1.50\text{H})(6.00\times10^{-5}\text{H})}}\sin\left(\frac{0.0230\text{s}}{\sqrt{(1.50\text{ H})(6.00\times10^{-5}\text{H})}}\right) = -0.0499\text{A}.$$

Positive charge flowing away from plate which had positive charge at $t = 0$.

f) Capacitor: $U_C = \dfrac{q^2}{2C} = \dfrac{(5.43\times10^{-4}\,\text{C})^2}{2(6.00\times10^{-5}\text{F})} = 2.46\times10^{-3}\,\text{J}.$

Inductor: $U_L = \dfrac{1}{2}Li^2 = \dfrac{1}{2}(1.50\text{H})(0.0499\,\text{A})^2 = 1.87\times10^{-3}\,\text{J}.$

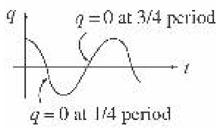
30.30: (a) Energy conservation says $U_L(\text{max}) = U_C(\text{max})$

$$\frac{1}{2}Li_{\text{max}}^2 = \frac{1}{2}\text{CV}^2$$

$$i_{\text{max}} = V\sqrt{C/L} = (22.5\text{V})\sqrt{\frac{18\times10^{-6}\,\text{F}}{12\times10^{-3}\,\text{H}}} = 0.871\text{A}$$

The charge on the capacitor is zero because all the energy is in the inductor.

(b)



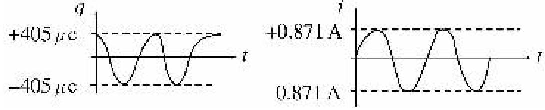
$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{LC}$$

at 1/4 period: $\dfrac{1}{4}T = \dfrac{1}{4}(2\pi\sqrt{LC}) = \dfrac{\pi}{2}\sqrt{(12\times10^{-3}\text{H})(18\times10^{-6}\,\text{F})}$

$= 7.30\times10^{-4}\,\text{s}$

at 3/4 period: $\dfrac{3}{4}T = 3(7.30\times10^{-4}\,\text{s}) = 2.19\times10^{-3}\,\text{s}$

(c) $q_0 = CV = (18\mu\text{F})(22.5\text{V}) = 405\mu C$



30.31: $C = \dfrac{Q}{V} = \dfrac{150\times10^{-9}\,\text{C}}{4.29\times10^{-3}\,\text{V}} = 30.0\ \mu\text{F}$

For an L-C circuit, $\omega = \sqrt{1/LC}$ and $T = 2\pi/\omega = 2\pi\sqrt{LC}$

$$L=\frac{(T/2\pi)^2}{C}=0.601\ \text{mH}$$

30.32: $\omega=\dfrac{1}{\sqrt{(0.0850\text{H})(3.20\times10^{-6}\text{F})}}=1917\ \text{rad/s}$

a) $i_{max}=\omega Q_{max}\Rightarrow Q_{max}=\dfrac{i_{max}}{\omega}=\dfrac{8.50\times10^{-4}\ \text{A}}{1917\ \text{rad/s}}=4.43\times10^{-7}\ \text{C}$

b) From Eq. 31.26 $q=\sqrt{Q^2-LCi^2}=\sqrt{(4.43\times10^{-7}\ \text{C})^2-\left(\dfrac{5.00\times10^{-4}\ \text{A}}{1917\ \text{s}^{-1}}\right)^2}$

$=3.58\times10^{-7}\ \text{C}.$

30.33: a) $\dfrac{d^2q}{dt^2}+\dfrac{1}{LC}q=0\Rightarrow q=LC\dfrac{di}{dt}=(0.640\ \text{H})(3.60\times10^{-6}\ \text{F})(2.80\ \text{A/s})$

$=6.45\times10^{-6}\ \text{C}.$

b) $\varepsilon=\dfrac{q}{C}=\dfrac{8.50\times10^{-6}\ \text{C}}{3.60\times10^{-6}\ \text{F}}=2.36\text{V}.$

30.34: a) $i_{max}=\omega Q_{max}\Rightarrow Q_{max}=\dfrac{i_{max}}{\omega}=i_{max}\sqrt{LC}.$

$\Rightarrow Q_{max}=(1.50\text{A})\sqrt{(0.400\text{H})(2.50\times10^{-10}\ \text{F})}=1.50\times10^{-5}\ \text{C}.$

$\Rightarrow U_{max}=\dfrac{Q^2{}_{max}}{2C}=\dfrac{(1.50\times10^{-5}\ \text{C})^2}{2(2.50\times10^{-10}\ \text{F})}=0.450\ \text{J}$

b) $2f=\dfrac{2\omega}{2\pi}=\dfrac{1}{\pi\sqrt{LC}}=\dfrac{1}{\pi\sqrt{(0.400\ \text{H})(2.50\times10^{-10}\ \text{F})}}=3.18\times10^4\ \text{s}^{-1}$

(must double the frequency since it takes the required value twice per period).

30.35: $[LC]=\text{H}\cdot\text{F}=\text{H}\cdot\dfrac{\text{C}}{\text{V}}=\Omega\cdot\text{s}\cdot\dfrac{\text{C}}{\text{V}}=\dfrac{\Omega}{\text{V}}\cdot\dfrac{\text{C}}{\text{s}}\cdot\text{s}^2=\dfrac{1}{\text{A}}\cdot\text{A}\cdot\text{s}^2=\text{s}^2\Rightarrow\left[\sqrt{LC}\right]=s.$

30.36: Equation (30.20) is $\dfrac{d^2q}{dt^2}+\dfrac{1}{LC}q=0.$ We will solve the equation using:

$$q = Q\cos(\omega t+\phi) \Rightarrow \frac{dq}{dt} = -\omega Q\ \sin(\omega t+\phi) \Rightarrow \frac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t+\phi).$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = -\omega^2 Q\ \cos(\omega t+\phi) + \frac{Q}{LC}\cos(\omega t+\phi) = 0 \Rightarrow \omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

30.37: a) $U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{Q^2\cos^2(\omega t+\phi)}{C}.$

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}L\omega^2 Q^2\sin^2(\omega t+\phi) = \frac{1}{2}\frac{Q^2\sin^2(\omega t+\phi)}{C}, \text{ since } \omega^2 = \frac{1}{\sqrt{LC}}.$$

b) $U_{\text{Total}} = U_C + U_L = \frac{1}{2}\frac{Q^2}{C}\cos^2(\omega t+\phi) + \frac{1}{2}L\omega^2 Q^2\sin^2(\omega t+\phi)$

$$= \frac{1}{2}\frac{Q^2}{C}\cos^2(\omega t+\phi) + \frac{1}{2}L\left(\frac{1}{LC}\right)Q^2\sin^2(\omega t+\phi)$$

$$= \frac{1}{2}\frac{Q^2}{C}(\cos^2(\omega t+\phi) + \sin^2(\omega t+\phi))$$

$$= \frac{1}{2}\frac{Q^2}{C} \Rightarrow U_{\text{Total}} \text{ is a constant.}$$

30.38: a) $q = Ae^{-(R/2L)t}\cos(\omega' t+\phi)$

$$\Rightarrow \frac{dq}{dt} = -A\frac{R}{2L}e^{-(R/2L)t}\cos(\omega' t+\phi) - \omega' Ae^{-(R/2L)t}\sin(\omega' t+\phi).$$

$$\Rightarrow \frac{d^2q}{dt^2} = A\left(\frac{R}{2L}\right)^2 e^{-(R/2L)t}\cos(\omega' t+\phi) + 2\omega' A\frac{R}{2L}e^{-(R/2L)t}\sin(\omega' t+\phi)$$
$$- \omega'^2 Ae^{-(R/2L)t}\cos(\omega' t+\phi).$$

$$\Rightarrow \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = q\left(\left(\frac{R}{2L}\right)^2 - \omega'^2 - \frac{R^2}{2L^2} + \frac{1}{LC}\right) = 0$$

$$\Rightarrow \omega'^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$$

b) $At\ t = 0, q = Q, i = \frac{dq}{dt} = 0:$

$$\Rightarrow q = A\cos\phi = Q \text{ and } \frac{dq}{dt} = -\frac{R}{2L}A\cos\phi - \omega' A\sin\phi = 0$$

$$\Rightarrow A = \frac{Q}{\cos\phi} \text{ and } -\frac{QR}{2L} - \omega' Q\tan\phi = -\frac{R}{2L\omega'}$$

$$= -\frac{R}{2L\sqrt{1/LC - R^2/4L^2}}.$$

30.39: Subbing $x \to q, m \to L, b \to R, k \to \frac{1}{C}$, we find:

a) Eq. (13.41): $\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{kx}{m} = 0 \to$ Eq.(30.27): $\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0.$

b) Eq. (13.43): $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} \to$ Eq.(30.28): $\omega' = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}.$

c) Eq. (13.42): $x = Ae^{-(b/2m)t}\cos(\omega' t + \phi) \to$ Eq.(30.28): $q = Ae^{-(R/2L)t}\cos(\omega' t + \phi).$

30.40: $\left[\frac{L}{C}\right] = \frac{\text{H}}{\text{F}} = \frac{\Omega \cdot \text{s}}{\text{C/V}} = \frac{\Omega \cdot \text{V}}{\text{A}} = \Omega^2 \Rightarrow \left[\sqrt{\frac{L}{C}}\right] = \Omega$

30.41: $\omega'^2 = \frac{1}{LC} - \frac{R^2}{4L^2} = \frac{1}{6LC} \Rightarrow R^2 = 4L^2\left(\frac{1}{LC} - \frac{1}{6LC}\right) \Rightarrow R = 2L\sqrt{\frac{1}{LC} - \frac{1}{6LC}}$

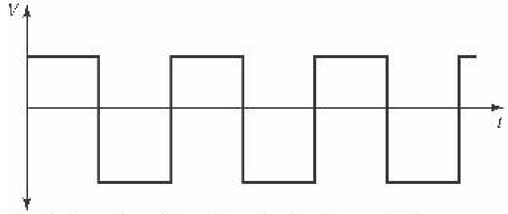
$$\Rightarrow R = 2(0.285\ \text{H})\sqrt{\frac{1}{(0.285\ \text{H})\,(4.60 \times 10^{-4}\ \text{F})} - \frac{1}{6(0.285\ \text{H})\,(4.60 \times 10^{-4}\ \text{F})}} = 45.4\ \Omega.$$

30.42: a) When $R = 0$, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.450\ \text{H})\,(2.50 \times 10^{-5}\ \text{F})}} = 298\ \text{rad/s}.$

b) We want $\frac{\omega}{\omega_0} = 0.95 \Rightarrow \frac{(1/LC - R^2/4L^2)}{1/LC} = 1 - \frac{R^2C}{4L} = (0.95)^2$

$$\Rightarrow R = \sqrt{\frac{4L}{C}(1 - (0.95)^2)} = \sqrt{\frac{4(0.450\ \text{H})\ \ (0.0975)}{(2.50 \times 10^{-5}\ \text{F})}} = 83.8\ \Omega$$

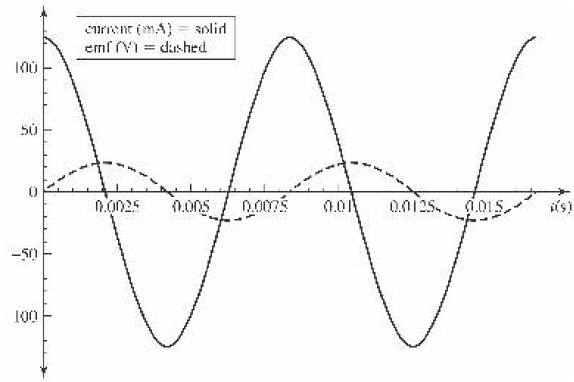
30.43: a)



b) Since the voltage is determined by the derivative of the current, the V versus t graph is indeed proportional to the derivative of the current graph.

30.44: a) $\varepsilon = -L\dfrac{di}{dt} = -L\dfrac{d}{dt}((0.124\text{ A})\cos[(240\,\pi/\text{s})t]$

$\Rightarrow \varepsilon = +(0.250\text{ H})\,(0.124\text{ A})\,(240\,\pi)\sin((240\,\pi/\text{s})t) = +(23.4\text{ V})\sin((240\pi/\text{s})t).$



b) $\varepsilon_{\max} = 23.4\text{ V}; i = 0$, since the emf and current are $90°$ out of phase.

c) $i_{\max} = 0.124\text{ A}; \varepsilon = 0$, since the emf and current are $90°$ out of phase.

30.45: a) $\Phi_B = \displaystyle\int_a^b B(hdr) = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right)(hdr) = \frac{\mu_0 Nih}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 Nih}{2\pi}\ln(b/a).$

b) $L = \dfrac{N\Phi_B}{i} = \dfrac{\mu_0 N^2 h}{2\pi}\ln(b/a).$

c) $\ln(b/a) = \ln(1-(b-a)/a) \approx \dfrac{b-a}{a} + \dfrac{(b-a)^2}{2a^2} + \cdots \Rightarrow \text{L} \approx \dfrac{\mu_0 N^2 h}{2\pi}\left(\dfrac{b-a}{a}\right).$

30.46: a) $M = \dfrac{N_2}{I}\Phi_{B_2} = \dfrac{N_2}{I}\dfrac{A_2}{A_1}\Phi_{B_1} = \dfrac{N_2 A_2}{IA_1}\dfrac{\mu_0 N_1 IA_1}{l_1} = \dfrac{\mu_0 N_1 N_2 A_2}{l_1} = \dfrac{\mu_0 N_1 N_2 \pi r_2^{\,2}}{l_1}.$

b) $|\varepsilon_2| = N_2\dfrac{d\Phi_{B_2}}{dt} = N_2\dfrac{\mu_0 N_1 A_2}{l_1}\dfrac{di_1}{dt} = \dfrac{\mu_0 N_1 N_2 \pi r_2^{\,2}}{l_1}\dfrac{di_1}{dt}.$

c) $|\varepsilon_1| = M_{12}\dfrac{di_2}{dt} = M\dfrac{di_2}{dt} = \dfrac{\mu_0 N_1 N_2 \pi r_2^{\,2}}{l_1}\dfrac{di_2}{dt}.$

30.47: a) $\varepsilon = -L\dfrac{di}{dt} \Rightarrow L = \varepsilon/(di/dt) = (30.0\text{ V})/(4.00\text{ A}/s) = 7.5\text{ H}.$

b) $\varepsilon = \dfrac{d\Phi}{dt} \Rightarrow \Phi_f - \Phi_i = \varepsilon\Delta t \Rightarrow \Phi_f = (30.0\text{ V})(12.0\text{ s}) = 360\text{ Wb}.$

c) $P_L = Li\dfrac{di}{dt} = (7.50\text{ H})(48.0\text{ A})(4.00\text{ A/s}) = 1440\text{ W}.$

$P_R = i^2R = (48.0\text{ A})^2(60.0\,\Omega) = 138240\text{ W} \Rightarrow \dfrac{P_L}{P_R} = 0.0104.$

30.48: a) $\varepsilon = L\dfrac{di}{dt} = (3.50\times10^{-3}\text{H})\dfrac{d}{dt}\left((0.680\text{ A})\cos(\pi t/0.0250\text{ s})\right)$

$$\Rightarrow \varepsilon_{\max} = (3.50\times10^{-3}\text{ H})(0.680\text{ A})\frac{\pi}{0.0250\text{ s}} = 0.299\text{ V}.$$

b) $\Phi_{B\max} = \dfrac{Li_{\max}}{N} = \dfrac{(3.50\times10^{-3}\text{H})(0.680\text{ A})}{400} = 5.95\times10^{-6}\text{ Wb}.$

c) $\varepsilon(t) = -L\dfrac{di}{dt} = -(3.50\times10^{-3}\text{ H})(0.680\text{ A})(\pi/0.0250\text{ s})\sin(\pi t/0.0250\text{ s}).$

$$\Rightarrow \varepsilon(t) = -(0.299\text{ V})\sin((125.6\text{ s}^{-1})t)$$
$$\Rightarrow \varepsilon(0.0180\text{ s}) = -(0.299\text{ V})\sin((125.6\text{ s}^{-1})(0.0180\text{ s}))$$
$$\Rightarrow \varepsilon(t) = 0.230\text{ V}$$

30.49: a) Series: $L_1\dfrac{di_1}{dt} + L_2\dfrac{di_2}{dt} = L_{\text{eq}}\dfrac{di}{dt},$

but $i_1 = i_2 = i$ for series components so $\dfrac{di_1}{dt} = \dfrac{di_2}{dt} = \dfrac{di}{dt}$, thus $L_1 + L_2 = L_{\text{eq}}$

b) Parallel: Now $L_1\dfrac{di_1}{dt} = L_2\dfrac{di_2}{dt} = L_{\text{eq}}\dfrac{di}{dt}$, where $i = i_1 + i_2$.

So $\dfrac{di}{dt} = \dfrac{di_1}{dt} + \dfrac{di_2}{dt}$. But $\dfrac{di_1}{dt} = \dfrac{L_{\text{eq}}}{L_2}\dfrac{di}{dt}$ and $\dfrac{di_2}{dt} = \dfrac{L_{\text{eq}}}{L_2}\dfrac{di}{dt}$

$$\Rightarrow \frac{di}{dt} = \frac{L_{\text{eq}}}{L_1}\frac{di}{dt} + \frac{L_{\text{eq}}}{L_2}\frac{di}{dt} \Rightarrow L_{\text{eq}} = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1}.$$

30.50: a) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encl}} \Rightarrow B2\pi r = \mu_0 i \Rightarrow B = \dfrac{\mu_0 i}{2\pi r}.$

b) $d\Phi_B = BdA = \dfrac{\mu_0 i}{2\pi r}l\,dr.$

c) $\Phi_B = \displaystyle\int_a^b d\Phi_B = \frac{\mu_0 il}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 il}{2\pi}\ln(b/a).$

d) $L = \dfrac{N\Phi_B}{i} = l\dfrac{\mu_0}{2\pi}\ln(b/a).$

e) $U=\frac{1}{2}Li^2=\frac{1}{2}l\frac{\mu_0}{2\pi}\ln(b/a)i^2=\frac{\mu_0 l i^2}{4\pi}\ln(b/a).$

30.51: a) $\oint \vec{B}\cdot d\vec{l}=\mu_0 I_{\text{encl}} \Rightarrow B2\pi r=\mu_0 i \Rightarrow B=\frac{\mu_0 i}{2\pi r}.$

b) $u=\frac{B^2}{2\mu_0} \Rightarrow dU=udV=u(l2\pi r dr)=\frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2\pi r}\right)^2(l2\pi r dr)=\frac{\mu_0 i^2 l}{4\pi r}dr.$

c) $U=\int_a^b dU=\frac{\mu_0 i^2 l}{4\pi}\int_a^b \frac{dr}{r}=\frac{\mu_0 i^2 l}{4\pi}\ln(b/a).$

d) $U=\frac{1}{2}Li^2 \Rightarrow L=\frac{2U}{i^2}=l\frac{\mu_0}{2\pi}\ln(b/a)$, which is the same as in Problem 30.50.

30.52: a) $L_1=\frac{N_1\Phi_{B_1}}{i_1}=\frac{N_1 A}{i_1}\left(\frac{\mu_0 N_1 i_1}{2\pi r}\right)=\frac{\mu_0 N_1{}^2 A}{2\pi r},$

$$L_2=\frac{N_2\Phi_{B_2}}{i_2}=\frac{N_2 A}{i_2}\left(\frac{\mu_0 N_2 i_2}{2\pi r}\right)=\frac{\mu_0 N_2{}^2 A}{2\pi r}$$

b) $M^2=\left(\frac{\mu_0 N_1 N_2 A}{2\pi r}\right)^2=\frac{\mu_0 N_1{}^2 A}{2\pi r}\frac{\mu_0 N_2{}^2 A}{2\pi r}=L_1 L_2.$

30.53: $u_B=u_E \Rightarrow \frac{\varepsilon_0 E^2}{2}=\frac{B^2}{2\mu_0} \Rightarrow B=\sqrt{\varepsilon_0\mu_0 E^2}=\sqrt{\mu_0\varepsilon_0}E$

$$=\sqrt{\varepsilon_0\mu_0}(650\ \text{V/m})=2.17\times10^{-6}\,\text{T}.$$

30.54: a) $R=\frac{V}{i_f}=\frac{12.0\ V}{6.45\times10^{-3}\ \text{A}}=1860\ \Omega.$

b) $i=i_f(1-e^{-(R/L)t}) \Rightarrow \frac{Rt}{L}=-\ln(1-i/i_f) \Rightarrow L=\frac{-Rt}{\ln(1-i/i_f)}$

$$\Rightarrow L=\frac{-(1860\ \Omega)(7.25\times10^{-4}\,\text{s})}{\ln(1-(4.86/6.45))}=0.963\ \text{H}.$$

30.55: a) After one time constant has passed:

$$i=\frac{\varepsilon}{R}(1-e^{-1})=\frac{6.00\ \text{V}}{8.00\ \Omega}(1-e^{-1})=0.474\ \text{A}$$

$$\Rightarrow U=\frac{1}{2}Li^2=\frac{1}{2}(2.50\ \text{H})(0.474\ \text{A})^2=0.281\ \text{J}.$$

Or, using Problem (30.25(c)):

$$U=\int P_L dt=(4.50\ W)\int_0^{3/7}(e^{-(3.20)t}-e^{-(6.40)t})dt.$$

$$=(4.50\ \text{W})\left(\frac{(1-e^{-1})}{3.20}-\frac{(1-e^{-2})}{6.40}\right)=0.281\ \text{J}$$

b) $U_{\text{tot}}=(4.50\ \text{W})\int_0^{L/R}(1-e^{-(R/L)t})dt=(4.50\ \text{W})\left(\frac{L}{R}+\frac{L}{R}(e^{-1}-1)\right)$

$$\Rightarrow U_{\text{tot}}=(4.50\ \text{W})\frac{2.50\ \text{H}}{8.00\ \Omega}e^{-1}=0.517\ \text{J}$$

c) $U_R=(4.50\ \text{W})\int_0^{L/R}(1-2e^{-(R/L)t}+e^{-2(R/L)t})dt$

$$=(4.50\ \text{W})\left(\frac{L}{R}+\frac{2L}{R}(e^{-1}-1)-\frac{L}{2R}(e^{-2}-1)\right)$$

$$\Rightarrow U_R=(4.50\ \text{W})\frac{2.50\ \text{H}}{8.00\ \Omega}(0.168)=0.236\ \text{J}.$$

The energy dissipated over the inductor (part (a)), plus the energy lost over the resistor (part (c)), sums to the total energy output (part (b)).

30.56: a) $U=\frac{1}{2}Li_0^{\,2}=\frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2=\frac{1}{2}(0.160\ \text{H})\left(\frac{60\ \text{V}}{240\ \Omega}\right)^2=5.00\times10^{-3}\ \text{J}.$

b) $i=\frac{\varepsilon}{R}e^{-(R/L)t}\Rightarrow\frac{di}{dt}=-\frac{R}{L}i\Rightarrow\frac{dU_L}{dt}=iL\frac{di}{dt}=-Ri^2=\frac{\varepsilon^2}{R}e^{-2(R/L)t}$

$$\Rightarrow\frac{dU_L}{dt}=-\frac{(60\ V)^2}{240\ \Omega}e^{-2(240/0.160)(4.00\times10^{-4})}=-4.52\ \text{W}.$$

c) In the resistor:

$$\frac{dU_R}{dt}=i^2R=\frac{\varepsilon^2}{R}e^{-2(R/L)t}=\frac{(60\ \text{V})^2}{240\ \Omega}e^{-2(240/0.160)(4.00\times10^{-4})}=4.52\ \text{W}.$$

d) $P_R(t) = i^2 R = \dfrac{\varepsilon^2}{R} e^{-2(R/L)t}$

$$\Rightarrow U_R = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2(R/L)t} = \frac{\varepsilon^2}{R}\frac{L}{2R} = \frac{(60\text{ V})^2(0.160\text{ H})}{2(240\ \Omega)^2} = 5.00\times10^{-3}\text{ J},$$

which is the same as part (a).

30.57: Multiplying Eq. (30.27) by i, yields:

$$i^2\text{R} + Li\frac{di}{dt} - \frac{q}{C}i = i^2R + Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = i^2R + \frac{d}{dt}\left(\frac{1}{2}Li^2\right) + \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right)$$
$$= P_R + P_L + P_C = 0.$$

That is, the rate of energy dissipation throughout the circuit must balance over all of the circuit elements.

30.58: a) If $t = \dfrac{3T}{8} \Rightarrow q = Q\cos(\omega t) = Q\cos\left(\dfrac{2\pi}{T}\dfrac{3T}{8}\right) = Q\cos\left(\dfrac{3\pi}{4}\right) = \dfrac{Q}{\sqrt{2}}$

$$\Rightarrow i = \frac{1}{\sqrt{LC}}(\sqrt{Q^2 - q^2}) = \frac{1}{\sqrt{LC}}(\sqrt{Q^2 - Q^2/2}) = \sqrt{\frac{Q^2}{2LC}}$$
$$\Rightarrow U_E = \frac{1}{2}Li^2 = \frac{1}{2}L\frac{Q^2}{2LC} = \frac{1}{2}\frac{Q^2}{2C} = \frac{q^2}{2C} = U_B.$$

b) The two energies are next equal when $q = \dfrac{Q}{\sqrt{2}} \Rightarrow \omega t = \dfrac{5\pi}{8} \Rightarrow t = \dfrac{5T}{8}$.

30.59: $V_C = 12.0\text{ V}; U_C = \frac{1}{2}CV_C^2$ so $C = 2U_C/V_C^2 = 2(0.0160\text{ J})/(12.0\text{ V})^2 = 222\ \mu F$

$f = \dfrac{1}{2\pi\sqrt{LC}}$ so $L = \dfrac{1}{(2\pi f)^2 C}$

$f = 3500$ Hz gives $L = 9.31\mu\text{H}$

30.60: a) $V_{\text{max}} = \dfrac{Q}{C} = \dfrac{6.00\times10^{-6}\text{ C}}{2.50\times10^{-4}\text{ F}} = 0.0240\text{ V}.$

b) $\frac{1}{2}Li_{\max}^2 = \frac{Q^2}{2C} \Rightarrow i_{\max} = \frac{Q}{\sqrt{LC}} = \frac{6.00\times10^{-6}}{\sqrt{(0.0600\text{ H})(2.50\times10^{-4}\text{F})}} = 1.55\times10^{-3}\text{ A}$

c) $U_{\max} = \frac{1}{2}Li_{\max}^2 = \frac{1}{2}(0.0600\text{ H})(1.55\times10^{-3}\text{A})^2 = 7.21\times10^{-8}\text{ J}.$

d) If $i = \frac{1}{2}i_{\max} \Rightarrow U_L = \frac{1}{4}U_{\max} = 1.80\times10^{-8}\text{ J} \Rightarrow U_C = \frac{3}{4}U_{\max} = \frac{\left(\sqrt{\frac{3}{4}}Q\right)^2}{2C} = \frac{q^2}{2C}$

$\Rightarrow q = \sqrt{\frac{3}{4}}Q = 5.20\times10^{-6}\text{C}.$

$U_{\max} = \frac{1}{2}Li^2 + \frac{1}{2}\frac{q^2}{C}$ for all times.

30.61: The energy density in the sunspot is $u_B = B^2/2\mu_0 = 6.366\times10^4\text{ J /m}^3.$

The total energy stored in the sunspot is $U_B = u_B V.$

The mass of the material in the sunspot is $m = \rho V.$

$K = U_B$ so $\frac{1}{2}mv^2 = U_B;\quad \frac{1}{2}\rho V v^2 = u_B V$

The volume divides out, and $v = \sqrt{2u_B/\rho} = 2\times10^4\text{ m/s}$

30.62: (a) The voltage behaves the same as the current. Since $V_R \propto i$, the scope must be across the $150\ \Omega$ resistor.

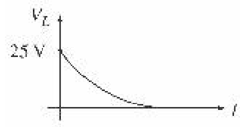
(b) From the graph, as $t \to \infty$, $V_R \to 25\text{V}$, so there is no voltage drop across the inductor, so its internal resistance must be zero.

$$V_R = V_{\max}(1 - e^{-t/\tau})$$

when $t = \tau$, $V_R = V_{\max}(1 - \frac{1}{e}) \approx 0.63V_{\max}$. From the graph, when $V = 0.63\ V_{\max} = 16\text{V},\ t \approx 0.5\text{ ms} = \tau$

$$L/R = 0.5\text{ ms} \to L = (0.5\text{ ms})(150\Omega) = 0.075\text{ H}$$

(c) Scope across the inductor:



30.63: a) In the *R-L* circuit the voltage across the resistor starts at zero and increases to the battery voltage. The voltage across the solenoid (inductor) starts at the battery voltage and decreases to zero. In the graph, the voltage drops, so the oscilloscope is across the solenoid.

b) At $t \to \infty$ the current in the circuit approaches its final, constant value. The voltage doesn't go to zero because the solenoid has some resistance R_L. The final voltage across the solenoid is IR_L, where I is the final current in the circuit.

c) The emf of the battery is the initial voltage across the inductor, 50 V. Just after the switch is closed, the current is zero and there is no voltage drop across any of the resistance in the circuit.

d) As $t \to \infty$, $\varepsilon - IR - IR_L = 0$

$\varepsilon = 50$ V and from the graph $I\,R_L = 15$ V (the final voltage across the inductor), so

$I\,R = 35$ V and $I = (35 \text{ V})/\text{R} = 3.5$ A

e) $I\,R_L = 15$ V, so $R_L = (15 \text{ V})/(3.5 \text{ A}) = 4.3\Omega$

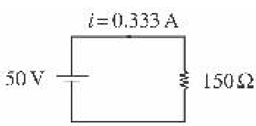
$\varepsilon - V_L - iR = 0$, where V_L includes the voltage across the resistance of the solenoid.

$$V_L = \varepsilon - iR, i = \frac{\varepsilon}{R_{\text{tot}}}(1 - e^{-t/\tau}), \text{ so } V_L = \varepsilon[1 - \frac{R}{R_{\text{tot}}}(1 - e^{-t/\tau})]$$

$\varepsilon = 50$ V, $R = 10\,\Omega$, $R_{\text{tot}} = 14.3\,\Omega$, so when $t = \tau, V_t = 27.9$ V

From the graph, V_L has this value when $t = 3.0$ ms (read approximately from the graph), so $\tau = L/R_{\text{tot}} = 3.0$ ms. Then $L = (3.0 \text{ ms})(14.3\,\Omega) = 43$ mH.

30.64: (a) Initially the inductor blocks current through it, so the simplified equivalent circuit is



$$i = \frac{\varepsilon}{R} = \frac{50 \text{ V}}{150\,\Omega} = 0.333 \text{ A}$$

$V_1 = (100\ \Omega)(0.333\ \text{A}) = 33.3\ \text{V}$
$V_4 = (50\ \Omega)(0.333\ \text{A}) = 16.7\ \text{V}$
$V_3 = 0$ since no current flows through it.
$V_2 = V_4 = 16.7\ \text{V}$ (inductor in parallel with $50\ \Omega$ resistor)
$A_1 = A_3 = 0.333\ \text{A},\ A_2 = 0$

(b) Long after S is closed, steady state is reached, so the inductor has no potential drop across it. Simplified circuit becomes

$$i = \varepsilon / R = \frac{50\ \text{V}}{130\ \Omega} = 0.385\ \text{A}$$

$V_1 = (100\ \Omega)(0.385\ \text{A}) = 38.5\ \text{V}\ ;V_2 = 0$

$V_3 = V_4 = 50\ \text{V} - 38.5\ \text{V} = 11.5\ \text{V}$

$$i_1 = 0.385\ \text{A}, i_2 = \frac{11.5\ \text{V}}{75\ \Omega} = 0.153\ \text{A}$$

$$i_3 = \frac{11.5\ \text{V}}{50\ \Omega} = 0.230\ \text{A}$$

30.65: a) Just after the switch is closed the voltage V_5 across the capacitor is zero and there is also no current through the inductor, so $V_3 = 0$. $V_2 + V_3 = V_4 = V_5$, and since $V_5 = 0$ and $V_3 = 0$, V_4 and V_2 are also zero. $V_4 = 0$ means V_3 reads zero.

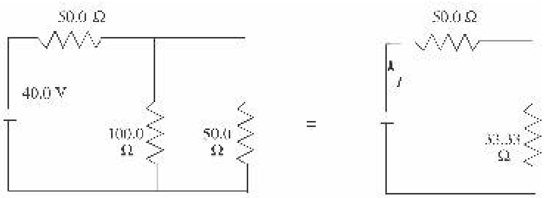
V_1 then must equal 40.0 V, and this means the current read by A_1 is $(40.0\ \text{V})/(50.0\ \Omega) = 0.800\ \text{A}$.

$A_2 + A_3 + A_4 = A_1$, but $A_2 = A_3 = 0$ so $A_4 = A_1 = 0.800\ \text{A}$.

$A_1 = A_4 = 0.800\ \text{A}$; all other ammeters read zero.

$V_1 = 40.0\ \text{V}$ and all other voltmeters read zero.

b) After a long time the capacitor is fully charged so $A_4 = 0$. The current through the inductor isn't changing, so $V_2 = 0$. The currents can be calculated from the equivalent circuit that replaces the inductor by a short-circuit.:



$I = (40.0 \text{ V})/(83.33\ \Omega) = 0.480 \text{ A}$; A_1 reads 0.480 A
$V_1 = I(50.0\ \Omega) = 24.0 \text{ V}$

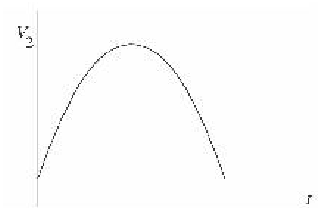
The voltage across each parallel branch is $40.0 \text{ V} - 24.0 \text{ V} = 16.0 \text{ V}$

$V_2 = 0, V_3 = V_4 = V_5 = 16.0 \text{ V}$

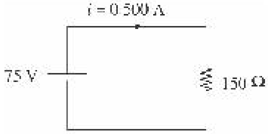
$V_3 = 16.0 \text{ V}$ means A_2 reads 0.160 A. $V_4 = 16.0 \text{ V}$ means A_3 reads 0.320 A. A_4 reads zero. Note that $A_2 + A_3 = A_1$.

c) $V_5 = 16.0 \text{ V}$ so $Q = CV = (12.0\ \mu\text{F})(16.0 \text{ V}) = 192\ \mu\text{C}$

d) At $t = 0$ and $t \to \infty$, $V_2 = 0$. As the current in this branch increases from zero to 0.160 A the voltage V_2 reflects the rate of change of current.



30.66: (a) Initially the capacitor behaves like a short circuit and the inductor like an open circuit. The simplified circuit becomes



$$i = \frac{\varepsilon}{R} = \frac{75 \text{ V}}{150\ \Omega} = 0.500 \text{ A}$$

$$V_1 = Ri = (50\ \Omega)(0.50 \text{ A}) = 25.0 \text{ V}$$
$$V_3 = 0,\ V_4 = (100\ \Omega)(0.50 \text{ A}) = 50.0 \text{ V}$$
$$V_2 = V_4 \text{ (in parallel)} = 50.0 \text{ V}$$
$$A_1 = A_3 = 0.500 \text{ A},\ A_2 = 0$$

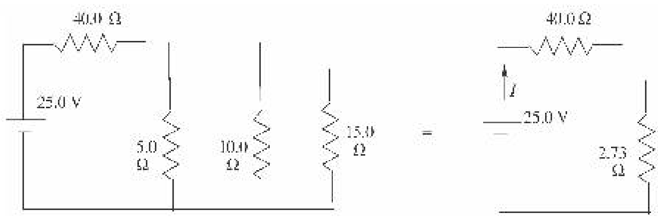
(b) Long after S is closed, capacitor stops all current. Circuit becomes

$V_3 = 75.0\text{ V}$ and all other meters read zero.

(c) $q = CV = (75\text{ nF})(75\text{ V}) = 5630\text{ nC},$ long after S is closed.

30.67: a) Just after the switch is closed there is no current through either inductor and they act like breaks in the circuit. The current is the same through the $40.0\ \Omega$ and $15.0\ \Omega$ resistors and is equal to $(25.0\text{ V})/(40.0\ \Omega + 15.0\ \Omega) = 0.455\text{ A}$. $A_1 = A_4 = 0.455\text{ A}$; $A_2 = A_3 = 0$.

b) After a long time the currents are constant, there is no voltage across either inductor, and each inductor can be treated as a short-circuit. The circuit is equivalent to:



$I = (25.0\text{ V})/(42.73\ \Omega) = 0.585\text{ A}$

A_1 reads 0.585 A. The voltage across each parallel branch is $25.0\text{ V} - (0.585\text{ A})(40.0\ \Omega) =$

1.60 V. A_2 reads $(1.60\text{ V})/(5.0\ \Omega) = 0.320\text{ A}$. A_3 reads $(1.60\text{ V})/10.0\ \Omega) = 0.160\text{ A}$. A_4 reads $(1.60\text{ V})/(15.0\ \Omega) = 0.107\text{ A}$.

30.68: (a) $\tau = L/R = \frac{10\text{ mH}}{25\ \Omega} = 0.40\text{ ms}$ since $0.50\text{ s} >> \tau,$ steady state has been reached, for all practical purposes.

$$i = \varepsilon/R = 50\text{ V}/25\ \Omega = 2.00\text{ A}$$

The upper limit of the energy that the capacitor can get is the energy stored in the inductor initially.

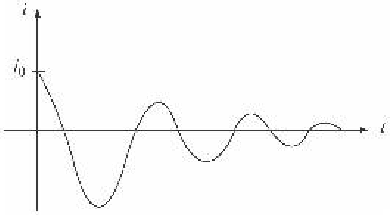
$$U_C = U_L \to \frac{Q_{\max}^2}{2C} = \frac{1}{2}Li_0^2 \to Q_{\max} = i_0\sqrt{LC}$$

$$Q_{\max} = (2.00\text{ A})\sqrt{(10\times 10^{-3}\text{ H})(20\times 10^{-6}\text{ F})} = 0.90\times 10^{-3}\text{ C}$$

(b) Eventually *all* the energy in the inductor is dissipated as heat in the resistor.

$$U_R = U_L = \frac{1}{2}Li_0^2 = \frac{1}{2}(10\times10^{-3}\,\mathrm{H})\,(2.00\,\mathrm{A})^2$$
$$= 2.0\times10^{-2}\ \mathrm{J}$$

(c)



30.69: a) At $t = 0$, all the current passes through the resistor R_1, so the voltage v_{ab} is the total voltage of 60.0 V.

b) Point a is at a higher potential than point b. c) $v_{cd} = 60.0\text{ V}$ since there is no current through R_2.

d) Point c is at a higher potential than point b.

e) After a long time, the switch is opened, and the inductor initially maintains the current of $i_{R_2} = \dfrac{\varepsilon}{R_2} = \dfrac{60.0\text{ V}}{25.0\ \Omega} = 2.40\text{ A}$. Therefore the potential between a and b is $v_{ab} = -i\text{R}_1 = -(2.40\text{ A})(40.0\ \Omega) = -96.0\text{ V}$.

f) Point b is at a higher potential than point a.

g) $v_{cd} = -i(R_1 + R_2) = -(2.40\text{ A})(40\ \Omega + 25\ \Omega) = -156\text{ V}$

h) Point d is at a higher potential than point c.

30.70: a) Switch is closed, then at some later time:

$$\frac{di}{dt} = 50.0\text{ A/s} \Rightarrow v_{cd} = L\frac{di}{dt} = (0.300\text{ H})(50.0\text{ A/s}) = 15.0\text{ V}.$$

The top circuit loop: $60.0\text{ V} = i_1 R_1 \Rightarrow i_1 = \dfrac{60.0\text{ V}}{40.0\ \Omega} = 1.50\text{ A}.$

The bottom loop: $60\text{ V} - i_2 R_2 - 15.0\text{ V} = 0 \Rightarrow i_2 = \dfrac{45.0\text{ V}}{25.0\ \Omega} = 1.80\text{ A}.$

b) After a long time: $i_2 = \dfrac{60.0\text{ V}}{25.0\ \Omega} = 2.40\text{ A}$, and immediately when the switch is opened, the inductor maintains this current, so $i_1 = i_2 = 2.40\text{ A}$.

30.71: a) Immediately after S_1 is closed, $i_0 = 0$, $v_{ac} = 0$, and $v_{cb} = 36.0\text{ V}$, since the inductor stops the current flow.

b) After a long time, $i_0 = \dfrac{\varepsilon}{R_0 + R} = \dfrac{36.0\text{ V}}{50\ \Omega + 150\ \Omega} = 0.180\text{ A}$,

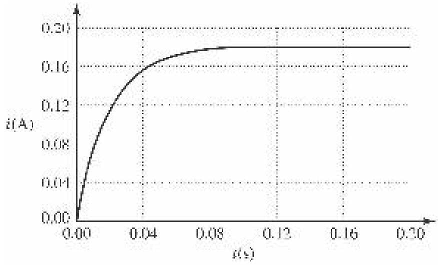
$v_{ac} = i_0 R_0 = (0.18\text{ A})(50\ \Omega) = 9.00\text{ V}$, and $v_{cb} = 36.0\text{ V} - 9.00\text{ V} = 27.0\text{ V}.$

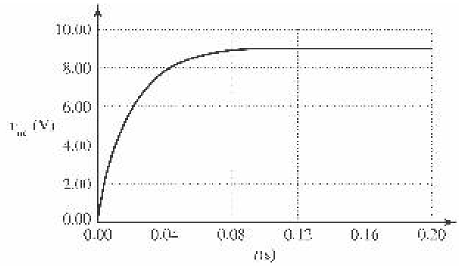
c) $i(t) = \dfrac{\varepsilon}{R_{\text{total}}}(1 - e^{-(R_{\text{total}}/L)t}) \Rightarrow i(t) = (0.180\ \text{A})\,(1 - e^{-(50\,s^{-1})t})$,

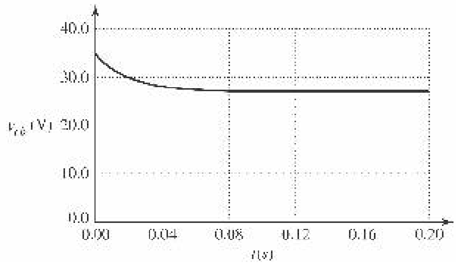
$v_{ac}(t) = i(t)\,R_0 = (9.00\ \text{V})\left(1 - e^{-(50\,s^{-1})t}\right)$ and

$v_{cb}(t) = \varepsilon - i(t)\,R_0 = 36.0\ \text{V} - (9.00\ \text{V})\left(1 - e^{-(50\,s^{-1})t}\right) = (9.00\ \text{V})\left(3 + e^{-(50\,s^{-1})t}\right)$.

Below are the graphs of current and voltage found above.







30.72: a) Immediately after S_2 is closed, the inductor maintains the current $i = 0.180$ A through R. The Kirchoff's Rules around the outside of the circuit yield:

$$\varepsilon + \varepsilon_L - iR - i_0 R_0 = 36.0\ \text{V} + (0.18)\,(150) - (0.18)\,(150) - i_0(50) = 0$$

$$\Rightarrow i_0 = \frac{36\ \text{V}}{50\ \Omega} = 0.720\ \text{A},\ v_{ac} = (0.72\ \text{A})\,(50\ \text{V}) = 36.0\ \text{V and } v_{cb} = 0.$$

b) After a long time, $v_{ac} = 36.0$ V, and $v_{cb} = 0$. Thus

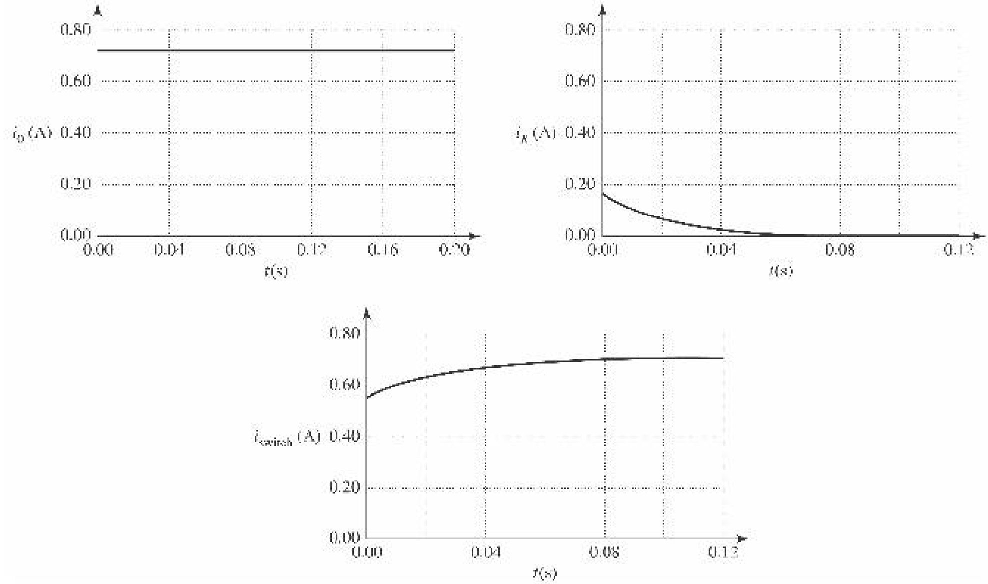
$$i_0 = \frac{\varepsilon}{R_0} = \frac{36.0\ \text{V}}{50\ \Omega} = 0.720\ \text{A},$$

$i_R = 0$, and $i_{s_2} = 0.720$ A

c) $i_0 = 0.720\ \text{A},\ i_R(t) = \dfrac{\varepsilon}{R_{\text{total}}} e^{-(R/L)t} \Rightarrow i_R(t) = (0.180\ \text{A})e^{-(12.5\,s^{-1})t}$, and

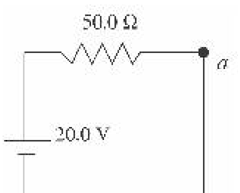
$i_{s_2}(t) = (0.720\ \text{A}) - (0.180\ A)e^{-(12.5\,s^{-1})t} = (0.180\ \text{A})\left(4 - e^{-(12.5\,s^{-1})t}\right)$

Below are the graphs of currents found above.



30.73: a) Just after the switch is closed there is no current in the inductors. There is no current in the resistors so there is no voltage drop across either resistor. A reads zero and V reads 20.0 V.

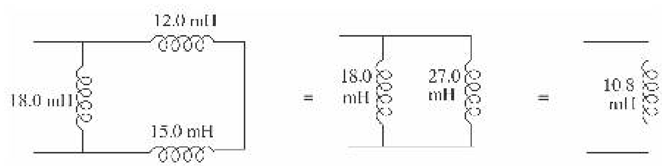
b) After a long time the currents are no longer changing, there is no voltage across the inductors, and the inductors can be replaced by short-circuits. The circuit becomes equivalent to



$$I^a = (20.0\ \text{V})/(75.0\ \Omega) = 0.267\ \text{A}$$

The voltage between points a and b is zero, so the voltmeter reads zero.

c) Use the results of problem 30.49 to combine the inductor network into its equivalent:



$R = 75.0\ \Omega$ is the equivalent resistance.

Eq.(30.14) says $i = (\varepsilon / R)(1 - e^{-t/\tau})$, with $\tau = L / R = (10.8 \text{ mH})/(75.0\ \Omega) = 0.144$ ms

$\varepsilon = 20.0\ V$, $R = 75.0\ \Omega$, $t = 0.115$ ms, so $i = 0.147$ A

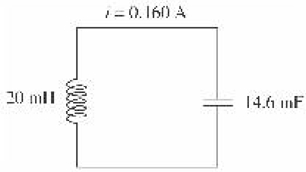
$V_R = iR = (0.147\text{ A})(75.0\ \Omega) = 11.0\text{ V}$

$20.0\text{ V} - V_R - V_L = 0$ so $V_L = 20.0\text{ V} - \text{V}_R = 9.0\text{ V}$

30.74: (a) Steady state: $i = \dfrac{\varepsilon}{R} = \dfrac{75.0\text{ V}}{125\ \Omega} = 0.600\text{ A}$

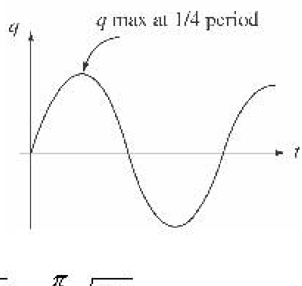
(b) Equivalent circuit:

$$\frac{1}{C_s} = \frac{1}{25\ \mu\text{F}} + \frac{1}{35\mu\text{F}}$$
$$C_s = 14.6\mu\text{F}$$



Energy conservation: $\dfrac{q^2}{2C} = \dfrac{1}{2}Li_0^2$

$$q = i_0\sqrt{LC} = (0.600\text{ A})\sqrt{(20\times10^{-3}\text{H})\,(14.6\times10^{-6}\text{F})}$$
$$= 3.24\times10^{-4}\,C$$



$$t = \frac{1}{4}T = \frac{1}{4}(2\pi\sqrt{LC}) = \frac{\pi}{2}\sqrt{LC}$$
$$t = \frac{\pi}{2}\sqrt{(20\times10^{-3}\text{H})\,(14.6\times10^{-6}\text{F})} = 8.49\times10^{-4}\text{s}$$

30.75: a) Using Kirchhoff's Rules: $\varepsilon - i_1R_1 = 0 \Rightarrow i_1 = \dfrac{\varepsilon}{R_1}$, and

$$\varepsilon - L\frac{di_2}{dt} - i_2R_2 = 0 \Rightarrow i_2 = \frac{\varepsilon}{R_2}(1 - e^{-(R_2/L)t}).$$

b) After a long time, $i_1 = \frac{\varepsilon}{R_1}$ still, and $i_2 = \frac{\varepsilon}{R_2}$.

c) After the switch is opened, $i_1 = i_2 = \frac{\varepsilon}{R_2} e^{-((R_1+R_2)/L)t}$, and the current drops off.

d) A 40-W light bulb implies $R = \frac{V^2}{P} = \frac{(120\ \text{V})^2}{40\ \text{W}} = 360\ \Omega$. If the switch is opened, and the current is to fall from 0.600A to 0.150 A in 0.0800 s,

then: $i_2 = (0.600\ \text{A})e^{-((R_1+R_2)/L)t} \Rightarrow 0.150\ \text{A} = (0.600\ \text{A})e^{-((360\ \Omega + R_2)/22.0\ \text{H})(0.0800\ s)}$

$$\Rightarrow \frac{22.0\ \text{H}}{0.0800\ s}\ln(4.00) = 360\ \Omega + R_2 \Rightarrow R_2 = 21.0\ \Omega$$

$$\Rightarrow \varepsilon = i_2 R_2 = (0.600\ \text{A})(21.2\ \Omega) = 12.7\ \text{V}.$$

e) Before the switch is opened, $i_0 = \frac{\varepsilon}{R_1} = \frac{12.7\ \text{V}}{360\ \Omega} = 0.0354\ \text{A}$

30.76: Series: $L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt} + M_{21}\frac{di_1}{dt} + M_{12}\frac{di_2}{dt} \equiv L_{eq}\frac{di}{dt}$.

But $i = i_1 + i_2 \Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$ and $M_{12} = M_{21} \equiv M$.

So $(L_1 + L_2 + 2M)\frac{di}{dt} = L_{eq}\frac{di}{dt}$,

or $L_{eq} = L_1 + L_2 + 2M$.

Parallel: We have $L_1 \frac{di_1}{dt} + M_{12}\frac{di_2}{dt} = L_{eq}\frac{di}{dt}$

and $L_2 \frac{di_2}{dt} + M_{21}\frac{di_1}{dt} = L_{eq}\frac{di}{dt}$,

with $\frac{di_1}{dt} + \frac{di_2}{dt} = \frac{di}{dt}$ and $M_{12} = M_{21} \equiv M$.

To simplify the algebra let $A = \frac{di_1}{dt}, B = \frac{di_2}{dt}$, and $C = \frac{di}{dt}$.

So $L_1 A + MB = L_{eq}C, L_2 B + MA = L_{eq}C, A + B = C$.

Now solve for A and B in terms of C.

$$\Rightarrow (L_1 - M)A + (M - L_2)B = 0 \text{ using } A = C - B.$$

$$\Rightarrow (L_1 - M)(C - B) + (M - L_2)B = 0$$

$$\Rightarrow (L_1 - M)C - (L_1 - M)B + (M - L_2)B = 0$$

$$\Rightarrow (2M - L_1 - L_2)B = (M - L_1)C \Rightarrow B = \frac{(M - L_1)}{(2M - L_1 - L_2)}C.$$

But $A = C - B = C - \frac{(M - L_1)C}{(2M - L_1 - L_2)} = \frac{(2M - L_1 - L_2) - M + L_1}{(2M - L_1 - L_2)}C$,

or $A = \dfrac{M - L_2}{2M - L_1 - L_2}C$. Substitute A in B back into original equation.

So $\dfrac{L_1(M - L_2)C}{2M - L_1 - L_2} + \dfrac{M(M - L_1)}{(2M - L_1 - L_2)}C = L_{eq}C$

$$\Rightarrow \frac{M^2 - L_1L_2}{2M - L_1 - L_2}C = L_{eq}C.$$

Finally, $L_{eq} = \dfrac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$

30.77: a) Using Kirchhoff's Rules on the top and bottom branches of the circuit:

$$\varepsilon - i_1R_1 - L\frac{di_1}{dt} = 0 \Rightarrow i_1 = \frac{\varepsilon}{R_1}(1 - e^{-(R_1/L)t}).$$

$$\varepsilon - i_2R_2 - \frac{q_2}{C} = 0 \Rightarrow -\frac{di_2}{dt}R_2 - \frac{i_2}{C} = 0 \Rightarrow i_2 = \frac{\varepsilon}{R_2}e^{-(1/R_2C)t})$$

$$\Rightarrow q_2 = \int_0^t i_2\,dt' = -\frac{\varepsilon}{R_2}R_2Ce^{-(1/R_2C)t'}\bigg|_0^t = \varepsilon C(1 - e^{-(1/R_2C)t}).$$

b) $i_1(0) = \dfrac{\varepsilon}{R_1}(1 - e^0) = 0, i_2 = \dfrac{\varepsilon}{R_2}e^0 = \dfrac{48.0\text{ V}}{5000\ \Omega} = 9.60 \times 10^{-3}\text{ A}.$

c) As $t \to \infty : i_1(\infty) = \dfrac{\varepsilon}{R_1}(1 - e^{-\infty}) = \dfrac{\varepsilon}{R_1} = \dfrac{48.0\text{ V}}{25.0\ \Omega} = 1.92\text{ A}, i_2 = \dfrac{\varepsilon}{R_2}e^{-\infty} = 0.$

A good definition of a "long time" is many time constants later.

d) $i_1 = i_2 \Rightarrow \dfrac{\varepsilon}{R_1}(1 - e^{-(R_1/L)t}) = \dfrac{\varepsilon}{R_2}e^{-(1_1/R_2C)t} \Rightarrow (1 - e^{-(R_1/L)t}) = \dfrac{R_1}{R_2}e^{-(1/R_2C)t}.$

Expanding the exponentials like $e^x = 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{3!} + \cdots$, we find :

$$\frac{R_1}{L}t - \frac{1}{2}\left(\frac{R_1}{L}\right)^2 t^2 + \cdots = \frac{R_1}{R_2}\left(1 - \frac{t}{RC} + \frac{t^2}{2R^2C^2} - \cdots\right)$$

$\Rightarrow t\left(\dfrac{R_1}{L} + \dfrac{R_1}{R_2^{\,2}C}\right) + O(t^2) + \cdots = \dfrac{R_1}{R_2}$, if we have assumed that $t << 1$. Therefore:

$$\Rightarrow t \approx \frac{1}{R_2}\left(\frac{1}{(1/L) + (1/R_2^{\,2}C)}\right) = \left(\frac{LR_2C}{L + R_2^{\,2}C}\right)$$

$$\Rightarrow t = \left(\frac{(8.0\text{ H})(5000\ \Omega)(2.0 \times 10^{-5}\text{ F})}{8.0\text{ H} + (5000\ \Omega)^2(2.0 \times 10^{-5}\text{ F})}\right) = 1.6 \times 10^{-3}\text{s}.$$

e) At $t = 1.57\times10^{-3}\,\text{s}: i_1 = \dfrac{\varepsilon}{R_1}(1-e^{-(R_1/L)t}) = \dfrac{48\ \text{V}}{25\ \Omega}(1-e^{-(25/8)t}) = 9.4\times10^{-3}\ \text{A}.$

f) We want to know when the current is half its final value. We note that the current i_2 is very small to begin with, and just gets smaller, so we ignore it and find:

$$i_{1/2} = 0.960\ \text{A} = i_1 = \frac{\varepsilon}{R_1}(1-e^{-(R_1/L)t}) = (1.92\ \text{A})(1-e^{-(R_1/L)t}).$$

$$\Rightarrow e^{-(R_1/L)t} = 0.500 \Rightarrow t = \frac{L}{R_1}\ln(0.5) = \frac{8.0\ \text{H}}{25\ \Omega}\ln(0.5) = 0.22\ \text{s}$$

30.78: a) Using Kirchoff's Rules on the left and right branches:

Left: $\varepsilon - (i_1+i_2)R - L\dfrac{di_1}{dt} = 0 \Rightarrow R(i_1+i_2) + L\dfrac{di_1}{dt} = \varepsilon.$

Right: $\varepsilon - (i_1+i_2)R - \dfrac{q_2}{C} = 0 \Rightarrow R(i_1+i_2) + \dfrac{q_2}{C} = \varepsilon.$

b) Initially, with the switch just closed, $i_1 = 0, i_2 = \dfrac{\varepsilon}{R}$ and $q_2 = 0.$

c) The substitution of the solutions into the circuit equations to show that they satisfy the equations is a somewhat tedious exercise in bookkeeping that is left to the reader.

We will show that the initial conditions are satisfied:

At $t = 0, q_2 = \dfrac{\varepsilon}{\omega R}e^{-\beta t}\sin(\omega t) = \dfrac{\varepsilon}{\omega R}\sin(0) = 0$

$$i_1(t) = \frac{\varepsilon}{R}(1-e^{-\beta t}[(2\omega RC)^{-1}\sin(\omega t)+\cos(\omega t)] \Rightarrow i_1(0) = \frac{\varepsilon}{R}(1-[\cos(0)]) = 0.$$

d) When does i_2 first equal zero? $\omega = \sqrt{\dfrac{1}{LC} - \dfrac{1}{(2RC)^2}} = 625\ \text{rad/s}$

$$i_2(t) = 0 = \frac{\varepsilon}{R}e^{-bt}[-(2\omega RC)^{-1}\sin(\omega t)+\cos(\omega t)] \Rightarrow -(2\omega RC)^{-1}\tan(\omega t)+1 = 0$$

$$\Rightarrow \tan(\omega t) = +2\omega RC = +2(625\ \text{rad/s})(400\ \Omega)(2.00\times10^{-6}\ \text{F}) = +1.00.$$

$$\Rightarrow \omega t = \arctan(+1.00) = +0.785 \Rightarrow t = \frac{0.785}{625\ \text{rad/s}} = 1.256\times10^{-3}\,\text{s}.$$

30.79: a) $\Phi_B = BA = B_L A_L + B_{\text{Air}}A_{\text{Air}} = \dfrac{\mu_0 Ni}{W}((D-d)W) + \dfrac{K\mu_0 Ni}{W}(dW) =$

$\mu_0 Ni[(D-d)+Kd]$

$$\Rightarrow L = \frac{N\Phi_B}{i} = \mu_0 N^2[(D-d)+Kd] = L_0 - L_0\frac{d}{D} + L_f\frac{d}{D} = L_0 + \left(\frac{L_f - L_0}{D}\right)d$$

$$\Rightarrow d = \left(\frac{L-L_0}{L_f - L_0}\right)D, \text{ where } L_0 = \mu_0 N^2 D, \text{ and } L_f = K\mu_0 N^2 D.$$

b) Using $K = \chi_m + 1$ we can find the inductance for any height $L = L_0\left(1 + \chi_m \frac{d}{D}\right)$.

Height of Fluid	Inductance of Liquid Oxygen	Inductance of Mercury
$d = D/4$	0.63024 H	0.63000 H
$d = D/2$	0.63048 H	0.62999 H
$d = 3D/4$	0.63072 H	0.62999 H
$d = D$	0.63096 H	0.62998 H

Where are used the values $\chi_m(O_2) = 1.52 \times 10^{-3}$ and $\chi_m(\text{Hg}) = -2.9 \times 10^{-5.}$

d) The volume gauge is much better for the liquid oxygen than the mercury because there is an easily detectable spread of values for the liquid oxygen, but not for the mercury.

31.1: a) $V_{rms} = \frac{V}{\sqrt{2}} = \frac{45.0 \text{ V}}{\sqrt{2}} = 31.8 \text{ V}.$

b) Since the voltage is sinusoidal, the average is zero.

31.2: a) $I = \sqrt{2}I_{rms} = \sqrt{2}(2.10 \text{ A}) = 2.97 \text{ A}.$

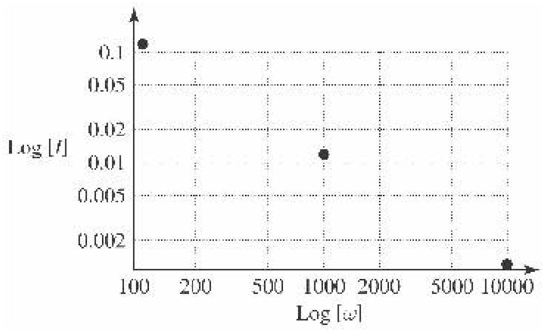
b) $I_{rav} = \frac{2}{\pi}I = \frac{2}{\pi}(2.97 \text{ A}) = 1.89 \text{ A}.$

c) The root-mean-square voltage is always greater than the rectified average, because squaring the current before averaging, then square-rooting to get the root-mean-square value will always give a larger value than just averaging.

31.3: a) $V = IX_L = I\omega L \Rightarrow I = \frac{V}{\omega L} = \frac{60.0 \text{ V}}{(100 \text{ rad/s})(5.00 \text{ H})} = 0.120 \text{ A}.$

b) $I = \frac{V}{\omega L} = \frac{60.0 \text{ V}}{(1000 \text{ rad/s})(5.00 \text{ H})} = 0.0120 \text{ A}.$

c) $I = \frac{V}{\omega L} = \frac{60.0 \text{ V}}{(10{,}000 \text{ rad/s})(5.00 \text{ H})} = 0.00120 \text{ A}.$

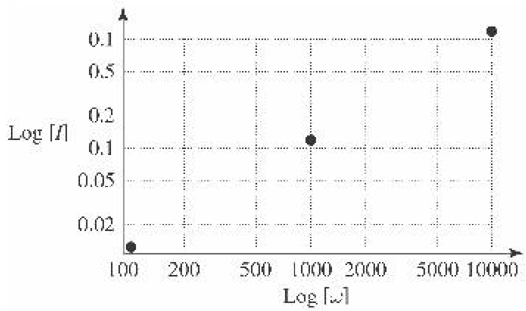


31.4: a) $V = IX_C = \dfrac{I}{\omega C} \Rightarrow I = V\omega C = (60.0\ \text{V})\,(100\ \text{rad/s})\,(2.20 \times 10^{-6}\ \text{F}) = 0.0132\ \text{A}.$

b) $I = V\omega C = (60.0\ \text{V})\,(10000\ \text{rad/s})\,(2.20 \times 10^{-6}\ \text{F}) = 0.132\ \text{A}.$

c) $I = V\omega C = (60.0\ \text{V})\,(10{,}000\ \text{rad/s})\,(2.20 \times 10^{-6}\ \text{F}) = 1.32\ \text{A}.$

d)



31.5: a) $X_L = \omega L = 2\pi fL = 2\pi(80\ \text{Hz})\,(3.00\ \text{H}) = 1508\ \Omega$

b) $X_L = \omega L = 2\pi fL \Rightarrow L = \dfrac{X_L}{2\pi f} = \dfrac{120\ \Omega}{2\pi(80\ \text{Hz})} = 0.239\ \text{H}.$

c) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(80\ \text{Hz})\,(4.0 \times 10^{-6}\ \text{F})} = 497\ \Omega$

d) $X_C = \dfrac{1}{2\pi fC} \Rightarrow C = \dfrac{1}{2\pi fX_C} = \dfrac{1}{2\pi(80\ \text{Hz})\,(120\ \Omega)} = 1.66 \times 10^{-5}\ \text{F}.$

31.6: a) $X_L = \omega L = 2\pi fL = 2\pi(60\ \text{Hz})(0.450\ \text{H}) = 170\Omega$ If $f = 600\ \text{Hz}$, $X_L = 1700\Omega$

b) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60\ \text{Hz})\,(2.50 \times 10^{-6}\ \text{F})} = 1061\ \Omega$ If $f = 600\ \text{Hz}$, $X_C =$ $106.1\ \Omega.$

c) $X_C = X_L \Rightarrow \dfrac{1}{\omega C} = \omega L \Rightarrow \omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.450\ \text{H})\,(2.50 \times 10^{-6}\ \text{Hz})}} = 943\ \text{rad/s},$

so $f = 150$ Hz.

31.7: $V_C = \dfrac{I}{\omega C} \Rightarrow C = \dfrac{I}{\omega V_C} = \dfrac{(0.850\ \text{A})}{2\pi(60\ \text{Hz})\ (170\ \text{V})} = 1.32 \times 10^{-5}\ \text{F}.$

31.8: $V_L = I\omega L \Rightarrow f = \dfrac{V_L}{2\pi IL} = \dfrac{(12.0\ \text{V})}{2\pi(2.60 \times 10^{-3}\ \text{A})\ (4.50 \times 10^{-4}\ \text{H})} = 1.63 \times 10^{6}\ \text{Hz}.$

31.9: a) $i = \dfrac{v}{R} = \dfrac{(3.80\ \text{V})\cos((720\ \text{rad/s})t)}{150\ \Omega} = (0.0253\ \text{A})\cos((720\ \text{rad/s})t).$

b) $X_L = \omega L = (720\ \text{rad/s})\ (0.250\ \text{H}) = 180\ \Omega.$

c) $v_L = L\dfrac{di}{dt} = -(\omega L)\ (0.0253\ \text{A})\sin((720\ \text{rad/s})t) = -(4.55\ \text{V})\sin((720\ \text{rad/s})t).$

31.10: a) $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(120\ \text{rad/s})\ (4.80 \times 10^{-6}\ \text{F})} = 1736\ \Omega.$

b) To find the voltage across the resistor we need to know the current, which can be found from the capacitor (remembering that it is out of phase by $90°$ from the capacitor's voltage).

$$i = \frac{v_C}{X_C} = \frac{v\cos(\omega t)}{X_C} = \frac{(7.60\ \text{V})\cos((120\ \text{rad/s})t)}{1736\ \Omega} = (4.38 \times 10^{-3}\ \text{A})\cos((120\ \text{rad/s})t)$$

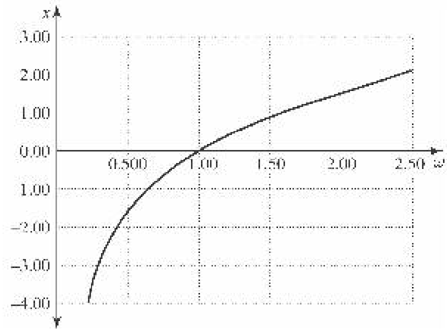
$$\Rightarrow v_R = iR = (4.38 \times 10^{-3}\ \text{A})\ (250\ \Omega)\cos((120\ \text{rad/s})t) = (1.10\ \text{V})\cos((120\ \text{rad/s})t).$$

31.11: a) If $\omega = \omega_0 = \dfrac{1}{\sqrt{LC}} \Rightarrow X = \omega L - \dfrac{1}{\omega C} \Rightarrow X = \dfrac{L}{\sqrt{LC}} - \dfrac{1}{C/\sqrt{LC}} = 0.$

b) When $\omega > \omega_0 \Rightarrow X > 0.$

c) When $\omega > \omega_0 \Rightarrow X < 0.$

d) The graph of X against ω is on the following page.



31.12: a) $Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{(200\ \Omega)^2 + ((250\ \text{rad/s})\ (0.400\ \text{H}))^2} = 224\ \Omega.$

b) $I = \dfrac{V}{Z} = \dfrac{30.0\ \text{V}}{224\ \Omega} = 0.134\ \text{A}$

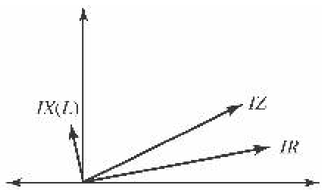
c) $V_R = IR = (0.134\ \text{A})\ (200\ \Omega) = 26.8\ \text{V};$

$V_L = I\omega L = (0.134\ \text{A})\ (250\ \text{rad/s})\ (0.400\ \text{H})$

$\Rightarrow V_L = 13.4\ \text{V}.$

d) $\phi = \arctan\left(\dfrac{v_L}{v_R}\right) = \arctan\left(\dfrac{13.4\ \text{V}}{26.8\ \text{V}}\right) = 26.6°,$ and the voltage leads the current.

e)



31.13: a) $Z = \sqrt{R^2 + (1/\omega C)^2} = \sqrt{(200\ \Omega)^2 + 1/((250\ \text{rad/s})\ (6.00\times 10^{-6}\ \text{F}))^2}$

$= 696\ \Omega.$

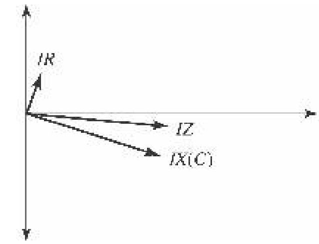
b) $I = \dfrac{V}{Z} = \dfrac{30.0\ \text{V}}{696\ \Omega} = 0.0431\ \text{A}.$

$V_R = IR = (0.0431\ \text{A})\ (200\ \Omega) = 8.62\ \text{V};$

c) $V_C = \dfrac{I}{\omega C} = \dfrac{(0.0431\ \text{A})}{(250\ \text{rad/s})\ (6.00\times 10^{-6}\ \text{F})} = 28.7\ \text{V}.$

d) $\phi = \arctan\left(\dfrac{V_C}{V_R}\right) = \arctan\left(\dfrac{28.7\ \text{V}}{8.62\ \text{V}}\right) = -73.3°,$ and the voltage lags the current.

e)



31.14: a)

$$Z=(\omega L-1/\omega C)=(250\text{ rad/s})\,(0.400\text{ H})-\frac{1}{(250\text{ ad/s})\,(6.00\times10^{-6}\text{ F})}=567\ \Omega$$

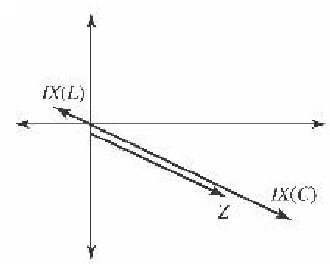
b) $I=\dfrac{V}{Z}=\dfrac{30.0\text{ V}}{567\ \Omega}=0.0529\text{ A}.$

c) $V_C=I\omega L=(0.0529)\,(250\text{ rad/s})\,(0.400\text{ H})=5.29\text{ V}$

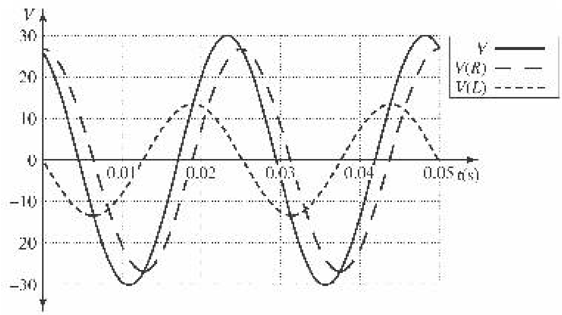
$$V_C=\frac{I}{\omega C}=\frac{(0.0529\text{ A})}{(250\text{ rad/s})\,(6.00\times10^{-6}\text{ F})}=35.3\text{ V}.$$

d) $\phi=\arctan\left(\dfrac{V_L-V_C}{V_R}\right)=\arctan(-\infty)=-90.0°$, and the voltage lags the current.

e)



31.15: a)



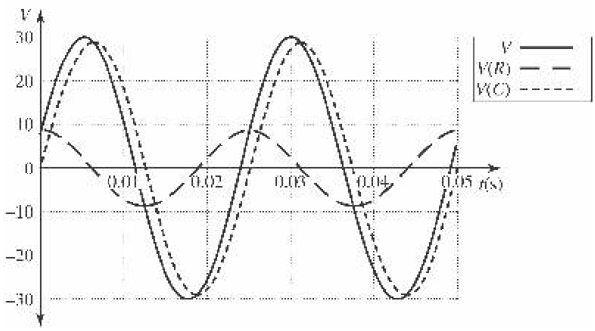
b) The different voltages are:

$$v = (30.0\text{ V})\cos(250t + 26.6°),\ v_R = (26.8\text{ V})\cos(250t),\ v_L = (13.4\text{ V})\cos(250t + 90°)$$

At $t = 20$ ms : $v = 20.5$ V, $v_R = 7.60$ V, $v_L = 12.85$ V. Note $v_R + v_L = v$.

c) At $t = 40$ ms : $v = -15.2$ V, $v \ = -22.49$ V, $v_L = 7.29$ V. Note $v_R + v_L = v$. Be careful with radians vs. degrees in above expressions!

31.16: a)



b) The different voltage are:

$$v = (30.0\text{ V})\cos(250t - 73.3°),\ v_R = (8.62\text{ V})\cos(250t),\ v_C = (28.7\text{ V})\cos(250t - 90°)$$

At $t = 20$ ms : $v = -25.1$ V, $v_R = 2.45$ V, $v_C = -27.5$ V. Note $v_R + v_C = v$.

c) At $t = 40$ ms : $v = -22.9$ V, $v_R = -7.23$ V, $v_C = -15.6$ V. Note $v_R + v_C = v$. Careful with radians vs. degrees!

31.17: a) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$$\Rightarrow Z = \sqrt{(200\ \Omega)^2 + ((250\text{ rad/s})\ (0.0400\text{ H}) - 1/\ ((250\text{ rad/s})\ (6.00 \times 10^{-6}\text{ F}))^2}$$

$= 601\ \Omega$.

b) $I = \dfrac{V}{Z} = \dfrac{30\text{ V}}{601\ \Omega} = 0.0499$ A.

c) $\phi = \arctan\left(\dfrac{\omega L - 1/\omega C}{R}\right) = \arctan\left(\dfrac{100\ \Omega - 667\ \Omega}{200\ \Omega}\right) = -70.6^\circ$, and the voltage lags the current.

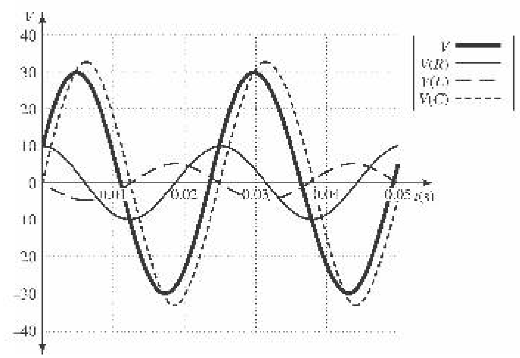
d) $V_R = IR = (0.0499\text{ A})\ (200\ \Omega) = 9.98$ V;

$$V_L = I\omega L = (0.0499\text{ A})\ (250\text{ rad/s})(0.400\text{ H}) = 4.99\text{ V};$$

$$V_C = \frac{I}{\omega C} = \frac{(0.0499\text{ A})}{(250\text{ rad/s})\ (6.00 \times 10^{-6}\text{ F})} = 33.3\text{ V}.$$

e) Because of the charge-storing nature of the capacitor, its voltage will tag the source voltage. That is, the capacitor's voltage will peak after the source voltage.

31.18: a)



The different voltages plotted above are:

$v = (30\text{ V})\cos(250t - 70.6°)$, $v_R = (9.98\text{ V})\cos(250t)$,

$v_L = (4.99\text{ V})\cos(250t + 90°) v_C = (33.3\text{ V})\cos(250t - 90°)$.

b) At $t = 20\text{ ms}: v = -24.3\text{ V}, v_R = 2.83\text{ V}, v_L = 4.79\text{ V}, v_C = -31.9\text{ V}$.

c) At $t = 40\text{ ms}: v = -23.8\text{ V}, v_R = -8.37\text{ V}, v_L = 2.71\text{ V}, v_C = -18.1\text{ V}$.

In both parts (b) and (c), note that the voltage equals the sum of the other voltages at the given instant. Be careful with degrees vs. radians!

31.19: a) Current largest at the resonance frequency

$f_0 = \dfrac{1}{2\pi\sqrt{LC}} = 113\text{ Hz}$. At resonance, $X_L = X_C$ and $Z = R$. $I = V/R = 15.0\text{ mA}$

b) $X_C = 1/\omega C = 500\ \Omega$; $X_L = \omega L = 160\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\ \Omega)^2 + (160\ \Omega - 500\ \Omega)^2} = 394.5\ \Omega$$

$I = V/Z = 7.61\text{ mA}$

$X_C > X_L$ so source voltage lags the current.

31.20: Using $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ and $\phi = \arctan\left(\dfrac{\omega L - 1/(\omega C)}{R}\right)$, along with the values $R = 200\ \Omega$, $L = 0.400\text{ H}$, and $C = 6.00 \times 10^{-6}\text{ F}$:

a) $\omega = 1000\text{ rad/s}: Z = 307\ \Omega, \phi = 49.4°$;

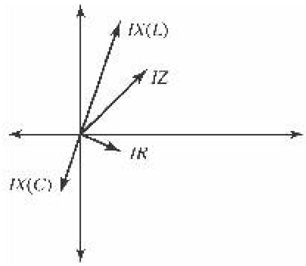
$\omega = 600$ rad/s : $Z = 204\ \Omega$, $\phi = -10.7°$;

$\omega = 200$ rad/s : $Z = 779\ \Omega$, $\phi = -75.1°$.

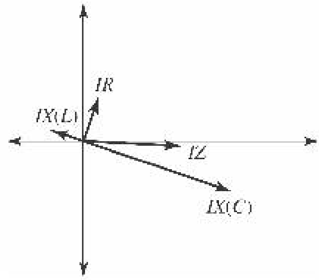
b) The current increases at first, then decreases again since $I = \dfrac{V}{Z}$.

c) The phase angle was calculated in part (a) for all frequencies.

d)



e)



31.21: $V^2 = V_R^2 + (V_L - V_C)^2$

$$V = \sqrt{(30.0\text{ V})^2 + (50.0\text{ V} - 90.0\text{ V})^2} = 50.0\text{ V}$$

31.22: a) First, let us find the phase angle between the voltage and the current

$$\tan(\phi) = \frac{\omega L - \frac{1}{\omega C}}{R} = \frac{2\pi(1.25\times10^3\text{ Hz})\,(20.0\times10^{-3}\text{ H}) - \frac{1}{2\pi(1.25\times10^3\text{ Hz})\,(140\times10^{-9}\text{ C})}}{350\ \Omega} \Rightarrow \phi = -65.1°$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} = \sqrt{(350\ \Omega)^2 = +(-752\ \Omega)^2} = 830\ \Omega.$$

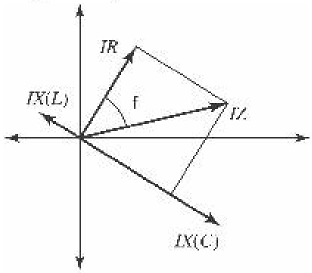
The average power provided by the supply is then

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\phi) = \frac{V_{\text{rms}}^2}{Z}\cos(\phi) = \frac{(120\text{ V})^2}{830\ \Omega}\cos(-65.1°) = 7.32\text{ W}$$

b) The average power dissipated by the resistor is $P_R = I_{\text{rms}}^2 R = \left(\frac{120\text{ V}}{830\ \Omega}\right)^2 (350\ \Omega) = 7.32\text{ W}$

31.23: a) Using the phasor diagram at right we can see:

$$\cos\phi = \frac{IR}{I\sqrt{R^2 + X_L{}^2 - X_C{}^2}} = \frac{R}{Z}.$$



b) $P_{av} = \frac{1}{2}\frac{V^2}{Z}\cos\phi = \frac{V_{rms}^2}{Z}\cos\phi$

$\Rightarrow P_{av} = \frac{V_{rms}^2}{Z}\frac{R}{Z} = I_{rms}^2 R.$

31.24: $$P_{av} = \frac{V_{rms}^2}{Z}\cos\phi = \frac{V_{rms}^2}{Z}\frac{R}{Z}$$
$$= \frac{V_{rms}^2}{Z^2}R = \frac{(80.0\text{ V})^2}{(105\ \Omega)^2}(75.0\ \Omega) = 43.5\text{ W}.$$

31.25: a) $\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

$$= \frac{240\ \Omega}{\sqrt{(240\ \Omega)^2 + \left(2\pi(400\text{ Hz})(0.120\text{ H}) - \frac{1}{2\pi(400\text{ Hz})(7.30\times10^{-6}\text{ F})}\right)^2}}$$

$= \frac{240\ \Omega}{344\ \Omega} = 0.698$

$\Rightarrow \phi = \cos^{-1}(0.698) = 45.8°.$

b) From (a), $Z = 344\ \Omega.$

c) $V_{rms} = I_{rms}Z = (0.450\text{ A})(344\ \Omega) = 155\text{ V}.$

d) $P_{av} = V_{rms}I_{rms}\cos\phi = (155\text{ V})(0.450\text{ A})(0.698) = 48.7\text{ W}.$

e) $P_R = P_{av} = 48.7\text{ W}.$

f) Zero.

g) Zero.

For pure capacitors and inductors there is no average energy flow.

31.26: a) The power factor equals:

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (\omega L)^2}} = \frac{(360\ \Omega)}{\sqrt{(360\ \Omega)^2 + (((2\pi)60\ \text{rad/s})\ (5.20\ \text{H}))^2}} = 0.181.$$

b)

$$P_{av} = \frac{1}{2}\frac{V^2}{Z}\cos\phi = \frac{1}{2}\frac{(240\ \text{V})^2}{\sqrt{(360\ \Omega)^2 + (((2\pi)60\ \text{rad/s})\ (5.20\ \text{H}))^2}}(0.181) = 2.62\ \text{W}.$$

31.27: a) At the resonance frequency, $Z = R$.

$V = IZ = IR = (0.500\ \text{A})\ (300\ \Omega) = 150\ \text{V}$

b) $V_R = IR = 150\ \text{V}$

$X_L = \omega L = L(1/\sqrt{LC} = \sqrt{L/C} = 2582\ \Omega;\ V_L = IX_L = 1290\ \text{V}$

$X_C = 1/(\omega C) = \sqrt{L/C} = 2582\ \Omega;$

$V_C = IX_C = 1290\ \text{V}$

c) $P_{av} = \frac{1}{2}V\ I\ \cos\phi = \frac{1}{2}I^2\ R$, since $V = IR$ and $\cos\phi = 1$ at resonance.

$P_{av} = \frac{1}{2}(0.500\ \text{A})^2\ (300\ \Omega) = 37.5\ \text{W}$

31.28: a) The amplitude of the current is given by

$$I = \frac{V}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}}$$

Thus, the current will have a maximum amplitude when

$\omega L = \frac{1}{\omega^2 C} \Rightarrow C = \frac{1}{\omega^2 L} = \frac{1}{(50.0\ \text{rad/s})^2\ (9.00\ \text{H})} = 44.4\ \mu\text{F}.$

b) With the capacitance calculated above we find that $Z = R$, and the amplitude of the current is $I = \frac{V}{R} = \frac{120\ V}{400\ \Omega} = 0.300\ \text{A}$. Thus, the amplitude of the voltage across the inductor is $V = I(\omega L) = (0.300\ \text{A})\ (50.0\ \text{rad/s})\ (9.00\ \text{H}) = 135\ \text{V}$.

31.29: a) At resonance, the power factor is equal to one, because the impedance of the circuit is exactly equal to the resistance, so $\frac{R}{Z} = 1$.

b) Average power: $P_{av} = \frac{V^2_{rms}}{R} = \frac{1}{2}\frac{(150\ \text{V})^2}{150\ \Omega} = 75\ \text{W}$.

c) If the capacitor is changed, and then resonance is again attained, the power factor again equals one. The average power still has no dependence on the capacitor, so $P_{av} = 75\ \text{W}$ again.

31.30: a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.350\text{ H})(1.20\times10^{-8}\text{ F})}} = 15.4\times10^3\text{ rad/s}.$

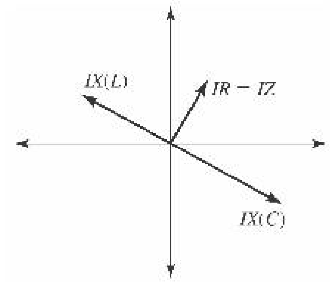
b) $V_C = \dfrac{I}{\omega C} \Rightarrow I = V_C\omega C = (550\text{ V})(15.4\times10^3\text{ rad/s})(1.20\times10^{-8}\text{ F}) = 0.102\text{ A}$

$\Rightarrow V_{\max(source)} = IR = (0.102\text{ A})(400\ \Omega) = 40.8\text{ V}.$

31.31: a) At resonance:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.400\text{ H})(6.00\times10^{-6}\text{ F})}}$$
$$\Rightarrow \omega_0 = 645.5\text{ rad/s} \Rightarrow 103\text{ Hz}.$$

b)



c) $V_1 = V_{\text{rms}(source)} = \dfrac{V}{\sqrt{2}} = \dfrac{30.0\text{ V}}{\sqrt{2}} = 21.2\text{ V}, I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{V_{\text{rms}}}{R} = \dfrac{21.2\text{ V}}{200\ \Omega}$

$= 0.106\text{ A}$

$V_2 = I_{\text{rms}}\omega_0 L = (0.106\text{ A})(645.5\text{ rad/s})(0.400\text{ H}) = 27.4\text{ V}.$

$V_3 = \dfrac{I_{\text{rms}}}{\omega_0 C} = \dfrac{(0.106\text{ A})}{(645.5\text{ rad/s})(6.00\times10^{-6}\text{ F})} = 27.4\text{ V} = V_2,$

$V_4 = 0$, since the capacitor and inductor's voltages cancel each other.

$V_5 = V_{\text{rms}(source)} = \dfrac{V}{\sqrt{2}} = \dfrac{30\text{ V}}{\sqrt{2}} 21.2\text{ V}.$

d) If the resistance is changed, that has no affect upon the resonance frequency:

$\omega_0 = 645.5\text{ rad/s} \Rightarrow 103\text{ Hz}$

e) $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{Z} = \dfrac{V_{\text{rms}}}{R} = \dfrac{21.2\text{ V}}{100\ \Omega} = 0.212\text{ A}.$

31.32: a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(0.280\text{ H})(4.00\times10^{-6}\text{ F})}} = 945\text{ rad/s}.$

b) $I = 1.20$ A at resonance, so: $R = Z = \dfrac{V}{I} = \dfrac{120\text{ V}}{1.70\text{ A}} = 70.6\ \Omega$

c) At resonance:

$$V_{\text{peak}}(R) = 120\text{ V},\ V_{\text{peak}}(L) = V_{\text{peak}}(C) = I\omega L = (1.70\text{ A})(945\text{ rad/s})(0.280\text{ H})$$
$$= 450\text{ V}.$$

31.33: a) $\dfrac{N_1}{N_2} = \dfrac{120}{12} = 10.$

b) $I_{\text{rms}} = \dfrac{V_{\text{rms}}}{R} = \dfrac{12.0\text{ V}}{5.00\ \Omega} = 2.40\text{ A}$

c) $P_{av} = I_{\text{rms}}V_{\text{rms}} = (2.40\text{ A})(12.0\text{ V}) = 28.8\text{ W}$.

d) $R = \dfrac{V^2{}_{\text{rms}}}{P} = \dfrac{(120\text{ V})^2}{28.8\text{ W}} = 500\ \Omega$, and note that this is the same as

$$(5.00\ \Omega)\left(\frac{N_1}{N_2}\right)^2 = (5.00\ \Omega)\left(\frac{120}{12.0}\right)^2 = 500\ \Omega.$$

31.34: a) $\dfrac{N_2}{N_1} = \dfrac{13000}{120} = 108.$

b) $P = I_2V_2 = (0.00850\text{ A})(13000\text{ V}) = 110.5\text{ W}$.

c) $I_1 = I_2\dfrac{N_2}{N_1} = (0.00850\text{ A})(108) = 0.918\text{ A}$.

31.35: a) $R_1 = R_2\left(\dfrac{N_1}{N_2}\right)^2 \Rightarrow \dfrac{N_1}{N_2} = \sqrt{\dfrac{R_1}{R_2}} = \sqrt{\dfrac{12.8\times10^3\ \Omega}{8.00\ \Omega}} = 40.$

b) $V_2 = V_1\left(\dfrac{N_2}{N_1}\right) = (60.0\text{ V})\dfrac{1}{40} = 1.50\text{ V}$

31.36: a) $Z_{\text{tweeter}} = \sqrt{R^2 + (1/\omega C)^2}$

b) $Z_{\text{woofer}} = \sqrt{R^2 + (\omega L)^2}$

c) If $Z_{\text{tweeter}} = Z_{\text{woofer}}$, then the current splits evenly through each branch.

d) At the crossover point, where currents are equal:

$$R^2 + (1/\omega C^2) = R^2 + (\omega L)^2 \Rightarrow \omega = \frac{1}{\sqrt{LC}}.$$

31.37: $\phi = \arctan\left(\dfrac{\omega L}{R}\right) \Rightarrow L = \dfrac{R}{\omega}\tan\phi = \dfrac{R}{2\pi f}\tan\phi$

$$= \left(\frac{48.0\ \Omega}{2\pi(80\ \text{Hz})}\right)\tan(52.3°) = 0.124\ \text{H}.$$

31.38: a) If $\omega = 200\ \text{rad/s} : Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$$\Rightarrow Z = \sqrt{(200\ \Omega)^2 + ((200\ \text{rad/s})(0.400\ \text{H}) - 1/((200\ \text{rad/s})(6.00\times10^{-6}\ \text{F})))^2} = 779\ \Omega.$$

$$\Rightarrow I = \frac{V}{Z} = \frac{30\ \text{V}}{779\ \Omega} = 0.0385\ \text{A} \Rightarrow I_{\text{rms}} = \frac{1}{\sqrt{2}} = 0.0272\ \text{A}.$$

So, $V_1 = I_{\text{rms}}R = (0.0272\ \text{A})(200\ \Omega) = 5.44\ \text{V}$,

$V_2 = I_{\text{rms}}X_L = I_{\text{rms}}\omega L = (0.0272\ \text{A})(200\ \text{rad/s})(0.400\ \text{H}) = 2.18\ \text{V}$,

$V_3 = I_{\text{rms}}X_C = \dfrac{I_{\text{rms}}}{\omega C} = \dfrac{(0.0272\ \text{A})}{(200\ \text{rad/s})(6.00\times10^{-6}\ \text{F})} = 22.7\ \text{V}$,

$V_4 = V_3 - V_2 = 22.7\ \text{V} - 2.18\ \text{V} = 20.5\ \text{V}$, and $V_5 = \varepsilon_{\text{rms}} = \frac{30.0}{\sqrt{2}}\ \text{V} = 21.2\ \text{V}$.

b) If $\omega = 1000\ \text{rad/s}$, using the same steps as above in part (a): $Z = 307\ \Omega, V_1 = 13.8\ \text{V}, V_2 = 27.6\ \text{V}, V_3 = 11.5\ \text{V}, V_4 = 16.1\ \text{V}, V_5 = 21.2\ \text{V}$.

31.39: a) $I_{\text{rav}} = 0$ when $\omega t = (n + 1/2)\pi \Rightarrow t_1 = \dfrac{\pi}{2\omega}, t_2 = \dfrac{3\pi}{2\omega} \Rightarrow t_2 - t_1 = \dfrac{\pi}{\omega}$.

b) $\displaystyle\int_{t_1}^{t_2} i\,dt = \int_{t_1}^{t_2} I\cos(\omega t)dt = \frac{I}{\omega}\sin(\omega t)\Big|_{t_1}^{t_2} = \frac{I}{\omega}[\sin(3\pi/2) - \sin(\pi/2)] = -\frac{2I}{\omega} = \frac{2I}{\omega}$,

since it is rectified.

c) So, $I_{\text{rav}}(t_2 - t_1) = \dfrac{2I}{\omega} \Rightarrow I_{\text{rav}} = \dfrac{\omega}{\pi}\dfrac{2I}{\omega} = \dfrac{2I}{\pi}$.

31.40: a) $X_L = \omega L \Rightarrow L = \dfrac{XL}{\omega} = \dfrac{250\ \Omega}{2\pi(120\ \text{Hz})} = 0.332\ \Omega$

b) $Z = \sqrt{R^2 + X_L{}^2} = \sqrt{(400\ \Omega)^2 + (250\ \Omega)^2} = 472\ \Omega, \cos\phi = \dfrac{R}{Z}$.

$$P_{av} = \frac{V^2{}_{\text{rms}}}{Z}\frac{R}{Z} \Rightarrow V_{\text{rms}} = Z\sqrt{\frac{P_{av}}{R}} = (472\ \Omega)\sqrt{\frac{800\ \text{W}}{400\ \Omega}} = 668\ \text{V}.$$

31.41: a) If the original voltage was lagging the circuit current, the addition of an inductor will help it "catch up," since a pure LR circuit would have the voltage

leading. This will increase the power factor, because it is largest when the current and voltage are in phase.

b) Since the voltage is lagging, the impedance is dominated by a capacitive element so we need an inductor such that $X_L = X_0$, where X_0 is the original capacitively dominated reactance (this could include inductors, but the capacitors "win").

$$R = 0.720\ Z = 0.720(60.0\ \Omega) = 43.2\ \Omega$$

$$\Rightarrow Z = \sqrt{R^2 + X_C^2} \Rightarrow X_0 = \sqrt{Z^2 - R^2} = \sqrt{(60\ \Omega)^2 - (43.2\ \Omega)^2} = 41.6\ \Omega.$$

$$X_L = X_C = 41.6\ \Omega = \omega L \Rightarrow L = \frac{X_C}{\omega} = \frac{41.6\ \Omega}{2\pi(50\ \text{Hz})} = 0.132\ \text{H}$$

31.42: $Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{240\ \text{V}}{3.00\ \text{A}} = 80.0\ \Omega = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (50.0\ \Omega)^2}$. Thus,

$R = \sqrt{(80.0\ \Omega)^2 - (50.0\ \Omega)^2} = 62.4\ \Omega$. The average power supplied to this circuit is equal to the power dissipated by the resistor, which is

$$P = I^2_{\text{rms}} R = (3.00\ \text{A})^2\ (62.4\ \Omega) = 562\ \text{W}$$

31.43: a) $\omega_0 = 1/\sqrt{LC} = 3162\ \text{rad/s}; \omega = 2\omega_0 = 6324\ \text{rad/s}$

$X_L = \omega L = 31.62\ \Omega$; $\qquad X_C = 1/(\omega C) = 7.906\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = X_L - X_C = 23.71\ \Omega$

$I = V/Z = (5.00 \times 10^{-3}\ \text{V})/(23.71\ \Omega) = 2.108 \times 10^{-4}\ \text{A}$

$V_C = IX_C = 1.667 \times 10^{-3}\ \text{V}$; this is the maximum voltage across the capacitor.

$Q = CV_C = (20.0 \times 10^{-6}\text{F})(1.667 \times 10^{-3}\ \text{V}) = 33.34\ \text{nC}$

b) In part (a) we found $I = 0.211$ mA

c) $X_L > X_C$ and $R = 0$ gives that the source and inductor voltages are in phase; the voltage across the capacitor lags the source and inductor voltages by 180°.

31.44: a) $X_{L_2} = \omega_2 L = 2\omega_1 L = 2\left(\frac{1}{\omega_1 C}\right) = 2\left(\frac{2}{\omega_2 C}\right) = 4X_{C_2} \Rightarrow \frac{X_{L_2}}{X_{C_2}} = 4$, and so the inductor's reactance is greater than that of the capacitor.

b) $X_{L_3} = \omega_3 L = \frac{\omega_1 L}{3} = \left(\frac{1}{3\omega_1 C}\right) = \left(\frac{1}{9\omega_3 C}\right) = \frac{1}{9} X_{C_3} \Rightarrow \frac{X_{L_3}}{X_{C_3}} = \frac{1}{9}$, and so the capacitor's reactance is greater than that of the inductor.

c) Since $X_L = X_C$ at ω_1, that is the resonance frequency.

31.45: $V_{\text{out}} = \sqrt{V_R^2 + V_L^2} = I\sqrt{R^2 + (\omega L)^2} = \frac{V_s}{Z}\sqrt{R^2 + (\omega L)^2}$

$$\Rightarrow \frac{V_{out}}{V_s} = \frac{\sqrt{R^2 + (\omega L)^2}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$$

It ω is small: $\frac{V_{out}}{V_s} \approx \frac{R}{\sqrt{R^2 + (1/\omega C)^2}} = \frac{\omega R}{\sqrt{\omega^2 R^2 + (1/C)^2}} \approx \omega RC.$

If ω is large: $\frac{V_{out}}{V_s} \approx \frac{\sqrt{(\omega L)^2}}{\sqrt{(\omega L)^2}} = 1.$

31.46: $V_{out} = V_C = \frac{I}{\omega C} \Rightarrow \frac{V_{out}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

If ω is large: $\frac{V_{out}}{V_s} = \frac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \approx \frac{1}{\omega C\sqrt{(\omega L)^2}} = \frac{1}{(LC)\omega^2}.$

If ω is small: $\frac{V_{out}}{V_s} \approx \frac{1}{\omega C\sqrt{(1/\omega C)^2}} = \frac{\omega C}{\omega C} = 1.$

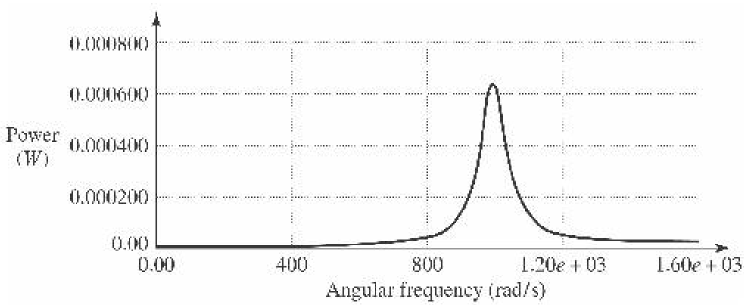
31.47: a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

b) $P_{av} = \frac{1}{2}I^2 R = \frac{1}{2}\left(\frac{V}{Z}\right)^2 R = \frac{V^2 R/2}{R^2 + (\omega L - 1/\omega C)^2}.$

c) The average power and the current amplitude are both greatest when the denominator is smallest, which occurs for $\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}.$

d) $P_{av} = \frac{(100\text{ V})^2 (200\ \Omega)/2}{(200\ \Omega)^2 + (\omega(2.00\text{ H}) - 1/\omega(5.00\times10^{-6}\text{ F}))^2}.$

$$\Rightarrow P_{av} = \frac{25\omega^2}{40{,}000\omega^2 + (2\omega^2 - 2{,}000{,}000)^2}.$$

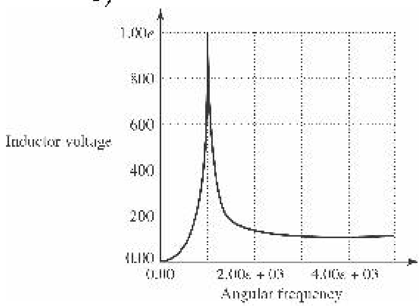


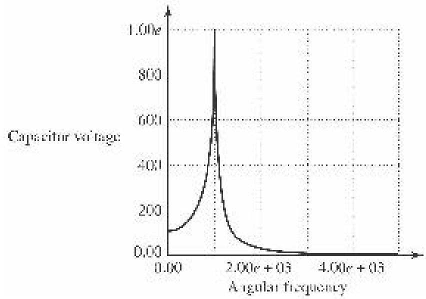
Note that as the angular frequency goes to zero, the power and current are zero, just as they are when the angular frequency goes to infinity. This graph exhibits the same strongly peaked nature as the light red curve in Fig. (31.15).

31.48: a) $V_L = I\omega\omega = \dfrac{V\omega L}{Z} = \dfrac{V\omega L}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

b) $V_C = \dfrac{I}{\omega C} = \dfrac{I}{\omega CZ} = \dfrac{1}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}.$

c)





d) When the angular frequency is zero, the inductor has zero voltage while the capacitor has voltage of 100 V (equal to the total source voltage). At very high frequencies, the capacitor voltage goes to zero, while the inductor's voltage goes to 100 V. At resonance, $\omega_0 = \dfrac{1}{\sqrt{LC}} = 1000\ \text{rad/s}$, the two voltages are equal, and are a maximum, 1000 V.

31.49: a) $U_B = \dfrac{1}{2}Li^2 \Rightarrow \langle U_B \rangle = \dfrac{1}{2}L\langle i^2 \rangle = \dfrac{1}{2}LI_{\text{rms}}{}^2 = \dfrac{1}{2}L\left(\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1}{4}LI^2.$

$$U_E = \frac{1}{2}Cv^2 \Rightarrow \langle U_E \rangle = \frac{1}{2}C\langle v^2 \rangle = \frac{1}{2}CV_{\text{rms}}{}^2 = \frac{1}{2}C\left(\frac{V}{\sqrt{2}}\right)^2 = \frac{1}{4}CV^2.$$

b) Using Problem (31.47a):

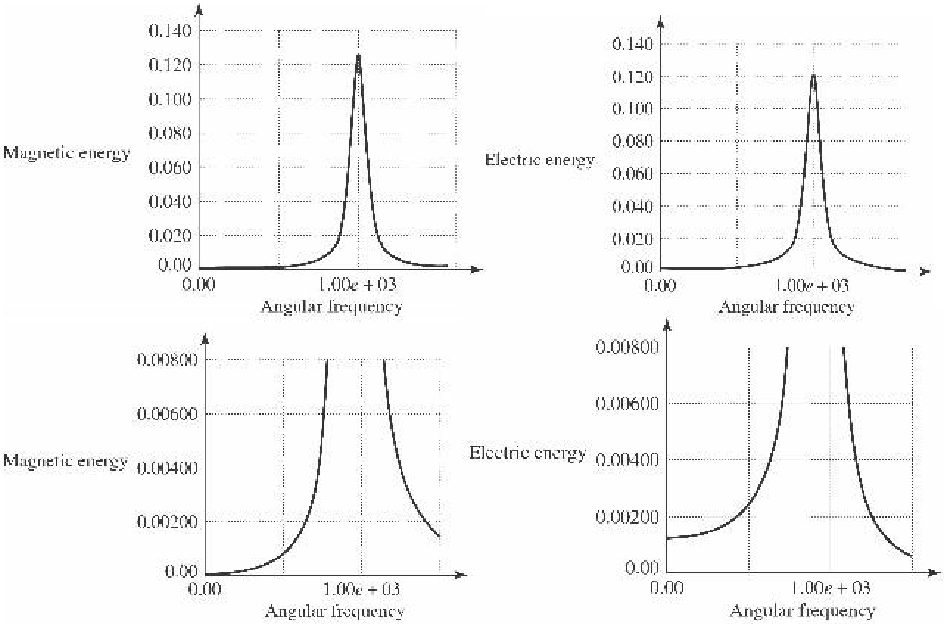
$$\langle U_B \rangle = \frac{1}{4}LI^2 = \frac{1}{4}L\left(\frac{V^2}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)^2 = \frac{LV^2}{4\left(R^2 + (\omega L - 1/\omega C)^2\right)}.$$

Using Problem (31.47b):

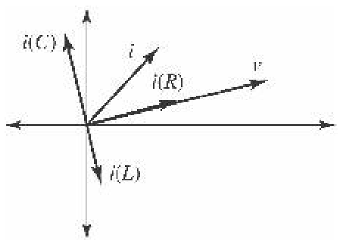
$$\langle U_E \rangle = \frac{1}{4}CV_C{}^2 = \frac{1}{4}C\,\frac{V^2}{\omega^2C^2\left(R^2 + (\omega L - 1/\omega C)^2\right)} = \frac{V^2}{4\omega^2C\left(R^2 + (\omega L - 1/\omega C)^2\right)}.$$

c) Below are the graphs of the magnetic and electric energies, the top two showing the general features, while the bottom two show the details close to angular frequency equal to zero.

d) When the angular frequency is zero, the magnetic energy stored in the inductor is zero, while the electric energy in the capacitor is $U_E = CV^2/4$. As the frequency goes to infinity, the energy noted in both inductor and capacitor go to zero. The energies equal each other at the resonant frequency where $\omega_0 = \dfrac{1}{\sqrt{LC}}$ and $U_B = U_E = \dfrac{LV^2}{4R^2}$.



31.50: a) Since the voltage drop between any two points must always be equal, the parallel LRC circuit must have equal potential drops over the capacitor, inductor and resistor, so $v_R = v_L = v_C = v$. Also, the sum of currents entering any junction must equal the current leaving the junction. Therefore, the sum of the currents in the branches must equal the current through the source: $i = i_R + i_L + i_C$.



b) $i_R = \frac{v}{R}$ is always in phase with the voltage. $i_L = \frac{v}{\omega L}$ lags the voltage by $90°$, and $i_C = v\omega C$ leads the voltage by $90°$.

c) From the diagram,

$$I^2 = I_R{}^2 + (I_C - I_L)^2 = \left(\frac{V}{R}\right)^2 + \left(V\omega C - \frac{V}{\omega L}\right)^2$$

d) From (c): $I = V\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}$. But

$$I = \frac{V}{Z} \Rightarrow \frac{1}{Z} = \sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}.$$

31.51: a) At resonance, $\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0 C = \frac{1}{\omega_0 L} \Rightarrow I_C = V\omega_0 C = \frac{V}{\omega_0 L} = I_L$ so $I = I_R$ and I is a minimum.

b) $P_{av} = \frac{V_{rms}^2}{Z}\cos\phi = \frac{V^2}{R}$ at resonance where $R < Z$ so power is a maximum.

c) At $\omega = \omega_0$, I and V are in phase, so the phase angle is zero, which is the same as a series resonance.

31.52: a) $V = \sqrt{2}V_{rms} = 311\,V$; $I_R = \frac{V}{R} = \frac{311\text{ V}}{400\ \Omega} = 0.778\text{ A}.$

b) $I_C = V\omega C = (311\text{ V})(360\text{ rad/s})(6.00\times10^{-6}\text{ F}) = 0.672\text{ A}$.

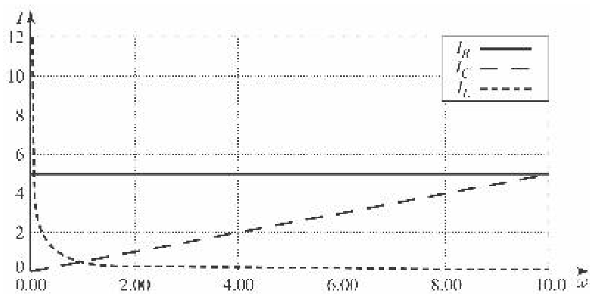
c) $\phi = \arctan\left(\frac{I_C}{I_R}\right) = \arctan\left(\frac{0.672\text{ A}}{0.778\text{ A}}\right) = 40.8°$, leading the voltage.

d) $I = \sqrt{I_R{}^2 + I_C{}^2} = \sqrt{(0.778\text{ A})^2 + (0.672\text{ A})^2} = 1.03\text{ A}$.

e) Leads since $\phi > 0$.

31.53: a) $I_R = \frac{V}{R}$; $I_C = V\omega C$; $I_L = \frac{V}{\omega L}$.

b)

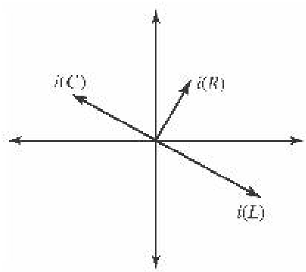


c) $\omega \to 0: I_C \to 0; I_L \to \infty$. $\omega \to \infty: I_C \to \infty; I_L \to 0$.

At low frequencies, the current is not changing much so the inductor's back-emf doesn't "resist." This allows the current to pass fairly freely. However, the current in the capacitor goes to zero because it tends to "fill up" over the slow period, making it less effective at passing charge.

At high frequency, the induced emf in the inductor resists the violent changes and passes little current. The capacitor never gets a chance to fill up so passes charge freely.

d) $\omega = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(2.0\text{ H})(0.50\times10^{-6}\ f)}} = 1000\text{ rad/sec} \Rightarrow f = 159\text{ Hz}$



e) $I = \sqrt{\left(\dfrac{V}{R}\right)^2 + (V\omega C - \dfrac{v}{\omega L})^2}$

$$= \sqrt{\left(\frac{100\text{V}}{200\ \Omega}\right)^2 + \left((100\text{ V})(1000\text{ s}^{-1})(0.50\times10^{-6}\text{ F}) - \frac{100\text{V}}{(1000\text{s}^{-1})(2.0\text{H})}\right)^2} = 0.50\text{A}$$

f) At resonance $I_L = I_C = V\omega C = (100\text{ v})(1000\text{ s}^{-1})(0.50\times10^{-6}\text{ F}) = 0.05\text{ A}$ and

$I_R = \dfrac{V}{R} = \dfrac{100\text{ V}}{200\ \Omega} = 0.50\text{ A}.$

31.54: a) Note that as $\omega \to \infty, \omega L \to \infty$ and $\dfrac{1}{\omega C} \to 0$. Thus, at high frequencies the current through R_1 is nearly zero and the power dissipated by the circuit is

$$P = \frac{V_{\text{rms}}^2}{R_2} = \frac{(240\text{ V})^2}{40.0\ \Omega} = 1.44\text{ kW}.$$

b) Now we let $\omega \to 0$, and so $\omega L \to 0$ and $\dfrac{1}{\omega C} \to \infty$. Thus, at low frequencies the current through R_2 is nearly zero and the power dissipated by the circuit is

$$P = \frac{V_{\text{rms}}^2}{R_1} = \frac{(240\text{ V})^2}{60.0\ \Omega} = 0.960\text{ kW}.$$

31.55: Connect the source, capacitor, resistor, and inductor in series.

31.56: a) $P_{av} = \dfrac{V_{rms}^2}{Z}\cos\phi \Rightarrow Z = \dfrac{V_{rms}^2 \cos\phi}{P_{av}} = \dfrac{(120\ \text{V})^2(0.560)}{(220\ \text{W})} = 36.7\ \Omega$

$$\Rightarrow R = Z\cos\phi = (36.7\ \Omega)(0.560) = 20.6\ \Omega.$$

b) $Z = \sqrt{R^2 + X_L{}^2} = X_L = \sqrt{Z^2 - R^2} = \sqrt{(36.7\ \Omega)^2 - (20.6\ \Omega)^2} = 30.4\ \Omega.$ But at $\phi = 0$ this is resonance, so the inductive and capacitive reactances equal each other. So:

$$X_C = \frac{1}{\omega C} \Rightarrow C = \frac{1}{\omega X_C} = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(50.0\ \text{Hz})(30.4\ \Omega)} = 1.05\times 10^{-4}\ F.$$

c) At resonance, $P = \dfrac{V^2}{R} = \dfrac{(120\ \text{V})^2}{20.6\ \Omega} = 699\ \text{W}.$

31.57: a) $\tan\phi = \dfrac{X_L - X_C}{R} \Rightarrow X_L = X_C + R\tan\phi.$

$$= 350\ \Omega + (180\ \Omega)\tan(-54°) = 102\ \Omega.$$

b) $P_{av} = I_{rms}^2 R \Rightarrow I_{rms} = \sqrt{\dfrac{P_{av}}{R}} = \sqrt{\dfrac{(140\ \text{W})}{(180\ \Omega)}} = 0.882\ \text{A}.$

c) $V_{rms} = I_{rms}Z = I_{rms}\sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow V_{rms} = (0.882\ \text{A})\sqrt{(180\ \Omega)^2 + (102\ \Omega - 350\ \Omega)^2} = 270\ \text{V}.$$

31.58: a) For $\omega = 800\,\text{rad/s}$, $Z = \sqrt{R^2 + (\omega \text{L} - 1/\omega \text{C})^2}$

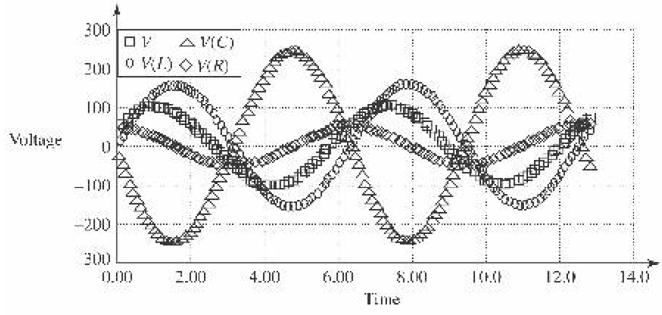
$$\Rightarrow Z = \sqrt{(500\ \Omega)^2 + ((800\ \text{rad/s})(2.0\ \text{H}) - 1/((800\ \text{rad/s})(5.0\times 10^{-7}\ \text{F})))^2} = 1030\ \Omega$$

$$\Rightarrow I = \frac{V}{Z} = \frac{100\ \text{V}}{1030\,\Omega} = 0.0971\ \text{A} \Rightarrow \text{V}_R = IR = (0.0971\ \text{A})(500\ \Omega) = 48.6\ \text{V}.$$

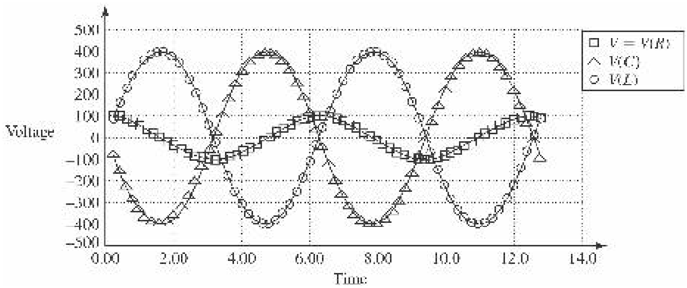
$$\text{V}_C = \frac{1}{\omega C} = \frac{0.0971\ \text{A}}{(800\ \text{rad/s})(5.0\times 10^{-7}\ \text{F})} = 243\ \text{V}.$$

$$\text{V}_L = I\omega L = (0.0971\ \text{A})(800\ \text{rad/s})(2.00\ \text{H}) = 155\ \text{V}.$$

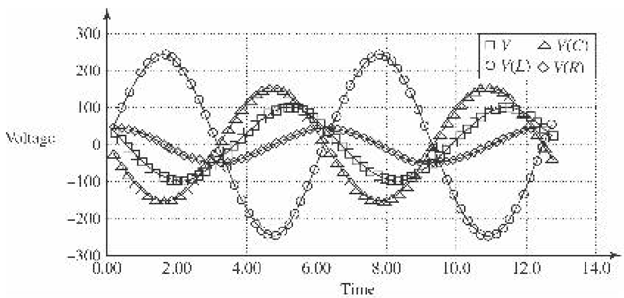
Also note $\phi = \arctan\left(\dfrac{\omega L - 1/(\omega C)}{R}\right) = -60.9°.$



b) Repeating exactly the same calculations as above for
$\omega = 1000\ \text{rad/s}: Z = R = 500\ \Omega; \phi = 0.; I = 0.200\ \text{A}; V_R = V = 100\ \text{V}; V_C = V_L = 400\ \text{V}.$



c) Repeating exactly the same calculations as part (a) for
$\omega = 1250\ \text{rad/s}: Z = R = 1030\ \Omega; \phi = +60.9°; I = 0.0971\ A; V_R = 48.6\ V; V_C = 155\ V; V_L = 243\ \text{V}$



31.59: a) $V_C = IX_C \Rightarrow I = \dfrac{V_C}{X_C} = \dfrac{360\ \text{V}}{480\ \Omega} = 0.75\ \text{A}.$

b) $Z = \dfrac{V}{I} = \dfrac{120\ \text{V}}{0.75\ \text{A}} = 160\ \Omega.$

c) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\Rightarrow X_L = X_C \pm \sqrt{Z^2 - R^2} = 480\ \Omega \pm \sqrt{(160\ \Omega)^2 - (80\ \Omega)^2}$$
$$\Rightarrow X_L = 619\ \Omega \text{ or } 341\ \Omega$$

d) If $\omega < \omega_0$ then $X_C = \dfrac{1}{\omega C} > X_L = \omega L$. For us, $X_L = 341\ \Omega$ if $\omega < \omega_0$.

31.60: We want $P_{av}(\omega_1) =$ maximum, $P_{av}(\omega_2) = 0.01\ P_{av}(\omega_1)$. Maximum power implies

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega_0^2} = \frac{1}{(1.0\times10^{-6}\ \text{H})[2\pi(94.1\times10^6\ \text{Hz})]^2} = 2.86\times10^{-12}\ \text{F}.$$

$$P_{av}(\omega_2) = 0.01 P_{av}(\omega_1) \Rightarrow \frac{V^2 R/2}{R^2 + (\omega L - 1/\omega C)^2} = \frac{1}{100}\left(\frac{V^2}{2R}\right)$$

$$\Rightarrow 100\ R^2 = R^2 + (\omega L - 1/\omega C)^2 \Rightarrow R = \sqrt{\frac{(\omega L - 1/\omega C)^2}{99}} = \frac{(\omega L - 1/\omega C)}{\sqrt{99}}$$

$$\Rightarrow R = \frac{1}{\sqrt{99}}\left(2\pi(94.0\times10^6\ \text{Hz})(1.00\times10^{-6}\ \text{H}) - \frac{1}{2\pi(94.0\times10^6\ \text{Hz})(2.86\times10\right.$$

$$\Rightarrow R = 0.126\ \Omega$$

This answer is very sensitive to the capacitance so you may have to carry the first part of the problem out to more significant figures.

31.61: The average current is zero because the current is symmetrical above and below the axis. We must calculate the rms-current:

$$I(t) = \frac{2I_0 t}{\tau} \Rightarrow I^2(t) = \frac{4I_0^2 t^2}{\tau^2} \Rightarrow \int_0^{\tau/2} I^2(t)dt = \frac{4I_0^2}{\tau^2}\left[\frac{t^3}{3}\right]_0^{\tau/2} = \frac{I_0^2\tau}{6}.$$

$$\Rightarrow \langle I^2 \rangle = \left(\frac{I_0^2\tau}{6}\right) \Big/ \left(\frac{\tau}{2}\right) = \frac{I_0^2}{3} \Rightarrow I_{rms} = \sqrt{\frac{I_0^2}{3}} = \frac{I_0}{\sqrt{3}}.$$

31.62: a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{(1.80\ \text{H})(9.00\times10^{-7}\ \text{F})}} = 786\ \text{rad/s}.$

b) $Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$

$$\Rightarrow Z = \sqrt{(300\ \Omega)^2 + ((786\ \text{rad/s})(1.80\ \text{H}) - 1/((786\ \text{rad/s})(9.00\times10^{-7}\ \text{F})))^2} = 300\ \Omega$$

$$\Rightarrow I_{\text{rms}_0} = \frac{V_{\text{rms}}}{Z} = \frac{60\ \text{V}}{300\ \Omega} = 0.200\ \text{A}.$$

c) We want

$$I = \frac{1}{2} I_{\text{rms}_0} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \Rightarrow R^2 + (\omega L - 1/\omega C)^2 = \frac{4V_{\text{rms}}^2}{I_{\text{rms}_0}^2}$$

$$\Rightarrow \omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} + R^2 - \frac{4V_{\text{rms}}^2}{I_{\text{rms}_0}^2} = 0$$

$$\Rightarrow (\omega^2)^2 L^2 + \omega^2 \left(R^2 - \frac{2L}{C} - \frac{4V_{\text{rms}}^2}{I_{\text{rms}_0}^2} \right) + \frac{1}{C^2} = 0.$$

Substituting in the values for this problem, the equation becomes: $(\omega^2)^2(3.24) +$ $\omega^2(-4.27 \times 10^6) + 1.23 \times 10^{12} = 0.$

Solving this quadratic equation in ω^2 we find $\omega^2 = 8.90 \times 10^5 \text{ rad}^2/\text{s}^2$ or $4.28 \times 10^5 \text{ rad}^2/\text{s}^2 \Rightarrow \omega = 943 \text{ rad/s}$ or 654 rad/s.

d) (i) $R = 300\ \Omega, I_{\text{rms}_0} = 0.200, |\omega_1 - \omega_2| = 289 \text{ rad/sec.}$ ii) $R = 30\ \Omega, I_{\text{rms}_0} = 2\text{A},$ $|\omega_1 - \omega_2| = 28 \text{ rad/sec.}$ (iii) $R = 3\ \Omega, I_{\text{rms}_0} = 20\text{A}, |\omega_1 - \omega_2| = 288.$

Width gets smaller as R gets smaller; I_{rms_0} gets larger as R gets smaller.

31.63: a) $I_0 = \frac{V}{Z} = \frac{V}{R}$ at resonance since $X_L = X_C$.

b) $\omega = \omega_0 + \Delta\omega$ is small compared to ω_0.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}.$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{1}{\omega^2 C^2}(\omega^2 LC - 1)^2.$$

$$\omega_0^2 = \frac{1}{LC} \text{ so } C^2 = \frac{1}{L^2 \omega_0^4}. \text{ Thus } \frac{1}{\omega^2 C^2} = \frac{L^2 \omega_0^4}{(\omega_0^2 + 2\omega_0 \Delta\omega + \Delta\omega^2)}$$

but $\Delta\omega^2$ is very small so

$$\frac{1}{\omega^2 C^2} \approx \frac{L^2 \omega_0^4}{(\omega_0^2 + 2\omega_0 \Delta\omega)} = \frac{L^2 \omega_0^2}{\left(1 + \frac{2\Delta\omega}{\omega_0}\right)} \approx L^2 \omega_0^2 \left(1 - \frac{2\Delta\omega}{\omega_0}\right).$$

$$\omega^2 LC - 1 = (\omega_0^2 + 2\omega_0 \Delta\omega + \Delta\omega^2)\left(\frac{1}{\omega_0^2}\right) - 1 = 1 + 2\frac{\Delta\omega}{\omega_0} + \frac{\Delta\omega^2}{\omega_0^2} - 1 = \frac{2\Delta\omega}{\omega_0} + \frac{\Delta\omega^2}{\omega_0{}^2}.$$

Again, $\Delta\omega^3$ is very small compared to $\omega_0{}^2$, so $\omega^2 LC - 1 \approx \frac{2\Delta\omega}{\omega_0}$.

Putting this together gives

$$\left(\omega L - \frac{1}{\omega C}\right)^2 \cong L^2 \omega_0{}^2 \left(1 - \frac{2\Delta\omega}{\omega_0}\right)\left(\frac{2\Delta\omega}{\omega_0}\right)^2 = 4L^2 \Delta\omega^2 - \frac{8L^2 \Delta\omega^3}{\omega_0}.$$

But $\Delta\omega^3$ is *much* smaller than ω_0. Finally

$$\left(\omega L - \frac{1}{\omega_c}\right)^2 \approx 4L^2\Delta\omega^2, \text{ so } Z \cong \sqrt{R^2 + 4L^2\Delta\omega^2}.$$

c) $I = \frac{1}{2}I_0 \Rightarrow \frac{V}{Z} = \frac{1}{2}\frac{V}{R}$ or $Z^2 = (2R)^2$.

$$R^2 + 4L^2\Delta\omega^2 = 4R^2 \Rightarrow \Delta\omega = \pm\sqrt{\frac{3R^2}{4L^2}} = \pm\sqrt{\frac{3}{4}}\frac{R}{L}.$$

$$\omega = \omega_0 \pm \sqrt{\frac{3}{4}}\frac{R}{L} \text{ but } \sqrt{\frac{3}{4}}\frac{R}{L} << \frac{1}{\sqrt{LC}} \Rightarrow R << \sqrt{\frac{4L}{3C}}.$$

d) $|\omega_1 - \omega_2| = 2\Delta\omega = \sqrt{3}\frac{R}{L}$. As R increases so does the width.

e) (i) $I_0 = \frac{120\ V}{15\ \Omega} = 8\ \text{A}; \omega_0 = \frac{1}{\sqrt{(2.50\ \text{H})(0.400\times10^{-6}\ \text{F})}} = 1000\ \text{rad/sec};$

$|\omega_1 - \omega_2| = \sqrt{3}\frac{15\ \Omega}{2.50\ \text{H}} = 10.4\ \text{rad/sec}.$ (ii) $I_0 = 80\text{A}$, $\omega_0 = 1000\ \text{rad/s}$, $|\omega_1 - \omega_2| = 1.04\ \text{rad/sec}.$

31.64: a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega c}\right)^2}}$ at resonance $\omega L = \frac{1}{\omega C}$. So $I_{\max} = \frac{V}{R}$.

b) $V_{C_{\max}} = I_{\max}X_C = \frac{V}{R\omega_0 C} = \frac{V}{R}\sqrt{\frac{L}{C}}.$

c) $V_{L_{\max}} = I_{\max}X_L = \frac{V}{R}\omega_0 L = \frac{V}{R}\sqrt{\frac{L}{C}}.$

d) $U_{C_{\max}} = \frac{1}{2}CV_{C_{\max}}^2 = \frac{1}{2}C\frac{V^2}{R^2}\frac{L}{C} = \frac{1}{2}L\frac{V^2}{R^2}.$

e) $U_{L_{\max}} = \frac{1}{2}LI_{\max}^2 = \frac{1}{2}L\frac{V^2}{R^2}.$

31.65: $\omega = \frac{\omega_0}{2}.$

a) $I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\frac{\omega_0 L}{2} - \frac{2}{\omega_0 C}\right)^2}} = \frac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

b) $V_{C_{\max}} = IX_C = \frac{2}{\omega_0 C}\frac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}} = \sqrt{\frac{L}{C}}\frac{2V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

c) $V_{L_{max}} = IX_L = \dfrac{\omega_0 L}{2} \dfrac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}} = \sqrt{\dfrac{L}{C}} \dfrac{V/2}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

d) $U_{C_{max}} = \dfrac{1}{2} C V_{C_{max}}^2 = \dfrac{2LV^2}{R^2 + \frac{9}{4}\frac{L}{C}}.$

e) $U_{L_{max}} = \dfrac{1}{2} L I^2 = \dfrac{1}{2} \dfrac{LV^2}{R^2 + \frac{9}{2}\frac{L}{C}}.$

31.66: $\omega = 2\omega_0$.

a) $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (2\omega_0 L - \frac{1}{2\omega_0 C})^2}} = \dfrac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

b) $V_{C_{max}} = IX_C = \dfrac{1}{2\omega_0 C} \dfrac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}} = \sqrt{\dfrac{L}{C}} \dfrac{V/2}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

c) $V_{L_{max}} = IX_L = 2\omega_0 L \dfrac{V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}} = \sqrt{\dfrac{L}{C}} \dfrac{2V}{\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

d) $U_{C_{max}} = \dfrac{1}{2} C V_{C_{max}}^2 = \dfrac{LV^2}{8\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

e) $U_{L_{max}} = \dfrac{1}{2} L I^2 = \dfrac{LV^2}{2\sqrt{R^2 + \frac{9}{4}\frac{L}{C}}}.$

31.67: a) $p_R = i^2 R = I^2 \cos^2(\omega t) R = V_R I \cos^2(\omega t)) = \dfrac{1}{2} V_R I (1 + \cos(2\omega t))$

$$\Rightarrow P_{av}(R) = \frac{1}{T}\int_0^T p_R dt = \frac{V_R I}{2T}\int_0^T (1 + \cos(2\omega t))dt = \frac{V_R I}{2T}[t]_0^T = \frac{1}{2} V_R I.$$

b) $p_L = Li\dfrac{di}{dt} = -\omega L I^2 \cos(\omega t)\sin(\omega t) = -\dfrac{1}{2} V_L I \sin(2\omega t).$

But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{av}(L) = 0.$

c) $p_c = \dfrac{dU}{dt} = \dfrac{d}{dt}\left(\dfrac{q^2}{2C}\right) = \dfrac{q}{C} i = v_C i = V_C I \sin(\omega t)\cos(\omega t) = \dfrac{1}{2} V_C I \sin(2\omega t).$

But $\int_0^T \sin(2\omega t)dt = 0 \Rightarrow P_{av}(C) = 0.$

d) $p = p_R + p_L + p_c = V_R I\cos^2(\omega t) - \dfrac{1}{2}V_L I\sin(2\omega t) + \dfrac{1}{2}V_C I\sin(2\omega t)$

$\Rightarrow P = I\cos(\omega t)(V_R\cos(\omega t) - V_L\sin(\omega t) + V_C\sin(\omega t)).$

But $\cos\phi = \dfrac{V_R}{V}$ and $\sin\phi = \dfrac{V_L - V_C}{V} \Rightarrow p = VI\cos(\omega t)(\cos\phi\cos(\omega t) - \sin\phi\sin(\omega t)),$ at any instant of time.

31.68: a) $V_R =$ maximum when $V_C = V_L \Rightarrow \omega = \omega_0 = \dfrac{1}{\sqrt{LC}}.$

b) From Problem (31.48a), $V_L =$ maximum when $\dfrac{dV_L}{d\omega} = 0.$ Therefore:

$$\frac{dV_L}{d\omega} = 0 = \frac{d}{d\omega}\left(\frac{V\omega L}{\sqrt{R^2 + \omega L - 1/\omega C)^2}}\right)$$

$$\Rightarrow 0 = \frac{VL}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V\omega^2 L(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}$$

$$\Rightarrow R^2 + (\omega L - 1/\omega C)^2 = \omega^2(L^2 - 1/\omega^4 C^2)$$

$$\Rightarrow R^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} = -\frac{1}{\omega^2 C^2} \Rightarrow \frac{1}{\omega^2} = LC - \frac{R^2 C^2}{2} \Rightarrow \omega = \frac{1}{\sqrt{LC - R^2 C^2/2}}.$$

c) From Problem (31.48b), $V_C =$ maximum when $\dfrac{dV_C}{d\omega} = 0.$ Therefore:

$$\frac{dV_C}{d\omega} = 0 = \frac{d}{d\omega}\left(\frac{V}{\omega C\sqrt{R^2 + (\omega L - 1/\omega C)^2}}\right)$$

$$\Rightarrow 0 = -\frac{V}{\omega^2 C\sqrt{R^2 + (\omega L - 1/\omega C)^2}} - \frac{V(L - 1/\omega^2 C)(L + 1/\omega^2 C)}{C(R^2 + (\omega L - 1/\omega C)^2)^{3/2}}$$

$$\Rightarrow R^2 + (\omega L - 1/\omega C)^2 = -\omega^2(L^2 - 1/\omega^4 C^2)$$

$$\Rightarrow R^2 + \omega^2 L^2 - \frac{2L}{C} = -\omega^2 L^2 \Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{2L^2}}.$$

31.69: a) From the current phasors we know that

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\Rightarrow Z = \sqrt{(400\ \Omega)^2 + \left((1000\ \text{rad/s})(0.50\ \text{H}) - \frac{1}{(1000\ \text{rad/s})(1.25\times10^{-6}\ \text{F})}\right)^2} = 500\ \Omega.$$

$$\Rightarrow I = \frac{V}{Z} = \frac{200\ \text{V}}{500\ \Omega} = 0.400\ \text{A}.$$

b) $\phi = \arctan\left(\dfrac{\omega L - 1/(\omega C)}{R}\right)$

$$\Rightarrow \phi = \arctan\left(\frac{(1000\text{ rad/s})(0.500\text{ H}) - 1/(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}{400\ \Omega}\right) = +36.9°$$

c) $Z_{\text{cpx}} = R + i\left(\omega L - \dfrac{1}{\omega C}\right)$

$$\Rightarrow Z_{\text{cpx}} = 400\ \Omega - i\left((1000\text{ rad/s})(0.50\text{ H}) - \frac{1}{(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})}\right)$$

$$= 400\ \Omega - 300\ \Omega i$$

$$\Rightarrow Z = \sqrt{(400\ \Omega)^2 + (-300\ \Omega)^2} = 500\ \Omega$$

d) $I_{\text{cpx}} = \dfrac{V}{Z_{\text{cpx}}} = \dfrac{200\text{ V}}{(400-300i)\ \Omega} = \left(\dfrac{8+6i}{25}\right)\text{A}$

$$\Rightarrow I = \sqrt{\left(\frac{8+6i}{25}\right)\left(\frac{8-6i}{25}\right)} = 0.400\text{ A.}$$

e) $\tan\phi = \dfrac{\text{Im}(I_{\text{cpx}})}{\text{Re}(I_{(\text{cpx})}} = \dfrac{6/25}{8/25} = 0.75 \Rightarrow \phi = +36.9°.$

f) $V_{R_{\text{cpx}}} = I_{\text{cpx}}R = \left(\dfrac{8+6i}{25}\right)(400\ \Omega) = (128+96i)\text{V.}$

$$V_{L_{\text{cpx}}} = iI_{\text{cpx}}\omega L = i\left(\frac{8+6i}{25}\right)(1000\text{ rad/s})(0.500\text{ H}) = (-120+160i)\text{V.}$$

$$\text{V}_{Lcpx} = i\frac{I_{\text{cpx}}}{\omega C} = i\left(\frac{8+6i}{25}\right)\frac{1}{(1000\text{ rad/s})(1.25\times10^{-6}\text{ F})} = (+192-256i)\text{V}\cdot$$

g) $V_{\text{cpx}} = V_{R_{\text{cpx}}} + V_{L_{\text{cpx}}} + V_{L_{\text{cpx}}} = (128+96i)V + (-120+160i)\text{V}$
$+(192-256i)\text{V} = 200\text{ V.}$

32.1: a) $t = \dfrac{d}{c} = \dfrac{3.84\times10^{8}\text{ m}}{3.00\times10^{8}\text{ m/s}} = 1.28\text{ s.}$

b) Light travel time is:

$$8.61\text{ years} = (8.61\text{ years})\frac{(365\text{ days})}{(1\text{ year})}\frac{(24\text{ hours})}{(1\text{ day})}\frac{(3600\text{ s})}{(1\text{ hour})} = 2.72\times10^{8}\text{ s}$$

$$d = ct = (3.0\times10^{8}\text{ m/s})\,(2.72\times10^{8}\text{ s}) = 8.16\times10^{16}\text{ m} = 8.16\times10^{13}\text{ km.}$$

32.2: $d = c\Delta t = (3.0\times10^{8}\text{ m/s})\,(6.0\times10^{-7}\text{ s}) = 180\text{ m.}$

32.3: $\vec{B}(z,t) = B_{\max}\cos(kz-\omega t)\hat{j} = B_{\max}\cos\left(2\pi f\left(\dfrac{z}{c}-t\right)\right)\hat{j}$

$$\Rightarrow \vec{B}(z,t) = (5.80\times10^{-4}\text{ T})\cos\left(2\pi(6.10\times10^{14}\text{ Hz})\left(\frac{z}{(3.00\times10^{8}\text{ m/s})} - t\right)\right)\hat{j}$$

$$\Rightarrow \vec{B}(z,t) = (5.80\times10^{-4}\ \text{T})\cos((1.28\times10^{7}\ \text{m}^{-1})z - (3.83\times10^{15}\ \text{rad/s})t)\hat{j}.$$

$$\vec{E}(z,t) = (B_y(z,t)\hat{j})\times(c\hat{k})$$

$$\Rightarrow \vec{E}(z,t) = (1.74\times10^{5}\ \text{V/m})\cos((1.28\times10^{7}\ \text{m}^{-1})z - (3.83\times10^{15}\ \text{rad/s})t)\hat{i}.$$

32.4: a) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{4.35\times10^{-7}\ \text{m}} = 6.90\times10^{14}\ \text{Hz}.$

b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{2.70\times10^{-3}\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}} = 9.00\times10^{-12}\ \text{T}.$

c) The electric field is in the x-direction, and the wave is propagating in the $-z$-direction. So the magnetic field is in the $-y$-direction, since $\vec{S}\propto\vec{E}\times\vec{B}$. Thus:

$$\vec{E}(z,t) = E_{\max}\cos(kz+\omega t)\hat{i} = E_{\max}\cos\left(2\pi f\left(\frac{z}{c}+t\right)\right)\hat{i}$$

$$\Rightarrow \vec{E}(z,t) = (2.70\times10^{-3}\ \text{V/m})\cos\left(2\pi(6.90\times10^{14}\ \text{Hz})\left(t+\frac{z}{3.00\times10^{8}\ \text{m/s}}\right)\right)\hat{i}$$

$$\Rightarrow \vec{E}(z,t) = (2.70\times10^{-3}\ \text{V/m})\cos((1.45\times10^{7}\ \text{m}^{-1})z + (4.34\times10^{15}\ \text{rad/s})t)\hat{i}.$$

And $\vec{B}(z,t) = \dfrac{-E(z,t)}{c}\hat{j} = -(9.00\times10^{-12}\ \text{T})\cos((1.45\times10^{7}\ \text{m}^{-1})z + (4.34\times10^{15}\text{rad/s})t)\hat{j}.$

32.5: a) $+y$ direction.

b) $\omega = 2\pi f = \dfrac{2\pi c}{\lambda} \Rightarrow \lambda = \dfrac{2\pi c}{\omega} = \dfrac{2\pi(3.00\times10^{8}\ \text{m/s})}{(2.65\times10^{12}\ \text{rad/s})} = 7.11\times10^{-4}\ \text{m}.$

c) Since the electric field is in the $-z$-direction, and the wave is propagating in the $+y$-direction, then the magnetic field is in the $-x$-direction $(\vec{S}\propto\vec{E}\times\vec{B})$. So:

$$\vec{B}(y,t) = \frac{-E(y,t)}{c}\hat{i} = \frac{-E_0}{c}\sin(ky-\omega t)\hat{i} = \frac{-E_0}{c}\sin\left(\frac{\omega}{c}y-\omega t\right)\hat{i}$$

$$\Rightarrow \vec{B}(y,t) = -\left(\frac{3.10\times10^{5}\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}}\right)\sin\left(\frac{(2.65\times10^{12}\ \text{rad/s})}{(3.00\times10^{8}\ \text{m/s})}y - (2.65\times10^{12}\ \text{rad/s})t\right)\hat{i}$$

$$\Rightarrow \vec{B}(y,t) = -(1.03\times10^{-3}\ \text{T})\sin((8.83\times10^{3}\ \text{m})y - (2.65\times10^{12}\ \text{rad/s})t)\hat{i}.$$

32.6: a) $-x$ direction.

b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi f}{c} \Rightarrow f = \dfrac{kc}{2\pi} = \dfrac{(1.38\times10^{4}\ \text{rad/m})(3.0\times10^{8}\ \text{m/s})}{2\pi} = 6.59\times10^{11}\ \text{Hz}.$

c) Since the magnetic field is in the $+y$-direction, and the wave is propagating in the $-x$-direction, then the electric field is in the $+z$-direction $(\vec{S}\propto\vec{E}\times\vec{B})$. So:

$$\vec{E}(x,t) = +cB(x,t)\hat{k} = +cB_0 \sin(kx + 2\pi f)\hat{k}$$

$$\Rightarrow \vec{E}(x,t) = +(c(3.25\times10^{-9}\ \text{T}))\sin((1.38\times10^{4}\ \text{rad/m})x + (4.14\times10^{12}\ \text{rad/s})t)\hat{k}$$

$$\Rightarrow \vec{E}(y,t) = +(2.48\ \text{V/m})\sin((1.38\times10^{4}\ \text{rad/m})x + (4.14\times10^{12}\ \text{rad/s})t)\hat{k}.$$

32.7: a) $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{8.30\times10^{5}\ \text{Hz}} = 361\ \text{m}.$

b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{361\ \text{m}} = 0.0174\ \text{m}^{-1}$

c) $\omega = 2\pi f = 2\pi(8.30\times10^{5}\ \text{Hz}) = 5.21\times10^{6}\ \text{rad/s}.$

$E_{\max} = cB_{\max} = (3.00\times10^{8}\ \text{m/s})\ (4.82\times10^{-11}\ \text{T}) = 0.0145\ \text{V/m}.$

32.8: $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{3.85\times10^{-3}\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}} = 1.28\times10^{-11}\ \text{T}.$

So $\dfrac{B_{\max}}{B_{\text{earth}}} = \dfrac{1.28\times10^{-11}\ \text{T}}{5\times10^{-5}\ \text{T}} = 2.56\times10^{-7}$, and thus $B_{\max}$ is much weaker than B_{earth}.

32.9: $E = vB = \dfrac{B}{\sqrt{\varepsilon\mu}} = \dfrac{B}{\sqrt{K_E\varepsilon_0 K_B\mu_0}} = \dfrac{cB}{\sqrt{K_E K_B}}$

$$\Rightarrow E = \frac{(3.00\times10^{8}\ \text{m/s})\ (3.80\times10^{-9}\ \text{T})}{\sqrt{(1.74)\ (1.23)}} = 0.779\ \text{V/m}.$$

32.10: a) $v = \dfrac{c}{\sqrt{K_E K_B}} = \dfrac{(3.00\times10^{8}\text{m/s})}{\sqrt{(3.64)\ (5.18)}} = 6.91\times10^{7}\ \text{m/s}.$

b) $\lambda = \dfrac{v}{f} = \dfrac{6.91\times10^{7}\ \text{m/s}}{65.0\ \text{Hz}} = 1.06\times10^{6}\ \text{m}.$

c) $B = \dfrac{E}{v} = \dfrac{7.20\times10^{-3}\ \text{V/m}}{6.91\times10^{7}\ \text{m/s}} = 1.04\times10^{-10}\ \text{T}.$

d) $I = \dfrac{EB}{2K_B\mu_0} = \dfrac{(7.20\times10^{-3}\ \text{V/m})(1.04\times10^{-10}\ \text{T})}{2(5.18)\mu_0} = 5.75\times10^{-8}\ \text{W/m}^2.$

32.11: a) $\lambda = \dfrac{v}{f} = \dfrac{2.17\times10^{8}\ \text{m/s}}{5.70\times10^{14}\ \text{Hz}} = 3.81\times10^{-7}\ \text{m}.$

b) $\lambda_0 = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{5.70\times10^{14}\ \text{Hz}} = 5.26\times10^{-7}\ \text{m}.$

c) $n = \dfrac{c}{v} = \dfrac{3.00\times10^{8}\ \text{m/s}}{2.17\times10^{8}\ \text{m/s}} = 1.38.$

d) $v = \dfrac{c}{\sqrt{K_E}} \Rightarrow K_E = \dfrac{c^2}{v^2} = n^2 = (1.38)^2 = 1.90.$

32.12: a) $v = f\lambda = (3.80\times10^{7}\ \text{Hz})(6.15\ \text{m}) = 2.34\times10^{8}\ \text{m/s}.$

b) $K_E = \dfrac{c^2}{v^2} = \dfrac{(3.00\times10^{8}\ \text{m/s})^2}{(2.34\times10^{8}\ \text{m/s})^2} = 1.64.$

32.13: a) $I = \dfrac{1}{2}\varepsilon_0 c E_{\text{max}}^2$; $E_{\text{max}} = 0.090\ \text{V/m}$, so $I = 1.1\times10^{-5}\ \text{W/m}^2$

b) $E_{\text{max}} = cB_{\text{max}}$ so $B_{\text{max}} = E_{\text{max}}/c = 3.0\times10^{-10}\ \text{T}$

c) $P_{\text{av}} = I(4\pi r^2) = (1.075\times10^{-5}\ \text{W/m}^2)(4\pi)(2.5\times10^{3}\ \text{m})^2 = 840\ \text{W}$

d) Calculation in part (c) assumes that the transmitter emits uniformly in all directions.

32.14: The intensity of the electromagnetic wave is given by Eqn. 32.29: $I = \frac{1}{2}\varepsilon_0 c E_{\text{max}}^2 = \varepsilon_0 c E_{\text{rms}}^2$. Thus the total energy passing through a window of area A during a time t is

$\varepsilon_0 c E_{\text{rms}}^2 At = (8.85\times10^{-12}\text{F/m})\,(3.00\times10^{8}\ \text{m/s})\,(0.0200\ \text{V/m})^2(0.500\ \text{m}^2)(30.0\ \text{s}) = 15.9\mu\text{J}$

32.15: $P_{\text{av}} = I(4\pi r^2) = (5.0\times10^{3}\ \text{W/m}^2)(4\pi)(2.0\times10^{10}\text{m})^2 = 2.5\times10^{25}\ \text{J}$

32.16: a) The average power from the beam is

$$P = IA = (0.800\ \text{W/m}^2)(3.0\times10^{-4}\ \text{m}^2) = 2.4\times10^{-4}\,\text{W}$$

b) We have, using Eq. 32.29, $I = \frac{1}{2}\varepsilon_0 c E_{\text{max}}^2 = \varepsilon_0 c E_{\text{rms}}^2$. Thus,

$$E_{\text{rms}} = \sqrt{\frac{I}{\varepsilon_0 c}} = \sqrt{\frac{0.800\ \text{W/m}^2}{(8.85\times10^{-12}\ \text{F/m})(3.00\times10^{8}\ \text{m/s})}} = 17.4\ \text{V/m}$$

32.17: $p_{\text{rad}} = I/c$ so $I = cp_{\text{rad}} = 2.70\times10^{3}\ \text{W/m}^2$

Then $P_{av} = I(4\pi r^2) = (2.70\times10^{3}\ \text{W/m}^2)\,(4\pi)\,(5.0\ \text{m})^2 = 8.5\times10^{5}\ \text{W}$

32.18: a) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^{8}\ \text{m/s}}{0.354\ \text{m}} = 8.47\times10^{8}\ \text{Hz}.$

b) $B_{\text{max}} = \dfrac{E_{\text{max}}}{c} = \dfrac{0.0540\ \text{V/m}}{3.00\times10^{8}\ \text{m/s}} = 1.80\times10^{-10}\,\text{T}.$

c) $I = S_{av} = \dfrac{EB}{2\mu_0} = \dfrac{(0.0540\ \text{V/m})\,(1.80\times10^{-10}\,\text{T})}{2\mu_0} = 3.87\times10^{-6}\ \text{W/m}^2.$

32.19: $P = S_{av}A = \dfrac{E_{max}^2}{2c\mu_0}\cdot(4\pi r^2) \Rightarrow E_{max} = \sqrt{\dfrac{Pc\mu_0}{2\pi r^2}}$

$$\Rightarrow E_{max} = \sqrt{\frac{(60.0\ \text{W})\,(3.00\times 10^8\ \text{m/s})\mu_0}{2\pi(5.00\ \text{m})^2}} = 12.0\ \text{V/m}.$$

$$\Rightarrow B_{max} = \frac{E_{max}}{c} = \frac{12.0\ \text{V/m}}{3.00\times 10^8\ \text{m/s}} = 4.00\times 10^{-8}\,\text{T}.$$

32.20: a) The electric field is in the $-y$-direction, and the magnetic filed is in the $+z$-direction, so $\hat{\boldsymbol{S}} = \hat{\boldsymbol{E}}\times\hat{\boldsymbol{B}} = (-\hat{\boldsymbol{j}})\times\hat{\boldsymbol{k}} = -\hat{\boldsymbol{i}}$. That is, the Poynting vector is in the $-x$-direction.

b) $S(x,t) = \dfrac{E(x,t)B(x,t)}{\mu_0} = -\dfrac{E_{max}B_{max}}{\mu_0}\cos\,(kx+\omega t)$

$= -\dfrac{E_{max}B_{max}}{2\mu_0}(1+\cos(2(\omega t + kx))).$

But over one period, the cosine function averages to zero, so we have:

$$|S_{av}| = \frac{E_{max}B_{max}}{2\mu_0}.$$

32.21: a) The momentum density $\dfrac{dp}{dV} = \dfrac{S_{av}}{c^2} = \dfrac{780\ \text{W/m}^2}{(3.0\times 10^8\ \text{m/s})^2} = 8.7\times 10^{-15}\ \text{kg/m}^2\cdot\text{s}.$

b) The momentum flow rate $\dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{S_{av}}{c} = \dfrac{780\ \text{W/m}^2}{3.0\times 10^8\ \text{m/s}} = 2.6\times 10^{-6}\,\text{Pa}.$

32.22: a) Absorbed light: $p_{rad} = \dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{S_{av}}{c} = \dfrac{2500\ \text{W/m}^2}{3.0\times 10^8\ \text{m/s}} = 8.33\times 10^{-6}\ \text{Pa}.$

$$\Rightarrow p_{rad} = \frac{8.33\times 10^{-6}\ \text{Pa}}{1.013\times 10^5\ \text{Pa/atm}} = 8.23\times 10^{-11}\,\text{atm}$$

b) Reflecting light: $p_{rad} = \dfrac{1}{A}\dfrac{dp}{dt} = \dfrac{2S_{av}}{c} = \dfrac{2(2500\ \text{W/m}^2)}{3.0\times 10^8\ \text{m/s}} = 1.67\times 10^{-5}\ \text{Pa}.$

$\Rightarrow p_{rad} = \dfrac{1.67\times 10^{-5}\ \text{Pa}}{1.013\times 10^5\ \text{Pa/atm}} = 1.65\times 10^{-10}\,\text{atm}$. The factor of 2 arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

c) The momentum density $\dfrac{dp}{dV} = \dfrac{S_{av}}{c^2} = \dfrac{2500\ \text{W/m}^2}{(3.0\times 10^8\ \text{m/s})^2} = 2.78\times 10^{-14}\ \text{kg/m}^2\cdot\text{s}.$

32.23: $S = \dfrac{\varepsilon_0}{\sqrt{\varepsilon_0 \mu_0}} E^2 = \sqrt{\dfrac{\varepsilon_0}{\mu_0}} E^2 = \sqrt{\dfrac{\varepsilon_0}{\mu_0}} Ec\dfrac{E}{c} = c\sqrt{\dfrac{\varepsilon_0}{\mu_0}} EB = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} \sqrt{\dfrac{\varepsilon_0}{\mu_0}} EB =$

$\dfrac{EB}{\mu_0} = \dfrac{E^2}{\mu_0 c} = \varepsilon_0 c E^2.$

32.24: Recall that $\vec{S} \propto \vec{E} \times \vec{B}$, so :

a) $\hat{S} = \hat{i} \times (-\hat{j}) = -\hat{k}.$

b) $\hat{S} = \hat{j} \times \hat{i} = -\hat{k}.$

c) $\hat{S} = (-\hat{k}) \times (-\hat{i}) = \hat{j}.$

d) $\hat{S} = \hat{i} \times (-\hat{k}) = \hat{j}.$

32.25: $B_{\max} = E_{\max} / c = 1.33 \times 10^{-8}\ \text{T}$

$\vec{E} \times \vec{B}$ is in the direction of propagation. For $\vec{E}$ in the $+x$-direction, $\vec{E} \times \vec{B}$ is in the $+z$-direction when $\vec{B}$ is in the $+y$-direction.

32.26: a) $\Delta x = \dfrac{\lambda}{2} = \dfrac{c}{2f} = \dfrac{3.00 \times 10^8\ \text{m/s}}{2\,(75.0 \times 10^6\ \text{Hz})} = 2.00\ \text{m}.$

b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength $= \dfrac{\lambda}{4} = \dfrac{\Delta x}{2} = \dfrac{2.00\ \text{m}}{2} = 1.00\ \text{m}.$

32.27: a) The node-antinode distance $= \dfrac{\lambda}{4} = \dfrac{v}{4f} = \dfrac{2.10 \times 10^8\ \text{m/s}}{4(1.20 \times 10^{10}\ \text{Hz})} = 4.38 \times 10^{-3}\text{m}.$

b) The distance between the electric and magnetic antinodes is one-quarter of a wavelength $= \dfrac{\lambda}{4} = \dfrac{v}{4f} = \dfrac{2.10 \times 10^8\ \text{m/s}}{4\,(1.20 \times 10^{10}\ \text{Hz})} = 4.38 \times 10^{-3}\ \text{m}.$

c) The distance between the electric and magnetic nodes is also one-quarter of a wavelength =

$\dfrac{\lambda}{4} = \dfrac{v}{4f} = \dfrac{2.10 \times 10^8\ \text{m/s}}{4(1.20 \times 10^{10}\ \text{Hz})} = 4.38 \times 10^{-3}\ \text{m}.$

32.28: $\Delta x_{\text{nodes}} = \dfrac{\lambda}{2} = \dfrac{c}{2f} = \dfrac{3.00 \times 10^8\ \text{m/s}}{2(7.50 \times 10^8\ \text{Hz})} = 0.200\ \text{m} = 20.0\ \text{cm}.$ There must be nodes at the planes, which are 80.0 cm apart, and there are two nodes between the planes, each 20.0 cm from a plane. It is at 20 cm, 40 cm, and 60 cm that a point charge will remain at rest, since the electric fields there are zero.

32.29: a) $\Delta x = \frac{\lambda}{2} \Rightarrow \lambda = 2\Delta x = 2(3.55 \text{ mm}) = 7.10 \text{ mm}.$

b) $\Delta x_E = \Delta x_B = 3.55 \text{ mm}.$

c) $v = f\lambda = (2.20\times 10^{10} \text{ Hz})(7.10\times 10^{-3} \text{ m}) = 1.56\times 10^{8} \text{ m/s}.$

32.30: a) $\frac{\partial^2 E_y(x,t)}{\partial x^2} = \frac{\partial^2}{\partial x^2}(-2E_{\max}\sin kx \sin \omega t) = \frac{\partial}{\partial x}(-2kE_{\max}\cos kx \sin \omega t)$

$\Rightarrow \frac{\partial^2 E_y(x,t)}{\partial x^2} = 2k^2 E_{\max}\sin kx \sin \omega t = \frac{\omega^2}{c^2}2E_{\max}\sin kx \sin \omega t = \varepsilon_0\mu_0 \frac{\partial^2 E_y(x,t)}{\partial t^2}.$

Similarly: $\frac{\partial^2 B_z(x,t)}{\partial x^2} = \frac{\partial^2}{\partial x^2}(-2B_{\max}\cos kx \cos \omega t) = \frac{\partial}{\partial x}(+2kB_{\max}\sin kx \cos \omega t)$

$\Rightarrow \frac{\partial^2 B_z(x,t)}{\partial x^2} = 2k^2 B_{\max}\cos kx \cos \omega t = \frac{\omega^2}{c^2}2B_{\max}\cos kx \cos \omega t = \varepsilon_0\mu_0 \frac{\partial^2 B_z(x,t)}{\partial t^2}.$

b) $\frac{\partial E_y(x,t)}{\partial x} = \frac{\partial}{\partial x}(-2E_{\max}\sin kx \sin \omega t) = -2kE_{\max}\cos kx \sin \omega t$

$\Rightarrow \frac{\partial E_y(x,t)}{\partial x} = -\frac{\omega}{c}2E_{\max}\cos kx \sin \omega t = -\omega 2\frac{E_{\max}}{c}\cos kx \sin \omega t = -\omega 2B_{\max}\cos kx \sin \omega t.$

$\Rightarrow \frac{\partial E_y(x,t)}{\partial x} = +\frac{\partial}{\partial t}(2B_{\max}\cos kx \cos \omega t) = -\frac{\partial B_z(x,t)}{\partial t}.$

Similarly: $-\frac{\partial B_z(x,t)}{\partial x} = \frac{\partial}{\partial x}(+2B_{\max}\cos kx \cos \omega t) = -2kB_{\max}\sin kx \cos \omega t$

$\Rightarrow -\frac{\partial B_z(x,t)}{\partial x} = -\frac{\omega}{c}2B_{\max}\sin kx \cos \omega t = -\frac{\omega}{c^2}2cB_{\max}\sin kx \cos \omega t$

$\Rightarrow -\frac{\partial B_z(x,t)}{\partial x} = -\varepsilon_0\mu_0\omega 2E_{\max}\sin kx \cos \omega t = \varepsilon_0\mu_0\frac{\partial}{\partial t}(-2E_{\max}\sin kx \sin \omega t) = \varepsilon_0\mu_0\frac{\partial E_y(x,t)}{\partial t}.$

32.31: a) Gamma rays: $\lambda = \frac{c}{f} = \frac{3.00\times 10^{8} \text{ m/s}}{6.50\times 10^{21} \text{ Hz}} = 4.62\times 10^{-14} \text{ m} = 4.62\times 10^{-5} \text{ nm}.$

b) Green light: $\lambda = \frac{c}{f} = \frac{3.00\times 10^{8} \text{ m/s}}{5.75\times 10^{14} \text{ Hz}} = 5.22\times 10^{-7} \text{ m} = 522 \text{ nm}.$

32.32: a) $f = \frac{c}{\lambda} = \frac{3.0\times 10^{8} \text{ m/s}}{5000 \text{ m}} = 6.0\times 10^{4} \text{ Hz}.$

b) $f = \frac{c}{\lambda} = \frac{3.0\times 10^{8} \text{ m/s}}{5.0 \text{ m}} = 6.0\times 10^{7} \text{ Hz}.$

c) $f = \frac{c}{\lambda} = \frac{3.0\times 10^{8} \text{ m/s}}{5.0\times 10^{-6} \text{ m}} = 6.0\times 10^{13} \text{ Hz}.$

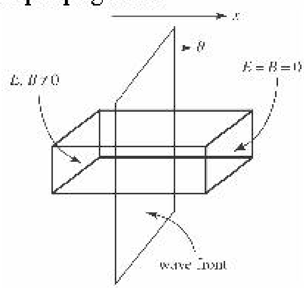
d) $f = \dfrac{c}{\lambda} = \dfrac{3.0 \times 10^{8}\ \text{m/s}}{5.0 \times 10^{-9}\ \text{m}} = 6.0 \times 10^{16}\ \text{Hz}.$

32.33: Using a Gaussian surface such that the front surface is ahead of the wave front (no electric or magnetic fields) and the back face is behind the wave front (as shown at right), we have:

$$\oint \vec{E} \cdot d\vec{A} = E_x A = \frac{Q_{\text{encl}}}{\varepsilon_0} = 0 \Rightarrow E_x = 0.$$

$$\oint \vec{B} \cdot d\vec{A} = B_x A = 0 \Rightarrow B_x = 0.$$

So the wave must be transverse, since there are no components of the electric or magnetic field in the direction of propagation.



32.34: Assume $\vec{E} = E_{\max}\hat{\jmath}\sin(kx - \omega t)$ *and* $\vec{B} = B_{\max}\hat{k}\sin(kx - \omega t + \phi)$, with $-\pi < \phi < \pi$.

Then Eq. (32.12) implies:

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} x \Rightarrow +kE_{\max}\cos(kx - \omega t) = +\omega B_{\max}\cos(kx - \omega t + \phi) \Rightarrow \phi = 0.$$

$$\Rightarrow kE_{\max} = \omega B_{\max} \Rightarrow E_{\max} = \frac{\omega}{k} B_{\max} = \frac{2\pi f}{2\pi / \lambda} B_{\max} = f\lambda B_{\max} = cB_{\max}.$$

Similarly for Eq.(32.14)

$$-\frac{\partial B_z}{\partial x} = \varepsilon_0 \mu_0 \frac{\partial E_y}{\partial t} \Rightarrow -kB_{\max}\cos(kx - \omega t + \phi) = -\varepsilon_0 \mu_0 \omega E_{\max}\cos(kx - \omega t) \Rightarrow \phi = 0.$$

$$\Rightarrow kB_{\max} = \varepsilon_0 \mu_0 \omega E_{\max} \Rightarrow B_{\max} = \frac{\varepsilon_0 \mu_0 \omega}{k} E_{\max} = \frac{2\pi f}{c^2 2\pi / \lambda} E_{\max} = \frac{f\lambda}{c^2} E_{\max} = \frac{1}{c} E_{\max}.$$

32.35: From Eq. (32.12): $\dfrac{\partial}{\partial t}\left(\dfrac{\partial E_y(x,t)}{\partial x}\right) = \dfrac{\partial}{\partial t}\left(-\dfrac{\partial B_z(x,t)}{\partial t}\right) = -\dfrac{\partial^2 B_z(x,t)}{\partial t^2}$

But also from Eq. (32.14): $-\dfrac{\partial}{\partial x}\left(\dfrac{\partial B_z(x,t)}{\partial x}\right) = \dfrac{\partial}{\partial x}\left(\varepsilon_0 \mu_0 \dfrac{\partial E_y(x,t)}{\partial t}\right) =$

$$-\varepsilon_0 \mu_0 \frac{\partial^2 B_z(x,t)}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 B_z(x,t)}{\partial x^2} = \varepsilon_0\mu_0 \frac{\partial^2 B_z(x,t)}{\partial t^2}.$$

32.36: $E_y(x,t) = E_{\max}\cos(kx - \omega t) \Rightarrow u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 E_{\max}^2 \cos(kx - \omega t)$

$$\Rightarrow u_E = \frac{\varepsilon_0 c^2}{2}\left(\frac{E_{\max}}{c}\right)^2 \cos(kx - \omega t) = \frac{1}{2\mu_0} B_{\max}^2 \cos(kx - \omega t) = \frac{B_z^2}{2\mu_0} = u_B$$

32.37: a) The energy incident on the mirror is $Pt = IAt = \frac{1}{2}\varepsilon_0 cE^2 At$

$$\Rightarrow E = \frac{1}{2}\varepsilon_0 (3.00\times10^8 \text{ m/s})(0.028 \text{ V/m})^2 (5.00\times10^{-4} \text{ m}^2)(1.00 \text{ s}) = 5.20\times10^{-10}\text{ J}.$$

b) The radiation pressure $p_{\text{rad}} = \frac{2I}{c} = \varepsilon_0 E^2 = \varepsilon_0 (0.0280 \text{ V/m})^2 = 6.94\times10^{-15}$ Pa.

c) Power $P = I\cdot 4\pi R^2 = cp_{\text{rad}} 2\pi R^2$

$$\Rightarrow P = 2\pi(3.00\times10^8 \text{ m/s})(6.94\times10^{-15} \text{ Pa})(3.20 \text{ m})^2 = 1.34\times10^{-4} \text{ W}.$$

32.38: a) $f = \frac{c}{\lambda} = \frac{3.00\times10^8 \text{ m/s}}{0.0384 \text{ m}} = 7.81\times10^9\text{ Hz}.$

b) $B_{\max} = \frac{E_{\max}}{c} = \frac{1.35 \text{ V/m}}{3.00\times10^8 \text{ m/s}} = 4.50\times10^{-9}$ T.

c) $I = \frac{1}{2}\varepsilon_0 c E_{\max}^2 = \frac{1}{2}\varepsilon_0 (3.00\times10^8 \text{ m/s})(1.35 \text{ V/m})^2 = 2.42\times10^{-3} \text{ W/m}^2.$

d) $F = pA = \frac{IA}{c} = \frac{EBA}{2\mu_0 c} = \frac{(1.35 \text{ V/m})(4.50\times10^{-9} \text{ T})(0.240 \text{ m}^2)}{2\mu_0 (3.00\times10^8 \text{ m/s})} = 1.93\times10^{-12}$ N.

32.39: a) The laser intensity $I = \frac{P}{A} = \frac{4P}{\pi D^2} = \frac{4(3.20\times10^{-3} \text{ W})}{\pi(2.50\times10^{-3} \text{ m})^2} = 652 \text{ W/m}^2.$

But $I = \frac{1}{2}\varepsilon_0 cE^2 \Rightarrow E = \sqrt{\frac{2I}{\varepsilon_0 c}} = \sqrt{\frac{2(652 \text{ W/m}^2)}{\varepsilon_0 (3.00\times10^8 \text{ m/s})}} = 701$ V/m.

And $B = \frac{E}{c} = \frac{701\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 2.34\times10^{-6}\ \text{T}$.

b) $u_{B_{av}} = u_{E_{av}} = \frac{1}{4}\varepsilon_0 E_{\max}^2 = \frac{1}{4}\varepsilon_0 (701\ \text{V/m})^2 = 1.09\times10^{-6}\ \text{J/m}^3$. Note the extra factor of $\frac{1}{2}$ since we are averaging.

c) In one meter of the laser beam, the total energy is:

$$E_{\text{tot}} = u_{\text{tot}}\text{Vol} = 2u_E(AL) = 2u_E\pi D^2 L/4$$

$$\Rightarrow E_{\text{tot}} = 2(1.09\times10^{-6}\ \text{J/m}^3)\pi\,(2.50\times10^{-3}\text{m})^2\ (1.00\ \text{m})/4 = 1.07\times10^{-11}\ \text{J}.$$

32.40: a) The change in the momentum vector determines p_{rad}. If W is the fraction absorbed, $\Delta\vec{P} = \vec{P}_{\text{out}} - \vec{P}_{\text{in}} = (1-W)p - (-p) = (2-W)p$. Here, $(1-W)$ is the fraction reflected. The positive direction was chosen in the direction of reflection. p is the magnitude of the incoming momentum. With Eq. 32.31, and taking the average, we get $p_{\text{rad}} = (2-W)\frac{I}{C}$. Be careful not to confuse p, the momentum of the incoming wave, with p_{rad}, the radiation pressure.

b) (i) totally absorbing $W = 1$ so $p_{\text{rad}} = \frac{I}{C}$

(ii) totally reflecting $W = 0$ so $p_{\text{rad}} = \frac{2}{C}$

These are just equations 32.32 and 32.33.

c) $W = 0.9, I = 1.40\times10^2\text{W/m}^2 \Rightarrow p_{\text{rad}} = \frac{(2-0.9)(1.40\times10^3\ \text{W/m}^2)}{3.00\times10^8\ \frac{\text{m}}{\text{s}}} = 5.13\times10^{-6}\text{Pa}$

$W = 0.1, I = 1.40\times10^3\ \text{W/m}^2 \Rightarrow p_{\text{rad}} = \frac{(2-0.1)(1.40\times10^2\ \text{W/m}^2)}{3.00\times10^8\ \frac{\text{m}}{\text{s}}} = 8.87\times10^{-6}\ \text{Pa}$

32.41: a) At the sun's surface:

$$P = IA \Rightarrow I = \frac{P}{A} = \frac{P}{4\pi R^2} = \frac{3.9\times10^{26}\ \text{W}}{4\pi(6.96\times10^{8}\ \text{m})^2} = 6.4\times10^{7}\ \text{W/m}^2$$

$$\Rightarrow p_{\text{rad}} = \frac{I}{c} = \frac{6.4\times10^{7}\ \text{W/m}^2}{3.00\times10^{8}\ \text{m/s}} = 0.21\ \text{Pa}.$$

Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $p_{\text{rad}}(R_{\text{sun}}/2) = 0.85$ Pa.

At the top of the earth's atmosphere, the measured sunlight intensity is $1400\ \text{W/m}^2 =$

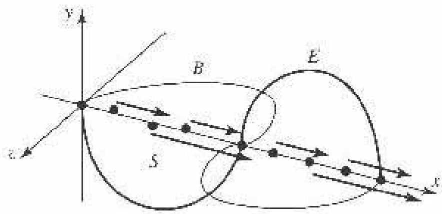
5×10^{-6} Pa, which is about 100,000 times less than the values above.

b) The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about $6\ \times10^{13}$ times greater than the radiation, pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.42: a) $\vec{S}\,(x,t) = \frac{E_{\max}B_{\max}}{2\mu_0}(1-\cos 2\,(kx-\omega t))\hat{i} \Rightarrow S(x,t) < 0 \Rightarrow \cos 2(kx-\omega t) > 1,$

which never happens. So the Poynting vector is always positive, which makes sense since the direction of wave propagation by definition is the direction of energy flow.

b)



32.43: a) $B = \mu_0 ni \Rightarrow \frac{dB}{dt} = \mu_0 n\frac{di}{dt} \Rightarrow \frac{d\Phi_B}{dt} = \frac{dB}{dt}A = \mu_0 nA\frac{di}{dt}.$

$$\text{So, } \oint \vec{E}\cdot d\vec{l} = -\frac{d\Phi_B}{dt} \Rightarrow E2\pi r = -\mu_0 nA\frac{di}{dt} = -\mu_0 n\pi r^2\frac{di}{dt}$$

$$\Rightarrow E = -\frac{\mu_0 nr}{2}\frac{di}{dt}.$$

b) The direction of the Poynting vector is radially inward, since the magnetic field is along the solenoid's axis and the electric filed is circumferential. It's magnitude

$$S = \frac{EB}{\mu_0} = \frac{\mu_0 n^2 ri}{2}\frac{di}{dt}.$$

c) $u = \frac{B^2}{2\mu_0} = \frac{(\mu_0 ni)^2}{2\mu_0} = \frac{\mu_0 n^2 i^2}{2} \Rightarrow U = u(lA) = ul\pi a^2 = \frac{\mu_0 \pi n^2 i^2 la^2}{2}$.

But also $U = \frac{Li^2}{2} \Rightarrow Li = \frac{2U}{i} = \frac{\mu_0 \pi n^2 i^2 la^2}{i} = \mu_0 \pi n^2 ila^2$, and so the rate of energy increase due to the increasing current is given by $P = Li\frac{di}{dt} = \mu_0 \pi n^2 ila^2 \frac{di}{dt}$.

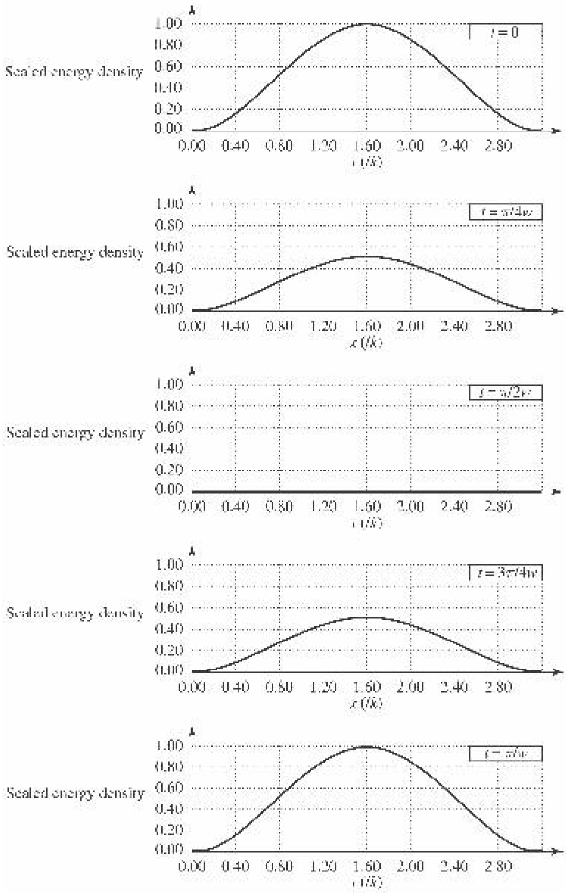
d) The in-flow of electromagnetic energy through a cylindrical surface located at the solenoid coils is $\int\int \vec{S} \cdot d\vec{A} = S2\pi al = \frac{\mu_0 n^2 ai}{2}\frac{di}{dt} \cdot 2\pi al = \mu_0 \pi n^2 ila^2 \frac{di}{dt}$.

e) The values from parts (c) and (d) are identical for the flow of energy, and hence we can consider the energy stored in a current carrying solenoid as having entered through its cylindrical walls while the current was attaining its steady-state value.

32.44: a) The energy density, as a function of *x*, for the equations for the electrical and magnetic fields of Eqs. (32.34) and (32.35) is given by:
$u = \varepsilon_0 E^2 = 4\varepsilon_0 E^2_{max} \sin^2 kx \sin \omega t$

b) At $t = \frac{\pi}{4\omega}$, $\cos \omega t = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ and $\sin \omega t = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$.

For $0 < x < \frac{\pi}{2k}$, $\sin kx > 0, \cos kx > 0 \Rightarrow \hat{S} = \hat{E} \times \hat{B} = -\hat{j} \times \hat{k} = -\hat{i}$.



And for $\frac{\pi}{2k} < x < \frac{\pi}{k}$, $\sin kx > 0, \cos kx < 0 \Rightarrow \hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = -\hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}}.$

At $t = \frac{3\pi}{4\omega}, \cos \omega t = \cos\frac{3\pi}{4} = -\frac{1}{\sqrt{2}}$ and $\sin \omega t = \sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$.

For $0 < x < \frac{\pi}{2k}$, $\sin kx > 0, \cos kx > 0 \Rightarrow \hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = \hat{\boldsymbol{j}} \times \hat{\boldsymbol{k}} = \hat{\boldsymbol{i}}.$

And for $\frac{\pi}{2k} < x < \frac{\pi}{k}$, $\sin kx > 0, \cos kx < 0 \Rightarrow \hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = \hat{\boldsymbol{j}} \times -\hat{\boldsymbol{k}} = -\hat{\boldsymbol{i}}.$

c) the plots from part (a) can be interpreted as two waves passing through each other in opposite directions, adding constructively at certain times, and destructively at others.

32.45: a) $E = \rho J = \frac{\rho I}{A} = \frac{\rho I}{\pi a^2}$, in the direction of the current.

b) $\oint \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{l}} = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi a}$, counterclockwise when looking into the current.

c) The direction of the Poynting vector $\hat{\boldsymbol{S}} = \hat{\boldsymbol{E}} \times \hat{\boldsymbol{B}} = \hat{\boldsymbol{k}} \times \hat{\boldsymbol{\phi}} = -\hat{\boldsymbol{\rho}}$, where we have used cylindrical coordinates, with the current in the z-direction.

Its magnitude is $S = \dfrac{EB}{\mu_0} = \dfrac{1}{\mu_0}\dfrac{\rho I}{\pi a^2}\dfrac{\mu_0 I}{2\pi a} = \dfrac{\rho I^2}{2\pi^2 a^3}$.

d) Over a length l, the rate of energy flowing in is $SA = \dfrac{\rho I^2}{2\pi^2 a^3} 2\pi a l = \dfrac{\rho l I^2}{\pi a^2}$.

The thermal power loss is $I^2 R = I^2 \dfrac{\rho l}{A} = \dfrac{\rho l I^2}{\pi a^2}$, which exactly equals the flow of electromagnetic energy.

32.46: $B = \dfrac{\mu_0 i}{2\pi r}$, and $\oint_S \vec{E}\cdot d\vec{A} = EA = \dfrac{q}{\varepsilon_0} \Rightarrow E = \dfrac{q}{\pi\varepsilon_0 r^2}$, so the magnitude of the Poynting vector is $S = \dfrac{EB}{\mu_0} = \dfrac{qi}{2\varepsilon_0\pi^2 r^3} = \dfrac{q}{2\varepsilon_0\pi^2 r^3}\dfrac{dq}{dt}$.

Now, the rate of energy flow into the region between the plates is:

$$\int\int \vec{S}\cdot d\vec{A} = S(2\pi r l) = \frac{lq}{\varepsilon_0 \pi r^2}\frac{dq}{dt} = \frac{1}{2}\frac{l}{\varepsilon_0 \pi r^2}\frac{d(q^2)}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{l}{\varepsilon_0 A}q^2\right) = \frac{d}{dt}\left(\frac{q^2}{2C}\right) = \frac{dU}{dt}.$$

This is just rate of increase in electrostatic energy U stored in the capacitor.

32.47: The power from the antenna is $P = IA = \dfrac{cB_{\max}^2}{2\mu_0}4\pi r^2$. So

$$\Rightarrow B_{\max} = \sqrt{\frac{2\mu_0 P}{4\pi r^2 c}} = \sqrt{\frac{2\mu_0(5.50\times 10^4\ \text{W})}{4\pi(2500\ \text{m})^2(3.00\times 10^8\ \text{m/s})}} = 2.42\times 10^{-9}\ \text{T}$$

$$\Rightarrow \frac{dB}{dt} = \omega B_{\max} = 2\pi f B_{\max} = 2\pi(9.50\times 10^7\ \text{Hz})\,(2.42\times 10^{-9}\ \text{T}) = 1.44\ \text{T/s}$$

$$\Rightarrow \varepsilon = -\frac{d\Phi}{dt} = -A\frac{dB}{dt} = \frac{\pi D^2}{4}\frac{dB}{dt} = \frac{\pi(0.180\ \text{m})^2(1.44\ \text{T/s})}{4} = 0.0366\ \text{V}.$$

32.48: $I = \dfrac{P}{A} = \dfrac{1}{2}\varepsilon_0 c E^2 \Rightarrow E = \sqrt{\dfrac{2I}{\varepsilon_0 c}} = \sqrt{\dfrac{2(2.80\times 10^3\ \text{W}/36\ \text{m}^2)}{\varepsilon_0(3.00\times 10^8\ \text{m/s})}} = 242\ \text{V/m}.$

32.49: a) Find the force on you due to the momentum carried off by the light:
$p_{\text{rad}} = I/c$ and $F = p_{\text{rad}}\,A$ gives $F = I\,A/c = P_{\text{av}}/c$

$a_x = F/m = P_{\text{av}}/(mc) = (200\ \text{W})/[(150\ \text{kg})(3.00\times 10^8\ \text{m/s})] = 4.44\times 10^{-9}\ \text{m/s}^2$

Then $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$ gives $t = \sqrt{2(x-x_0)/a_x} = \sqrt{2(16.0\ \text{m})/(4.44\times 10^{-9}\ \text{m/s}^2)} =$

$8.49\times 10^4\ \text{s} = 23.6\ \text{h}$

The radiation force is very small. In the calculation we have ignored any other forces on you.

b) You could throw the flashlight in the direction away from the ship. By conservation of linear momentum you would move toward the ship with the same magnitude of momentum as you gave the flashlight.

32.50: $P = IA \Rightarrow I = \frac{P}{A} = \frac{1}{2}\varepsilon_0 c E^2 \Rightarrow E = \sqrt{\frac{2P}{A\varepsilon_0 c}} = \sqrt{\frac{2Vi}{A\varepsilon_0 c}}$

$$\Rightarrow E = \sqrt{\frac{2Vi}{A\varepsilon_0 c}} = \sqrt{\frac{2(5.00\times 10^5\ \text{V})\,(1000\ \text{A})}{(100\ \text{m}^2)\varepsilon_0 (3.00\times 10^8\ \text{m/s})}} = 6.14\times 10^4\ \text{V/m}.$$

And

$$B = \frac{E}{c} = \frac{6.14\times 10^4\ \text{V/m}}{3.00\times 10^8\ \text{m/s}} = 2.05\times 10^4\ \text{T}.$$

32.51: a) $F_G = \frac{GM_S m}{r^2} = \frac{GM_S}{r^2}.\frac{4\pi R^3 \rho}{3} = \frac{4\pi G M_S R^3 \rho}{3r^2}.$

b) Assuming that the sun's radiation is intercepted by the particle's cross-section, we can write the force on the particle as:

$$F = \frac{IA}{c} = \frac{L}{4\pi r^2}.\frac{\pi R^2}{c} = \frac{LR^2}{4cr^2}.$$

c) So if the force of gravity and the force from the radiation pressure on a particle from the sun are equal, we can solve for the particle's radius:

$$F_G = F \Rightarrow \frac{4\pi G M_S R^3 \rho}{3r^2} = \frac{LR^2}{4cr^2} \Rightarrow R = \frac{3L}{16\pi G M_S \rho c}.$$

$$\Rightarrow R = \frac{3(3.9\times 10^{26}\ \text{W})}{16\pi\,(6.7\times 10^{-11}\ \ \text{N}\cdot\text{m}^2/\text{kg}^2)\,(2.0\times 10^{30}\ \text{kg})\,(3000\ \text{kg/m}^3)\,(3.0\times 10^8\ \text{m/s})}$$

$$\Rightarrow R = 1.9\times 10^{-7}\ \text{m}.$$

d) If the particle has a radius smaller than that found in part (c), then the radiation pressure overcomes the gravitational force and results in an acceleration away from the sun, thus removing all such particles from the solar system.

32.52: a) The momentum transfer is always greatest when reflecting surfaces are used (consider a ball colliding with a wall—the wall exerts a greater force if the ball rebounds rather than sticks). So in solar sailing one would want to use a reflecting sail.

b) The equation for repulsion comes from balancing the gravitational force and the force from the radiation pressure. As seen in Problem 32.51, the latter is:

$$F_{\text{rad}} = \frac{2LA}{4\pi r^2 c}. \text{ Thus}: F_G = F_{\text{rad}} \Rightarrow \frac{GM_S m}{r^2} = \frac{2LA}{4\pi r^2 c} \Rightarrow A = \frac{4\pi GM_S mc}{2L}$$

$$\Rightarrow A = \frac{4\pi(6.7\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.0\times10^{30}\ \text{kg})(10000\ \text{kg})(3.0\times10^8\ \text{m/s})}{(2)3.9\times10^{26}\ \text{W}}$$

$$\Rightarrow A = 6.48\times10^6\ \text{m}^2 = 6.48\ \text{km}^2 = \frac{6.48\ \text{km}^2}{(1.6\ \text{km/mile})^2} = 2.53\ \text{mi}^2$$

c) This answer is independent of the distance from the sun since both the gravitational force and the radiation pressure go down like one over the distance squared, and thus the distance cancels out of the problem.

32.53: a) $\left[\frac{q^2a^2}{6\pi\varepsilon_0 c^3}\right] = \frac{C^2(\text{m/s}^2)^2}{(C^2/\text{N}\cdot\text{m}^2)(\text{m/s})^3} = \frac{\text{Nm}}{\text{s}} = \frac{\text{J}}{\text{s}} = \text{W} = \left[\frac{dE}{dt}\right].$

b) For a proton moving in a circle, the acceleration can be rewritten:

$$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(6.00\times10^6\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})}{(1.67\times10^{-27}\ \text{kg})(0.75\ \text{m})} = 1.53\times10^{15}\ \text{m/s}^2.$$

The rate at which it emits energy because of its acceleration is:

$$\frac{dE}{dt} = \frac{q^2a^2}{6\pi\varepsilon_0 c^3} = \frac{(1.6\times10^{-19}\text{C})^2(1.53\times10^{15}\ \text{m/s}^2)^2}{6\pi\varepsilon_0(3.0\times10^8\ \text{m/s})^3} = 1.33\times10^{-23}\ \text{J/s}$$

$$= 8.32\times10^{-5}\ \text{eV/s}.$$

So the fraction of its energy that it radiates every second is:

$$\frac{(dE/dt)(1\ \text{s})}{E} = \frac{8.32\times10^{-5}\ \text{eV}}{6.00\times10^6\ \text{eV}} = 1.39\times10^{-11}.$$

c) Carrying out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses:

$$E_e = E_p\frac{m_e}{m_p} = (6.00\times10^6\ \text{eV})\frac{(9.11\times10^{-31}\ \text{kg})}{(1.67\times10^{-27}\ \text{kg})} = 3273\ \text{eV}.$$

So the fraction of its energy that it radiates every second is:

$$\frac{(dE/dt)(1\ \text{s})}{E} = \frac{8.32\times10^{-5}\ \text{eV}}{3273\ \text{eV}} = 2.54\times10^{-8}.$$

32.54: For the electron in the classical hydrogen atom, its acceleration is:

$$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(13.6\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{(9.11\times10^{-31}\ \text{kg})(5.29\times10^{-11}\ \text{m})} = 9.03\times10^{22}\ \text{m/s}^2.$$

Then using the formula for the rate of energy emission given in Pr. (33-49):

$$\frac{dE}{dt} = \frac{q^2a^2}{6\pi\varepsilon_0 c^3} = \frac{(1.60\times10^{-19}\text{C})^2(9.03\times10^{22}\ \text{m/s}^2)^2}{6\pi\varepsilon_0(3.00\times10^8\ \text{m/s})^3}$$

$\Rightarrow \frac{dE}{dt} = 4.64 \times 10^{-8}$ J/s $= 2.89 \times 10^{11}$ eV/s, which means that the electron would almost immediately lose all its energy!

32.55: a) $E_y(x, t) = E_{\max} e^{-k_c x} \sin(k_c x - \omega t)$.

$$\frac{\partial E_y}{\partial x} = E_{\max}(-k_c)e^{-k_c x}\sin(k_c x - \omega t) + E_{\max}(+k_c)e^{-k_c x}\cos(k_c x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max}(+k_c^2)e^{-k_c x}\sin(k_c x - \omega t) + E_{\max}(-k_c^2)e^{-k_c x}\cos(k_c x - \omega t)$$
$$+ E_{\max}(-k_c^2)e^{-k_c x}\cos(k_c x - \omega t) + E_{\max}(-k_c^2)e^{-k_c x}\sin(k_c x - \omega t)$$
$$= -2E_{\max}k_c^2 e^{-k_c x}\cos(k_c x - \omega t).$$

$$\frac{\partial E_y}{\partial t} = E_{\max}e^{-k_c x}\,\omega\cos(k_c x - \omega t).$$

Set $\frac{\partial^2 E_y}{\partial x^2} = \frac{\mu \partial E_y}{\rho \partial t} \Rightarrow 2E_{\max}k_c^2 e^{-k_c x}\cos(k_c x - \omega t) = E_{\max}e^{-k_c x}\,\omega\cos(k_c x - \omega t)$. This will only be true if $\frac{2k_c^2}{\omega} = \frac{\mu}{\rho}$ or $k_c = \sqrt{\frac{\omega\mu}{2\rho}}$.

b) The hint basically answers the question.

c) $E_y = \frac{E_{y0}}{e} \Rightarrow k_c x = 1, x = \frac{1}{k_c} = \sqrt{\frac{2\rho}{\omega\mu}} = \sqrt{\frac{2(1.72 \times 10^{-8}\ \Omega\text{m})}{2\pi(1.0 \times 10^6\ \text{Hz})\mu_0}} = 6.60 \times 10^{-5}$ m.

33.1: a) $v = \frac{c}{n} = \frac{3.00 \times 10^8\ \text{m/s}}{1.47} = 2.04 \times 10^8\ \text{m/s}$.

b) $\lambda = \frac{\lambda_0}{n} = \frac{(6.50 \times 10^{-7}\ \text{m})}{1.47} = 4.42 \times 10^{-7}$ m.

33.2: a) $\lambda_{\text{vacuum}} = \frac{c}{f} = \frac{3.00 \times 10^8\ \text{m/s}}{5.80 \times 10^{14}\ \text{Hz}} = 5.17 \times 10^{-7}$ m.

b) $\lambda_{\text{glass}} = \frac{c}{fn} = \frac{3.00 \times 10^8\ \text{m/s}}{(5.80 \times 10^{14}\ \text{Hz})(1.52)} = 3.40 \times 10^{-7}$ m.

33.3: a) $n = \frac{c}{v} = \frac{3.00 \times 10^8\ \text{m/s}}{1.94 \times 10^8\ \text{m/s}} = 1.54.$

b) $\lambda_0 = n\lambda = (1.54)(3.55 \times 10^{-7}\ \text{m}) = 5.47 \times 10^{-7}$ m.

33.4: $\lambda_{\text{water}}n_{\text{water}} = \lambda_{\text{Benzene}}n_{\text{Benzene}} \Rightarrow \lambda_{CS_2} = \frac{\lambda_{\text{water}}n_{\text{water}}}{n_{\text{Benzene}}} = \frac{(4.38 \times 10^{-7}\ \text{m})(1.333)}{1.501}$

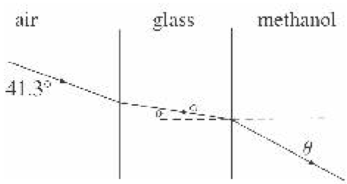
33.5 : a) Incident and reflected angles are always equal $\Rightarrow \theta_r' = \theta_a' = 47.5°$.

b) $\theta_b' = \dfrac{\pi}{2} - \theta_b = \dfrac{\pi}{2} - \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \dfrac{\pi}{2} - \arcsin\left(\dfrac{1.00}{1.66}\sin 42.5°\right) = 66.0°$.

33.6:
$$v = \frac{d}{t} = \frac{2.50\ \text{m}}{11.5\times10^{-9}\ \text{s}} = 2.17\times10^{8}\ \text{m/s}$$
$$n = \frac{c}{v} = \frac{3.00\times10^{8}\ \text{m/s}}{2.17\times10^{8}\ \text{m/s}} = 1.38$$

33.7:
$$n_a \sin\theta_a = n_b \sin\theta_b$$
$$n_b = n_a\left(\frac{\sin\theta_a}{\sin\theta_b}\right) = 1.00\left(\frac{\sin 62.7°}{\sin 48.1°}\right) = 1.194$$
$$n = c/v \text{ so } v = c/n = (3.00\times10^{8}\ \text{m/s})/1.194 = 2.51\times10^{8}\ \text{m/s}$$

33.8 (a)



Apply Snell's law at both interfaces.

At the air-glass interface:

$$(1.00)\sin 41.3° = n_{\text{glass}} \sin\alpha$$

At the glass-methanol interface:

$$n_{\text{glass}} \sin\alpha = (1.329)\sin\theta \qquad (2)$$

Combine (1) and (2):

$$\sin 41.3° = 1.329 \sin\theta$$

$$\theta = 29.8°$$

(b) Same figure as for (a), except $\theta = 20.2°$.

$$(1.00)\sin 41.3° = n \sin 20.2°$$

$$n = 1.91$$

33.9: a) Let the light initially be in the material with refractive index n_a and let the third and final slab have refractive index n_b Let the middle slab have refractive index n_1

1st interface : $n_a \sin\theta_a = n_1 \sin\theta_1$

2nd interface : $n_1 \sin\theta_1 = n_b \sin\theta_b$

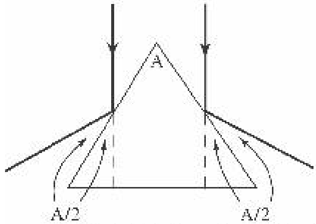
Combining the two equations gives $n_a \sin\theta_a = n_b \sin\theta_b$.

b) For N slabs, where the first slab has refractive index n_a and the final slab has refractive index $n_b, n_a \sin\theta_a = n_1 \sin\theta_1, n_1 \sin\theta_1 = n_2 \sin\theta_2, \ldots, n_{N-2} \sin\theta_{N-2} = n_b \sin\theta_b$. This gives $n_a \sin\theta_a = n_b \sin\theta_b$. The final direction of travel depends on the angle of incidence in the first slab and the indicies of the first and last slabs.

33.10: a) $\theta_{\text{water}} = \arcsin\left(\dfrac{n_{\text{air}}}{n_{\text{water}}}\sin\theta_{\text{air}}\right) = \arcsin\left(\dfrac{1.00}{1.33}\sin 35.0°\right) = 25.5°.$

b) This calculation has no dependence on the glass because we can omit that step in the chain : $n_{\text{air}} \sin\theta_{\text{air}} = n_{\text{glass}} \sin\theta_{\text{glass}} = n_{\text{water}} \sin\theta_{\text{water}}$.

33.11: As shown below, the angle between the beams and the prism is A/2 and the angle between the beams and the vertical is A, so the total angle between the two beams is 2A.



33.12: Rotating a mirror by an angle θ while keeping the incoming beam constant leads to an increase in the incident angle ϕ by θ. Therefore the angle between incoming and outgoing beams becomes $2\theta + 2\phi$ where an additional deflection of 2θ arose from the mirror rotation.

33.13: $\theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.70}{1.58}\sin 62.0°\right) = 71.8°.$

33.14: $\theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.33}{1.52}\sin 45.0°\right) = 38.2.$ But this is the angle from the normal to the surface, so the angle from the vertical is an additional 15° because of the tilt of the surface. Therefore the angle is 53.2°.

33.15: a) Going from the liquid into air:

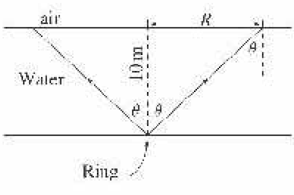
$$\frac{n_b}{n_a} = \sin\theta_{\text{crit}} \Rightarrow n_a = \frac{1.00}{\sin 42.5°} = 1.48.$$

$$\text{So}: \theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.48}{1.00}\sin 35.0°\right) = 58.1°.$$

b) Going from air into the liquid:

$$\theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.00}{1.48}\sin 35.0°\right) = 22.8°.$$

33.16:



If $\theta >$ critical angle, no light escapes,

so for the largest circle, $\theta = \theta_c$

$n_w \sin\theta_c = n_{air} \sin 90° = (1.00)(1.00) = 1.00$

$\theta_c = \sin^{-1}(1/n_w) = \sin^{-1}\dfrac{1}{1.333} = 48.6°$

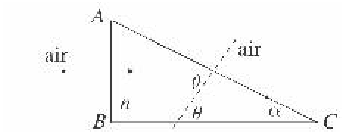
$\tan\theta_c = R/10.0 \text{ m} \rightarrow R = (10.0 \text{ m}) \tan 48.6° = 11.3 \text{ m}$

$A = \pi R^2 = \pi\ (13.3 \text{ m})^2 = 401 \text{ m}^2$

33.17: For glass $\rightarrow$ water, $\theta_{crit} = 48.7°$

$$n_a \sin\theta_{crit} = n_b \sin 90°, \text{ so } n_a = \frac{n_b}{\sin\theta_{crit}} = \frac{1.333}{\sin 48.7°} = 1.77$$

33.18: (a)



Total internal reflection occurs at AC: $n \sin\theta = (1.00)\sin 90° = 1.00$

$$(1.52)\sin\theta = 1.00$$
$$\theta = 41.1°$$
$$\alpha + \theta = 90° \rightarrow \alpha = 90° - 41.1° = 48.9°$$

If α is larger, θ is smaller and thus less than the critical angle, so this answer is the *largest* that α can be.

(b) Same approach as in (a), except AC is now a glass-water boundary.

$$n \sin\theta = n_w \sin 90° = 1.333$$
$$1.52 \sin\theta = 1.333$$
$$\theta = 61.3°$$
$$\alpha = 90° - 61.3° = 28.7°$$

33.19: a) The slower the speed of the wave, the larger the index of refraction—so air has a larger index of refraction than water.

b) $\theta_{crit} = \arcsin\left(\dfrac{n_b}{n_a}\right) = \arcsin\left(\dfrac{v_{air}}{v_{water}}\right) = \arcsin\left(\dfrac{344 \text{ m/s}}{1320 \text{ m/s}}\right) = 15.1°.$

c) Air. For total internal reflection, the wave must go from higher to lower index of refraction—in this case, from air to water.

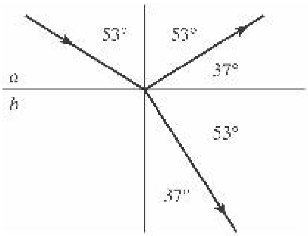
33.20: $\theta_{crit} = \arcsin\left(\dfrac{n_b}{n_a}\right) = \arcsin\left(\dfrac{1.00}{2.42}\right) = 24.4°.$

33.21 : a) $\tan\theta_p = \dfrac{n_b}{n_a} = \tan 54.5° = 1.40 \Rightarrow n_b = 1.40.$

b) $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.00}{1.40}\sin 54.5°\right) = 35.6°.$

33.22 : From the picture on the next page, $\theta_r = 37.0°$, and so :

$$n_b = n_a \frac{\sin\theta_a}{\sin\theta_b} = 1.33\frac{\sin 53°}{\sin 37°} = 1.77.$$



33.23 : a) $\tan\theta_p = \dfrac{n_b}{n_a} \Rightarrow n_a = \dfrac{n_b}{\tan\theta_p} = \dfrac{1.00}{\tan 31.2°} = 1.65.$

b) $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{1.65}{1.00}\sin 31.2°\right) = 58.7°.$

33.24 : a) In air $\theta_p = \arctan\left(\dfrac{n_b}{n_a}\right) = \arctan\left(\dfrac{1.66}{1.00}\right) = 58.9°.$

b) In water $\theta_p = \arctan\left(\dfrac{n_b}{n_a}\right) = \arctan\left(\dfrac{1.66}{1.33}\right) = 51.3°.$

33.25 : a) Through the first filter : $I_1 = \dfrac{1}{2}I_0.$

The second filter : $I_2 = \dfrac{1}{2}I_0\cos^2(41.0°) = 0.285 I_0.$

b) The light is linearly polarized.

33.26 : a) $I = I_{max}\cos^2\phi \Rightarrow I = I_{max}\cos^2(22.5°) = 0.854 I_{max}.$

b) $I = I_{max}\cos^2\phi \Rightarrow I = I_{max}\cos^2(45.0°) = 0.500 I_{max}.$

c) $I = I_{max}\cos^2\phi \Rightarrow I = I_{max}\cos^2(67.5°) = 0.146 I_{max}.$

33.27 : After the first filter the intensity is $\frac{1}{2}I_0 = 10.0\ \text{W/m}^2$ and the light is polarized along the axis of the first filter. The intensity after the second filter is $I = I_0\cos^2\phi$, where $I_0 = 10.0\ \text{W/m}^2$ and $\omega = 62.0° - 25.0° = 37.0°$. Thus, $I = 6.38\ \text{W/m}^2$.

33.28: Let the intensity of the light that exits the first polarizer be I_1, then, according to repeated application of Malus' law, the intensity of light that exits the third polarizer is

$$75.0\ \text{W/cm}^2 = I_1\cos^2(23.0°)\cos^2(62.0° - 23.0°).$$

So we see that $I_1 = \dfrac{75.0\ \text{W/cm}^2}{\cos^2(23.0°)\cos^2(62.0° - 23.0°)}$, which is also the intensity incident on the third polarizer after the second polarizer is removed. Thus, the intensity that exits the third polarizer after the second polarizer is removed is

$$\frac{75.0\ \text{W/cm}^2 \cos^2(62.0°)}{\cos^2(23.0°)\cos^2(62.0° - 23.0°)} = 32.3\ \text{W/cm}^2.$$

33.29 : a) $I_1 = \dfrac{1}{2}I_0,\ I_2 = \dfrac{1}{2}I_0\cos^2(45.0°) = 0.250I_0,\ I_3 = I_2\cos^2(45.0°) = 0.125I_0.$

b) $I_1 = \dfrac{1}{2}I_0,\ I_2 = \dfrac{1}{2}I_0\cos^2(90.0°) = 0.$

33.30: a) All the electric field is in the plane perpendicular to the propagation direction, and maximum intensity through the filters is at 90° to the filter orientation for the case of minimum intensity. Therefore rotating the second filter by 90° when the situation originally showed the maximum intensity means one ends with a dark cell.

b) If filter P_1 is rotated by 90°, then the electric field oscillates in the direction pointing toward the P_2 filter, and hence no intensity passes through the second filter: see a dark cell.

c) Even if P_2 is rotated back to its original position, the new plane of oscillation of the electric field, determined by the first filter, allows zero intensity to pass through the second filter.

33.31: Consider three mirrors, M_1 in the (x,y)-plane, M_2 in the (y,z)-plane, and M_3 in the (x,z)-plane. A light ray bouncing from M_1 changes the sign of the z-component of the velocity, bouncing from M_2 changes the x-component, and from M_3 changes the y-component. Thus the velocity, and hence also the path, of the light beam flips by 180°

33.32 : a) $\theta_b = \arcsin\left(\dfrac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\dfrac{v_b}{v_a}\sin\theta_a\right) = \arcsin\left(\dfrac{1480}{344}\sin 9.73°\right) = 46.6°.$

b) $\theta_{\text{crit}} = \arcsin\left(\dfrac{v_a}{v_b}\right) = \arcsin\left(\dfrac{344}{1480}\right) = 13.4°.$

33.33: a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$ and $n_2 \sin\theta_2 = n_3 \sin\theta_3$, so $n_1 \sin\theta_1 = n_3 \sin\theta_3$ $\sin\theta_3 = (n_1 \sin\theta_1)/n_3$ b) $n_3 \sin\theta_3 = n_2 \sin\theta_2$ and $n_2 \sin\theta_2 = n_1 \sin\theta_1$, so $n_1 \sin\theta_1 =$ $n_3 \sin\theta_3$ and the light makes the same angle with respect to the noral in the material with n_1 as it did in part (a).

c) For reflection, $\theta_r = \theta_a$. These angles are still equal if θ_r becomes the incident angle; reflected rays are also reversible.

33.34: It takes the light an additional 4.2 ns to travel 0.840 m after the glass slab is inserted into the beam. Thus,

$$\frac{0.840\text{ m}}{c/n}-\frac{0.840\text{ m}}{c}=(n-1)\frac{0.840\text{ m}}{c}=4.2\text{ ns.}$$

We can now solve for the index of refraction:

$$n=\frac{(4.2\times10^{-9}\text{ s})\,(3.00\times10^{8}\text{ m/s})}{0.840\text{ m}}+1=2.50.$$

The wavelength inside of the glass is

$$\lambda'=\frac{490\text{ nm}}{2.50}=196\text{ nm}\approx200\text{ nm}.$$

33.35: $\theta_b=90°-\arcsin\left(\dfrac{n_a}{n_b}\right)=90°-\arcsin\left(\dfrac{1.00}{1.38}\right)=43.6°.$

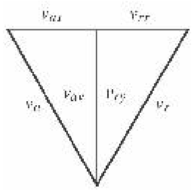
But $n_a\sin\theta_a=n_b\sin\theta_b\Rightarrow\theta_a=\arcsin\left(\dfrac{n_b\sin\theta_b}{n_a}\right)=\arcsin\left(\dfrac{1.38\sin(43.6°)}{1.00}\right)=72.1°.$

33.36: $n_a\sin\theta_a=n_b\sin\theta_b=n_b\sin\left(\dfrac{\theta_a}{2}\right)$

$$\Rightarrow(1.00)\sin\theta_a=\sin2\left(\frac{\theta_a}{2}\right)=2\sin\left(\frac{\theta_a}{2}\right)\cos\left(\frac{\theta_a}{2}\right)=(1.80)\sin\left(\frac{\theta_a}{2}\right)$$

$$\Rightarrow2\cos\left(\frac{\theta_a}{2}\right)=(1.80)\Rightarrow\theta_a=2\arccos\left(\frac{1.80}{2}\right)=51.7°.$$

33.37: The velocity vector "maps out" the path of the light beam, so the geometry as shown below leads to:



$v_a=v_r$ and $\theta_a=\theta_r\Rightarrow\arccos\left(\dfrac{v_{a_y}}{v_a}\right)=\arccos\left(\dfrac{v_{r_y}}{v_r}\right)\Rightarrow v_{a_y}=-v_{r_y}$, with the minus sign chosen by inspection. Similarly, $\Rightarrow\arcsin\left(\dfrac{v_{a_x}}{v_a}\right)=\arcsin\left(\dfrac{v_{r_x}}{v_r}\right)\Rightarrow v_{a_x}=v_{r_x}$.

33.38: $\#\lambda=(\#\lambda)_{\text{air}}+(\#\lambda)_{\text{glass}}=\dfrac{d_{\text{air}}}{\lambda}+\dfrac{d_{\text{glass}}}{\lambda}n=\dfrac{(0.0180\text{ m}-0.00250\text{ m})}{5.40\times10^{-7}\text{m}}+\dfrac{0.00250\text{ m}}{5.40\times10^{-7}\text{ m}}\times$ $(1.40)=3.52\times10^{4}.$

33.39: $\theta_{\text{crit}} = \arctan\left(\dfrac{(0.00534\text{ m})/2}{0.00310\text{ m}}\right) = 40.7° = \arcsin\left(\dfrac{n_b}{n_a}\right) = \arcsin\left(\dfrac{1.0}{n}\right) \Rightarrow n = \dfrac{1}{\sin(40.7°)} = 1$

Note: The radius is reduced by a factor of two since the beam must be incident at θ_{crit}, then refl on the glass-air interface to create the ring.

33.40: $\theta_a = \arctan\left(\dfrac{1.5\text{ m}}{1.2\text{ m}}\right) = 51°$

$$\Rightarrow \theta_b = \arcsin\left(\frac{n_a}{n_b}\sin\theta_a\right) = \arcsin\left(\frac{1.00}{1.33}\sin 51°\right) = 36°.$$

So the distance along the bottom of the pool from directly below where the light enters to where it hits the bottom is:

$$x = (4.0\text{ m})\tan\theta_b = (4.0\text{ m})\tan 36° = 2.9\text{ m}.$$

$$\Rightarrow x_{\text{total}} = 1.5\text{ m} + x = 1.5\text{ m} + 2.9\text{ m} = 4.4\text{ m}.$$

33.41 $\theta_a = \arctan\left(\dfrac{8.0\text{ cm}}{16.0\text{ cm}}\right) = 27°$ and $\theta_b = \arctan\left(\dfrac{4.0\text{ cm}}{16.0\text{ cm}}\right) = 14°$.

So, $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow n_b = \left(\dfrac{n_a \sin\theta_a}{\sin\theta_b}\right) = \left(\dfrac{1.00\sin 27°}{\sin 14°}\right) = 1.8.$

33.42: The beam of light will emerge at the same angle as it entered the fluid as seen by following what happens via Snell's Law at each of the interfaces. That is, the emergent beam is at $42.5°$ from the normal.

33.43: a) $\theta_i = \arcsin\left(\dfrac{n_a \sin 90°}{n_w}\right) = \arcsin\left(\dfrac{1.000}{1.333}\right) = 48.61°.$

The ice does not come into the calculation since $n_{\text{air}}\sin 90° = n_{\text{ice}}\sin\theta_c = n_w \sin\theta_i$.

b) Same as part (a).

33.44: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow n_a = \left(\dfrac{n_b \sin\theta_b}{\sin\theta_a}\right) = \left(\dfrac{1.33\sin 90°}{\sin 45°}\right) = 1.9.$

33.45: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow \theta_b = \arcsin\left(\dfrac{n_a \sin\theta_a}{n_b}\right)$

$$= \arcsin\left(\frac{1.66\sin(25.0°)}{1.00}\right) = 44.6°.$$

So the angle below the horizontal is $\theta_b - 25.0° = 44.6° - 25.0° = 19.6°$, and thus the angle between the two emerging beams is $39.2°$.

33.46: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow n_a = \left(\dfrac{n_b \sin\theta_b}{\sin\theta_a}\right) = \left(\dfrac{1.62 \sin 60°}{\sin 90°}\right) = 1.40.$

33.47: $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow n_a = \left(\dfrac{n_b \sin\theta_b}{\sin\theta_a}\right) = \left(\dfrac{1.52 \sin 57.2°}{\sin 90°}\right) = 1.28.$

33.48: a) For light in air incident on a parallel-faced plate, Snell's Law yields:
$n \sin\theta_a = n' \sin\theta'_b = n' \sin\theta_b = n \sin\theta'_a \Rightarrow \sin\theta_a = \sin\theta'_a \Rightarrow \theta_a = \theta'_a.$

b) Adding more plates just adds extra steps in the middle of the above equation that always cancel out. The requirement of parallel faces ensures that the angle $\theta'_n = \theta_{n'}$ and the chain of equations can continue.

c) The lateral displacement of the beam can be calculated using geometry:

$$d = L\sin(\theta_a - \theta'_b) \text{ and } L = \frac{t}{\cos\theta'_b} \Rightarrow d = \frac{t\sin(\theta_a - \theta'_b)}{\cos\theta'_b}.$$

d) $\theta'_b = \arcsin\left(\dfrac{n\sin\theta_a}{n'}\right) = \arcsin\left(\dfrac{\sin 66.0°}{1.80}\right) = 30.5°$

$$\Rightarrow d = \frac{(2.40\text{ cm})\sin(66.0° - 30.5°)}{\cos 30.5°} = 1.62\text{ cm}.$$

33.49: a) For sunlight entering the earth's atmosphere from the sun BELOW the horizon, we can calculate the angle δ as follows:

$n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow (1.00)\sin\theta_a = n\sin\theta_b$, where $n_b = n$ is the atmosphere's index of refraction. But the geometry of the situation tells us:

$$\sin\theta_b = \frac{R}{R+h} \Rightarrow \sin\theta_a = \frac{nR}{R+h} \Rightarrow \delta = \theta_a - \theta_b = \arcsin\left(\frac{nR}{R+h}\right) - \arcsin\left(\frac{R}{R+h}\right).$$

b) $\delta = \arcsin\left(\dfrac{(1.0003)(6.4\times10^6\text{ m})}{6.4\times10^6\text{ m} + 2.0\times10^4\text{ m})}\right) - \arcsin\left(\dfrac{6.4\times10^6\text{ m}}{64.\times10^6\text{ m} + 2.0\times10^4\text{ m}}\right) \Rightarrow$

$\delta = 0.22°$. This is about the same as the angular radius of the sun, $0.25°$.

33.50: A quarter-wave plate shifts the phase of the light by $\theta = 90°$. Circularly polarized light is out of phase by $90°$, so the use of a quarter-wave plate will bring it back into phase, resulting in linearly polarized light.

33.51: a) $I = \dfrac{1}{2}I_0\cos^2\theta\cos^2(90° - \theta) = \dfrac{1}{2}I_0(\cos\theta\sin\theta)^2 = \dfrac{1}{8}I_0\sin^2 2\theta.$

b) For maximum transmission, we need $2\theta = 90°$, so $\theta = 45°$.

33.52: a) The distance traveled by the light ray is the sum of the two diagonal segments:

$d = \left(x^2 + y_1^2\right)^{1/2} + \left((l - x)^2 + y_2^2\right)^{1/2}.$

Then the time taken to travel that distance is just:

$$t = \frac{d}{c} = \frac{(x^2 + y_1^2)^{1/2} + \left((l - x)^2 + y_2^2\right)^{1/2}}{c}$$

b) Taking the derivative with respect to x of the time and setting it to zero yields:

$$\frac{dt}{dx} = \frac{1}{c}\frac{d}{dt}\left[(x^2 + y_1^2)^{1/2} + \left((l - x)^2 + y_2^2\right)^{1/2}\right]$$

$$\Rightarrow \frac{dt}{dx} = \frac{1}{c}\left[x(x^2 + y_1^2)^{-1/2} - (l - x)\left((l - x)^2 + y_2^2\right)^{-1/2}\right] = 0$$

$$\Rightarrow \frac{x}{\sqrt{x^2 + y_1^2}} = \frac{(l - x)}{\sqrt{(l - x)^2 + y_2^2}} \Rightarrow \sin\theta_1 = \sin\theta_2 \Rightarrow \theta_1 = \theta_2.$$

33.53: a) The time taken to travel from point A to point B is just:

$$t = \frac{d_1}{v_1} + \frac{d_2}{v_2} = \frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l - x)^2}}{v_2}.$$

Taking the derivative with respect to x of the time and setting it to zero yields:

$$\frac{dt}{dx} = 0 = \frac{d}{dt}\left[\frac{\sqrt{h_1^2 + x^2}}{v_1} + \frac{\sqrt{h_2^2 + (l - x)^2}}{v_2}\right] = \frac{x}{v_1\sqrt{h_1^2 + x^2}} - \frac{(l - x)}{v_2\sqrt{h_2^2 + (l - x)^2}}.$$

But $v_1 = \dfrac{c}{n_1}$ and $v_2 = \dfrac{c}{n_2} \Rightarrow \dfrac{n_1 x}{\sqrt{h_1^2 + x^2}} = \dfrac{n_2 (l - x)}{\sqrt{h_2^2 + (l - x)^2}} \Rightarrow n_1 \sin\theta_1 = n_2 \sin\theta_2.$

33.54: a) n decreases with increasing λ, so n is smaller for red than for blue. So beam a is the red one.

b) The separation of the emerging beams is given by some elementary geometry.

$x = x_r - x_v = d\tan\theta_r - d\tan\theta_v \Rightarrow d = \dfrac{x}{\tan\theta_r - \tan\theta_v}$, where x is the vertical beam separation as they emerge from the glass $x = \dfrac{1.00\text{ mm}}{\sin 20°} = 2.92\text{ mm}$. From the ray geometry, we also have

$$\theta_r = \arcsin\left(\frac{\sin 70°}{1.61}\right) = 35.7° \text{ and } \theta_v = \arcsin\left(\frac{\sin 70°}{1.66}\right) = 34.5°, \text{ so :}$$

$$d = \frac{x}{\tan\theta_r - \tan\theta_v} = \frac{2.92\text{ mm}}{\tan 35.7° - \tan 34.5°} = 9\text{ cm}.$$

33.55: a) $n_a \sin\theta_a = n_b \sin\theta_b \Rightarrow \sin\theta_a = n_b \sin\dfrac{A}{2}.$

But $\theta_a = \dfrac{A}{2} + \alpha \Rightarrow \sin\left(\dfrac{A}{2} + \alpha\right) = \sin\dfrac{A + 2\alpha}{2} = n\sin\dfrac{A}{2}.$

At each face of the prism the deviation is α, so $2\alpha = \delta \Rightarrow \sin\dfrac{A+\delta}{2} = n\sin\dfrac{A}{2}$.

b) From part (a), $\delta = 2\arcsin\left(n\sin\dfrac{A}{2}\right) - A$

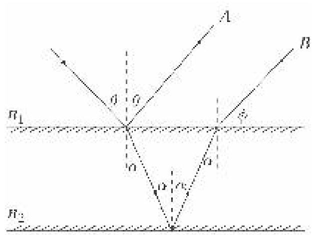
$$\Rightarrow \delta = 2\arcsin\left((1.52)\sin\frac{60.0°}{2}\right) - 60.0° = 38.9.$$

c) If two colors have different indices of refraction for the glass, then the deflection angles for them will differ:

$$\delta_{\text{red}} = 2\arcsin\left((1.61)\sin\frac{60.0°}{2}\right) - 60.0° = 47.2°$$

$$\delta_{\text{violet}} = 2\arcsin\left((1.66)\sin\frac{60.0°}{2}\right) - 60.0° = 52.2° \Rightarrow \Delta\delta = 52.2° - 47.2° = 5.0°.$$

33.56:



Direction of ray A: θ by law of reflection.

Direction of ray B:

At upper surface: $n_1 \sin\theta = n_2 \sin\alpha$

The lower surface reflects at α. Ray B returns to upper surface at angle of incidence $\alpha : n_2 \sin\alpha = n_1 \sin\phi$

Thus

$$n_1 \sin\theta = n_1 \sin\phi$$
$$\phi = \theta$$

Therefore rays A and B are parallel.

33.57: Both l-leucine and d-glutamic acid exhibit linear relationships between concentration and rotation angle. The dependence for l-leucine is:

Rotation angle $(°) = (-0.11°100\text{ ml/g})C(\text{g}/100\text{ ml})$, and for d - glutamic acid is :

Rotation angle $(°) = (0.124°100\text{ ml/g})C(\text{g}/100\text{ ml})$.

33.58: a) A birefringent material has different speeds (or equivalently, wavelengths) in two different directions, so:

$$\lambda_1 = \frac{\lambda_0}{n_1} \text{ and } \lambda_2 = \frac{\lambda_0}{n_2} \Rightarrow \frac{D}{\lambda_1} = \frac{D}{\lambda_2} + \frac{1}{4} \Rightarrow \frac{n_1 D}{\lambda_0} = \frac{n_2 D}{\lambda_0} + \frac{1}{4} \Rightarrow D = \frac{\lambda_0}{4(n_1 - n_2)}.$$

b) $D = \dfrac{\lambda_0}{4(n_1 - n_2)} = \dfrac{5.89\times10^{-7}\text{ m}}{4(1.875-1.635)} = 6.14\times10^{-7}\text{m.}$

33.59: a) The maximum intensity from the table is at $\theta = 35°$, so the polarized component of the wave is in that direction (or else we would not have maximum intensity at that angle).

b) At $\theta = 40° : I = 24.8\text{ W/m}^2 = \dfrac{1}{2}I_0 + I_p\cos^2(40° - 35°)$

$\Rightarrow 24.8\text{ W/m}^2 = 0.500\,I_0 + 0.996 I_p \quad (1).$

At $\theta = 120° : I = 5.2\text{ W/m}^2 = \dfrac{1}{2}I_0 + I_p\cos^2(120° - 35°)$

$\Rightarrow 5.2\text{ W/m}^2 = 0.500 I_0 + 7.60\times10^{-3} I_p \quad (2).$

Solving equations (1) and (2) we find:

$\Rightarrow 19.6\text{ W/m}^2 = 0.989 I_p \Rightarrow I_p = 19.8\text{ W/m}^2.$

Then if one subs this back into equation (1), we find:

$5.049 = 0.500 I_0 \Rightarrow I_0 = 10.1\text{ W/m}^2.$

33.60: a) To let the most light possible through N polarizers, with a total rotation of 90°, we need as little shift from one polarizer to the next. That is, the angle between successive polarizers should be constant and equal to $\dfrac{\pi}{2N}$. Then:

$$I_1 = I_0\cos^2\left(\frac{\pi}{2N}\right), I_2 = I_0\cos^4\left(\frac{\pi}{2N}\right), \cdots \Rightarrow I = I_N = I_0\cos^{2N}\left(\frac{\pi}{2N}\right).$$

b) If $n >> 1$, $\cos^n\theta = \left(1 - \dfrac{\theta^2}{2} + \cdots\right)^n = 1 - \dfrac{n}{2}\theta^2 + \cdots$

$$\Rightarrow \cos^{2N}\left(\frac{\pi}{2N}\right) \approx 1 - \frac{(2N)}{2}\left(\frac{\pi}{2N}\right)^2 = 1 - \frac{\pi^2}{4N} \approx 1, \text{ for large } N.$$

33.61: a) Multiplying Eq. (1) by $\sin\beta$ and Eq. (2) by $\sin\alpha$ yields:

$(1): \dfrac{x}{a}\sin\beta = \sin\omega t\cos\alpha\sin\beta - \cos\omega t\sin\alpha\sin\beta$

$(2): \dfrac{y}{a}\sin\alpha = \sin\omega t\cos\beta\sin\alpha - \cos\omega t\sin\beta\sin\alpha$

Subtracting yields: $\dfrac{x\sin\beta - y\sin\alpha}{a} = \sin\omega t(\cos\alpha\sin\beta - \cos\beta\sin\alpha).$

b) Multiplying Eq. (1) by $\cos\beta$ and Eq. (2) by $\cos\alpha$ yields:

$$(1): \frac{x}{a}\cos\beta = \sin\omega t\cos\alpha\cos\beta - \cos\omega t\sin\alpha\cos\beta$$

$$(2): \frac{y}{a}\cos\alpha = \sin\omega t\cos\beta\cos\alpha - \cos\omega t\sin\beta\cos\alpha$$

Subtracting yields: $\dfrac{x\cos\beta - y\cos\alpha}{a} = -\cos\omega t(\sin\alpha\cos\beta - \sin\beta\cos\alpha).$

(c) Squaring and adding the results of parts (a) and (b) yields:

$$(x\sin\beta - y\sin\alpha)^2 + (x\cos\beta - y\cos\alpha)^2 = a^2(\sin\alpha\ \cos\beta - \sin\beta\cos\alpha)^2$$

(d) Expanding the left-hand side, we have:

$$x^2(\sin^2\beta + \cos^2\beta) + y^2(\sin^2\alpha + \cos^2\alpha) - 2xy\ (\sin\alpha\sin\beta + \cos\alpha\ \cos\beta)$$

$$= x^2 + y^2 - 2xy\ (\sin\alpha\ \sin\beta + \cos\alpha\ \cos\beta) = x^2 + y^2 - 2xy\cos(\alpha - \beta).$$

The right-hand side can be rewritten:

$$a^2(\sin\alpha\cos\beta - \sin\beta\cos\alpha)^2 = a^2\sin^2(\alpha - \beta).$$

Therefore: $x^2 + y^2 - 2xy\cos(\alpha - \beta) = a^2\sin^2(\alpha - \beta).$

Or: $x^2 + y^2 - 2xy\cos\delta = a^2\sin^2\delta$, where $\delta = \alpha - \beta$.

(e) $\delta = 0: x^2 + y^2 - 2xy = (x-y)^2 = 0 \Rightarrow x = y,$ which is a straight diagonal line.

$\delta = \dfrac{\pi}{4}: x^2 + y^2 - \sqrt{2}xy = \dfrac{a^2}{2}$, which is an ellipse.

$\delta = \dfrac{\pi}{2}: x^2 + y^2 = a^2$, which is a circle.

This pattern repeats for the remaining phase differences.

33.62: a) By the symmetry of the triangles, $\theta_b^A = \theta_a^B$, and $\theta_a^C = \theta_r^B = \theta_a^B = \theta_b^A$.

Therefore, $\sin\theta_b^C = n\sin\theta_a^C = n\sin\theta_b^A = \sin\theta_a^A = \theta_b^C = \theta_a^A$.

b) The total angular deflection of the ray is:

$$\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_a^B + \theta_b^C - \theta_a^C = 2\theta_a^A - 4\theta_b^A + \pi.$$

c) From Snell's Law, $\sin\theta_a^A = n\sin\theta_b^A \Rightarrow \theta_b^A = \arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)$

$$\Rightarrow \Delta = 2\theta_a^A - 4\theta_b^A + \pi = 2\theta_a^A - 4\arcsin\left(\frac{1}{n}\sin\theta_a^A\right) + \pi.$$

d) $\dfrac{d\Delta}{d\theta_a^A} = 0 = 2 - 4\dfrac{d}{d\theta_a^A}\left(\arcsin\left(\dfrac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \dfrac{4}{\sqrt{1-\sin^2\theta_1/n^2}}\cdot\left(\dfrac{\cos\theta_1}{n}\right)$

$$\Rightarrow 4\left(1 - \frac{\sin^2\theta_1}{n^2}\right) = \left(\frac{16\cos^2\theta_1}{n^2}\right) \Rightarrow 4\cos^2\theta_1 = n^2 - 1 + \cos^2\theta_1$$

$$\Rightarrow 3\cos^2\theta_1 = n^2 - 1 \Rightarrow \cos^2\theta_1 = \frac{1}{3}(n^2 - 1).$$

e) For violet: $\theta_1 = \arccos\left(\sqrt{\frac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{3}(1.342^2-1)}\right) = 58.89°$

$$\Rightarrow \Delta_{\text{violet}} = 139.2° \Rightarrow \theta_{\text{violet}} = 40.8°.$$

For red: $\theta_1 = \arccos\left(\sqrt{\frac{1}{3}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{3}(1.330^2-1)}\right) = 59.58°$

$$\Rightarrow \Delta_{\text{red}} = 137.5° \Rightarrow \theta_{\text{red}} = 42.5°.$$

Therefore the color that appears higher is red.

33.63: a) For the secondary rainbow, we will follow similar steps to Pr. (34-51). The total angular deflection of the ray is:

$\Delta = \theta_a^A - \theta_b^A + \pi - 2\theta_b^A + \pi - 2\theta_b^A + \theta_a^A - \theta_b^A = 2\theta_a^A - 6\theta_b^A + 2\pi$, where we have used the fact from the previous problem that all the internal angles are equal and the two external equals are equal. Also using the Snell's Law relationship, we have:

$\theta_b^A = \arcsin\left(\frac{1}{n}\sin\theta_a^A\right)$.

$$\Rightarrow \Delta = 2\theta_a^A - 6\theta_b^A + 2\pi = 2\theta_a^A - 6\arcsin\left(\frac{1}{n}\sin\theta_a^A\right) + 2\pi.$$

b) $\frac{d\Delta}{d\theta_a^A} = 0 = 2 - 6\frac{d}{d\theta_a^A}\left(\arcsin\left(\frac{1}{n}\sin\theta_a^A\right)\right) \Rightarrow 0 = 2 - \frac{6}{\sqrt{1-\sin^2\theta_2/n^2}}\cdot\left(\frac{\cos\theta_2}{n}\right)$

$$\Rightarrow n^2(1-\sin^2\theta_2/n^2) = (n^2-1+\cos^2\theta_2) = 9\cos^2\theta_2 \Rightarrow \cos^2\theta_2 = \frac{1}{8}(n^2-1).$$

c) For violet: $\theta_2 = \arccos\left(\sqrt{\frac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{8}(1.342^2-1)}\right) = 71.55°$

$$\Rightarrow \Delta_{\text{violet}} = 233.2° \Rightarrow \theta_{\text{violet}} = 53.2°.$$

For red: $\theta_2 = \arccos\left(\sqrt{\frac{1}{8}(n^2-1)}\right) = \arccos\left(\sqrt{\frac{1}{8}(1.330^2-1)}\right) = 71.94°.$

$$\Rightarrow \Delta_{\text{red}} = 230.1° \Rightarrow \theta_{\text{red}} = 50.1°.$$

Therefore the color that appears higher is violet.

34.1: If up is the $+y$-direction and right is the $+x$-direction, then the object is at $(-x_0, -y_0)$, P_2' is at $(x_0, -y_0)$, and mirror 1 flips the y-values, so the image is at (x_0, y_0) which is P_3'.

34.2: Using similar triangles,

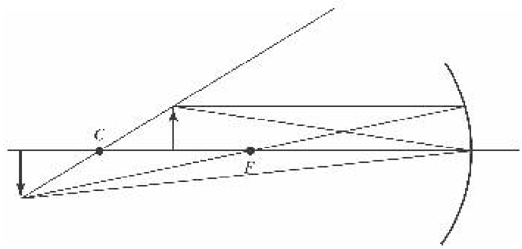
$$\frac{h_{\text{tree}}}{h_{\text{mirror}}} = \frac{d_{\text{tree}}}{d_{\text{mirror}}} \Rightarrow h_{\text{tree}} = h_{\text{mirror}}\frac{d_{\text{tree}}}{d_{\text{mirror}}} = 0.040\text{ m}\frac{28.0\text{ m} + 0.350\text{ m}}{0.350\text{ m}} = 3.24\text{ m}.$$

34.3: A plane mirror does not change the height of the object in the image, nor does the distance from the mirror change. So, the image is 39.2 cm to the right of the mirror, and its height is 4.85 cm.

34.4: a) $f = \dfrac{R}{2} = \dfrac{34.0 \text{ cm}}{2} = 17.0 \text{ cm}.$

b) If the spherical mirror is immersed in water, its focal length is unchanged—it just depends upon the physical geometry of the mirror.

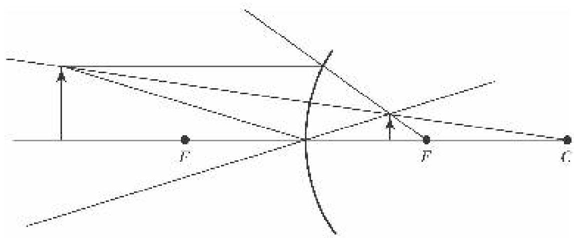
34.5: a)



b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s'} = \dfrac{2}{22.0 \text{ cm}} - \dfrac{1}{16.5 \text{ cm}} \Rightarrow s' = 33.0 \text{ cm},$ to the left of the mirror.

$y' = -y\dfrac{s'}{s} = -(0.600 \text{ cm})\dfrac{33.0 \text{ cm}}{16.5 \text{ cm}} = -1.20 \text{ cm},$ and the image is inverted and real.

34.6: a)



b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s'} = -\dfrac{2}{22.0 \text{ cm}} - \dfrac{1}{16.5 \text{ cm}} \Rightarrow s' = -6.60 \text{ cm},$ to the right of the mirror.

$y' = -y\dfrac{s'}{s} = -(0.600 \text{ cm})\dfrac{(-6.60 \text{ cm})}{16.5 \text{ cm}} = 0.240 \text{ cm},$ and the image is upright and virtual.

34.7: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{1}{1.75\text{ m}}-\frac{1}{5.58\times10^{10}\text{ m}}\Rightarrow s'=-1.75\text{ m}.$

$$\Rightarrow m=-\frac{1.75}{5.58\times10^{10}}=3.14\times10^{-11}\Rightarrow y'=my$$
$$=(3.14\times10^{-11})(6794\times10^{3}\text{ m})=2.13\times10^{-4}\text{ m}.$$

34.8: $R=-3.00\text{ cm},\ \frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=-\frac{2}{3.00\text{ cm}}-\frac{1}{21.0\text{ cm}}\Rightarrow s'=-1.40\text{ cm}$ (in the ball). The magnification is $m=-\frac{s'}{s}=-\frac{-1.40\text{ cm}}{21.0\text{ cm}}=0.0667.$

34.9: a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{fs}\Rightarrow s'=\frac{sf}{s-f}$. Also $m=-\frac{s'}{s}=\frac{f}{f-s}$.

b) For $f>0,\ s>f\Rightarrow s'>0,$ so the image is always on the outgoing side and is real. The magnification is $m=\frac{f}{f-s}<0,$ since $f<s.$

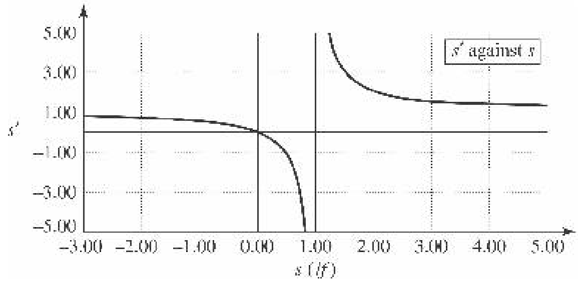
c) For $s\geq2f\Rightarrow|m|<\left|\frac{f}{-f}\right|=1,$ which means the image is always smaller and inverted since the magnification is negative.

For $f<s<2f\Rightarrow0<s-f<f\Rightarrow|m|>\frac{f}{f}=1.$

d) Concave mirror: $0<s<f\Rightarrow s'<0,$ and we have a virtual image to the right of the mirror. $|m|>\frac{f}{f}=1,$ so the image is upright and larger than the object.

34.10: For a convex mirror, $f<0\Rightarrow s'=\frac{sf}{s-f}=-\frac{s|f|}{s+|f|}<0.$ Therefore the image is always virtual. Also $m=\frac{f}{f-s}=\frac{-|f|}{-|f|-s}=\frac{|f|}{|f|+s}>0,$ so the image is erect, and $m<1$ since $|f|+s>|f|,$ so the image is smaller.

34.11: a)



b) $s' > 0$ for $s > f$, $s < 0$.

c) $s' < 0$ for $0 < s < f$.

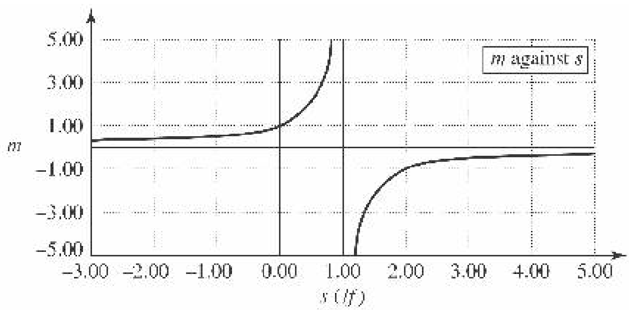
d) If the object is just outside the focal point, then the image position approaches positive infinity.

e) If the object is just inside the focal point, the image is at negative infinity, "behind" the mirror.

f) If the object is at infinity, then the image is at the focal point.

g) If the object is next to the mirror, then the image is also at the mirror.

h)



i) The image is erect if $s < f$.

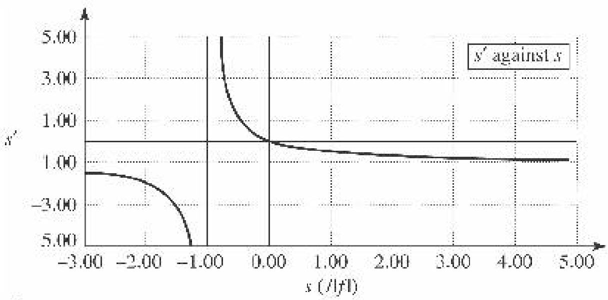
j) The image is inverted if $s > f$.

k) The image is larger if $0 < s < 2f$.

l) The image is smaller if $s > 2f$ or $s < 0$.

m) As the object is moved closer and closer to the focal point, the magnification INCREASES to infinite values.

34.12: a)



a) $s' > 0$ for $-|f| < s < 0$.

b) $s' < 0$ for $s < -|f|$ and $s < 0$.

c) If the object is at infinity, the image is at the outward going focal point.

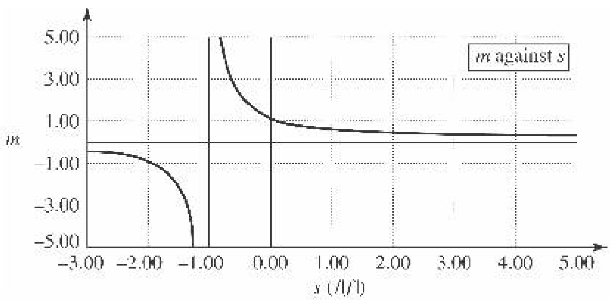
d) If the object is next to the mirror, then the image is also at the mirror. For the answers to (e), (f), (g), and (h), refer to the graph on the next page.

e) The image is erect (magnification greater than zero) for $s > -|f|$.

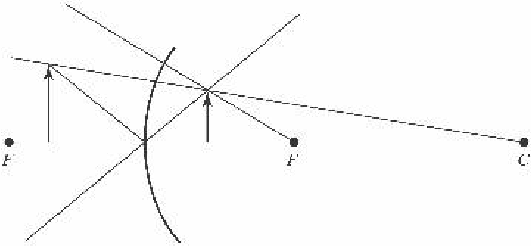
f) The image is inverted (magnification less than zero) for $s < -|f|$.

g) The image is larger than the object (magnification greater than one) for $-2|f| < s < 0$.

h) The image is smaller than the object (magnification less than one) for $s > 0$ and $s < -2|f|$.



34.13: a)



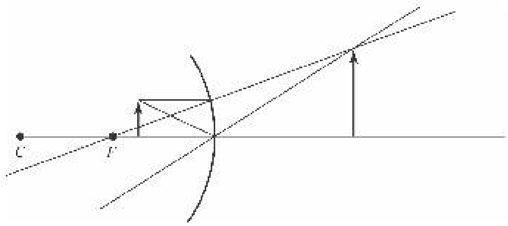
b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s'}=-\frac{2}{20.0\text{ cm}}-\frac{1}{12.0\text{ cm}} \Rightarrow s'=-5.45\text{ cm}$, to the right of the mirror.

$$y'=-y\frac{s'}{s}=-(0.9\text{ cm})\frac{-5.45\text{ cm}}{12.0\text{ cm}}=0.409\text{ cm},$$ and the image is upright and virtual.

34.14: a) $m=-\frac{s'}{s}=-\frac{-48.0}{12.0}=4.00$, where s' comes from part (b).

b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s'}=\frac{2}{32.0\text{ cm}}-\frac{1}{12.0\text{ cm}} \Rightarrow s'=-48.0\text{ cm}$. Since s' is negative, the image is virtual.

c)



34.15: $\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{1.309}{3.50\text{ cm}}+\frac{1.00}{s'}=0 \Rightarrow s'=-2.67\text{ cm}.$

34.16: a) $\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{1.33}{7.00\text{ cm}}+\frac{1.00}{s'}=0 \Rightarrow s'=-5.26\text{cm}$, so the fish appears 5.26 cm below the surface.

b) $\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{1.33}{33.0\text{ cm}}+\frac{1.00}{s'}=0 \Rightarrow s'=-24.8\text{cm}$, so the image of the fish appears 24.8 cm below the surface.

34.17: a) For $R > 0$ and $n_a > n_b$, with $\theta_a = \alpha + \phi$ and $\theta_b = \phi + \beta$, we have:

$$n_b\theta_b = n_a\theta_b \Rightarrow \theta_b = \phi + \beta = \frac{n_a}{n_b}(\alpha + \phi) \Rightarrow n_a\alpha - n_b\beta = (n_b - n_a)\phi.$$

But $\alpha = \frac{h}{s}$, $\beta = \frac{h}{-s'}$, and $\phi = \frac{h}{R}$, so subbing them in one finds:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{(n_b - n_a)}{R}.$$

Also, the magnification calculation yields:

$$\tan\theta_a \frac{y}{s'} \text{ and } \tan\theta_b \frac{y'}{-s'} \Rightarrow \frac{n_a y}{s'} = \frac{n_b y'}{s'} \Rightarrow m = \frac{y'}{y} = -\frac{n_a s'}{n_b s}.$$

b) For $R < 0$ and $n_a < n_b$, with $\theta_a = \alpha - \phi$ and $\theta_b = \beta - \phi$, we have: $n_b\beta - n_a\alpha = (n_b - n_a)\phi$. But $\alpha = \frac{h}{s}$, $\beta = \frac{h}{-s}$, and $\phi = \frac{h}{-R} \Rightarrow -\frac{n_a}{s} - \frac{n_b}{s'} = -\frac{(n_b - n_a)}{R}$, so subbing them in one finds: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{(n_b - n_a)}{R}$. Also, the magnification calculation yields:

$$n_a \tan\theta_a \approx n_b \tan\theta_b \Rightarrow \frac{n_a y}{s} = -\frac{n_b y'}{s'} \Rightarrow m = \frac{y'}{y} = -\frac{n_a s'}{n_a s}.$$

34.18: a) $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{\infty} + \frac{1.60}{s'} = \frac{0.60}{3.00\text{ cm}} \Rightarrow s' = 8.00\text{ cm}.$

b) $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{12.0\text{cm}} + \frac{1.60}{s'} = \frac{0.60}{3.00\text{ cm}} \Rightarrow s' = 13.7\text{ cm}.$

c) $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{2.00\text{ cm}} + \frac{1.60}{s'} = \frac{0.60}{3.00\text{ cm}} \Rightarrow s' = -5.33\text{ cm}.$

34.19: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow n_a = \frac{sR}{(s+R)}\frac{(s'-R)}{s'R}n_b = \frac{s}{s'}\frac{(s'-R)}{(s+R)}n_\text{b}.$

$$\Rightarrow n_a = \frac{90.0\text{ cm}}{160\text{ cm}}\frac{(160\text{ cm} - 3.00\text{ cm})}{(90.0\text{ cm} + 3.00\text{ cm})}(1.60) = 1.52.$$

34.20: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{24.0\text{ cm}} + \frac{1.60}{s'} = \frac{0.60}{4.00\text{ cm}} \Rightarrow s' = 14.8\text{ cm}.$

$y' = \left(\frac{-n_a s'}{n_b s}\right) y = \left(\frac{-14.8\text{ cm}}{(1.60)(24.0\text{ cm})}\right) 1.50\text{ mm} = -0.578\text{ mm}$, so the image height is 0.578 mm, and is inverted.

34.21: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{24.0\text{ cm}}+\frac{1.60}{s'}=\frac{-0.60}{4.00\text{ cm}} \Rightarrow s'=-8.35\text{ cm}.$

$y'=\left(\frac{-n_a s'}{n_b s}\right)y=\left(\frac{-(-8.35\text{ cm})}{(1.60)(24.0\text{ cm})}\right)1.50\text{ mm}=0.326\text{ mm}$, so the image height is 0.326 mm, and is erect.

34.22: a) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1.33}{14.0\text{ cm}}+\frac{1.00}{s'}=\frac{-0.33}{-14.0\text{ cm}} \Rightarrow s'=-14.0\text{ cm}$, so the fish appears to be at the center of the bowl.

$$m=\left(\frac{-n_a s'}{n_b s}\right)=\left(\frac{-(1.33)(-17.0\text{ cm})}{(1.00)(17.0\text{ cm})}\right)=+1.33.$$

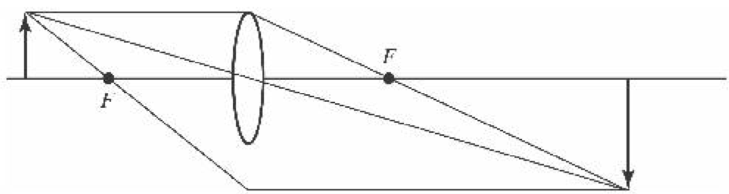
b) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1.00}{\infty}+\frac{1.33}{s'}=\frac{0.33}{+14.0\text{ cm}} \Rightarrow s'=56.4\text{ cm}$, which is outside the bowl.

34.23: For $s=18\text{ cm}$:

a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s'}=\frac{1}{14.0\text{ cm}}-\frac{1}{18.0\text{ cm}} \Rightarrow s'=63.0\text{ cm}.$

b) $m=-\frac{s'}{s}=-\frac{63.0}{18.0}=-3.50.$

c) and d) From the magnification, we see that the image is real and inverted.

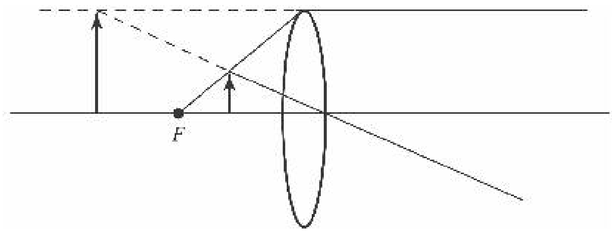


For $s=7.00\text{ cm}$:

a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{s'}=\frac{1}{14.0\text{ cm}}-\frac{1}{7.00\text{ cm}} \Rightarrow s'=-14.0\text{ cm}.$

b) $m=-\frac{s'}{s}=-\frac{-14.0}{7.00}=2.00.$

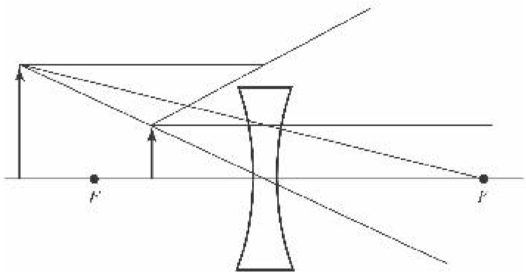
c) and d) From the magnification, we see that the image is virtual and erect.



34.24: a) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{f}=\dfrac{1}{16.0\text{ cm}}+\dfrac{1}{-12.0\text{ cm}}\Rightarrow f=-48.0\text{ cm}$, and the lens is diverging.

b) $y'=y\left(-\dfrac{s'}{s}\right)=(0.850\text{ cm})\left(-\dfrac{(-12.0)}{16.0}\right)=0.638\text{ cm}$, and is erect.

c)



34.25: $m=\dfrac{y'}{y}=\dfrac{1.30}{0.400}=3.25=-\dfrac{s'}{s}$. Also:

$\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s}=\dfrac{1}{7.00\text{ cm}}-\dfrac{1}{s'}\Rightarrow\dfrac{s'}{s}=\dfrac{s'}{7.00\text{ cm}}-1=-3.25\Rightarrow s'=-15.75\text{ cm}$ (to the left).

$\Rightarrow\dfrac{1}{s}=\dfrac{1}{7.0\text{ cm}}-\dfrac{1}{-15.75\text{ cm}}\Rightarrow s=4.85\text{ cm}$, and the image is virtual (since $s'<0$).

34.26: $m=\dfrac{y'}{y}=-\dfrac{4.50}{3.20}=-\dfrac{s'}{s}\Rightarrow s=0.711s'$. Also :

$\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow=\dfrac{1}{0.711\,s'}+\dfrac{1}{s'}=\dfrac{1}{90.0}\Rightarrow s'=217\text{ cm}$ (to the right).

$\Rightarrow s=0.711(217\text{ cm})=154\text{ cm}$, and the image is real (since $s'>0$).

34.27: $\dfrac{1}{s}+\dfrac{1}{s'}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)\Rightarrow \dfrac{1}{18.0\text{ cm}}+\dfrac{1}{s'}=(0.48)\left(\dfrac{1}{5.00\text{ cm}}-\dfrac{1}{3.50\text{ cm}}\right)$

$\Rightarrow s'=-10.3\text{ cm}$ (to the left of the lens).

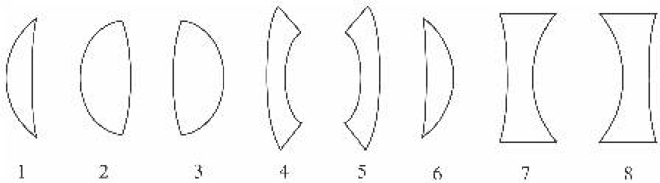
34.28: a) Given $s'=80.0s$, and $s+s'=6.00\text{ m}\Rightarrow 81.00s=6.00\text{ m}\Rightarrow s=0.0741\text{ m}$ and $s'=5.93\text{ m}$.

b) The image is inverted since both the image and object are real $(s'>0,\ s>0)$.

c) $\dfrac{1}{f}=\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{0.0741\text{ m}}+\dfrac{1}{5.93\text{ m}}\Rightarrow f=0.0732\text{ m}$, and the lens is converging.

34.29: $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)=(0.60)\left(\dfrac{1}{\pm 4.00\text{ cm}}-\dfrac{1}{\pm 8.00\text{ cm}}\right)$

$\Rightarrow f=\pm 4.44\text{ cm},\ \pm 13.3\text{ cm}.$



$f_1=+13.3\text{ cm};\ f_2=+4.44\text{ cm};\ f_3=4.44\text{ cm};\ f_4=-13.3\text{ cm};\ f_5=-13.3\text{ cm};\ f_6=+13.3\text{ cm};$
$f_7=-4.44\text{ cm};\ f_8=-4.44\text{ cm}.$

34.30: We have a converging lens if the focal length is positive, which requires:
$\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)>0\Rightarrow\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)>0.$ This can occur in one of three ways:

(i) $\{R_1<R_2\}\cup\{R_1,\ R_2>0\}$ (ii) $R_1>0,\ R_2<0$

(iii) $\{|R_1|>|R_2|\}\cup\{R_1,R_2<0\}$. Hence the three lenses in Fig. (35.29a).

We have a diverging lens if the focal length is negative, which requires:

$\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)<0\Rightarrow\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)<0.$ This can occur in one of three ways:

(i) $\{R_1>R_2\}\cup\{R_1,\ R_2>0\}$ (ii) $R_1>R_2>0$ (iii) $R_1<0,\ R_2>0$.

Hence the three lenses in Fig. (34.29b).

34.31: a) The lens equation is the same for both thin lenses and spherical mirrors, so the derivation of the equations in Ex. (34.9) is identical and one gets:

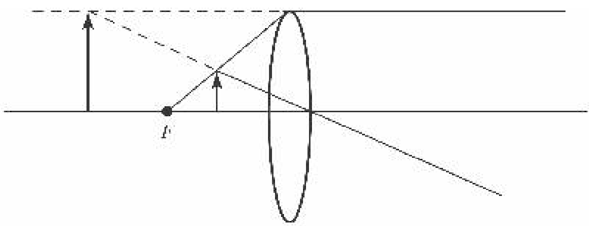
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{s-f}{fs}\Rightarrow s'=\frac{sf}{s-f}, \text{ and also } m=-\frac{s'}{s}=\frac{f}{f-s}.$$

b) Again, one gets exactly the same equations for a converging lens rather than a concave mirror because the equations are identical. The difference lies in the interpretation of the results. For a lens, the outgoing side is *not* that on which the object lies, unlike for a mirror. So for an object on the left side of the lens, a positive image distance means that the image is on the right of the lens, and a negative image distance means that the image is on the left side of the lens.

c) Again, for Ex. (34.10) and (34.12), the change from a convex mirror to a diverging lens changes nothing in the exercises, except for the interpretation of the location of the images, as explained in part (b) above.

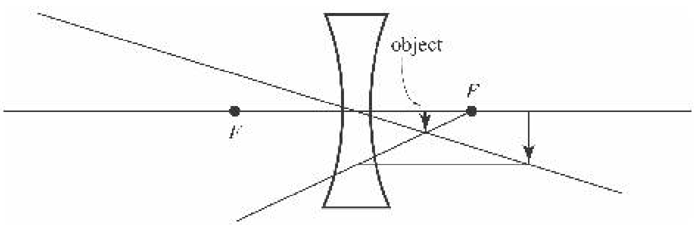
34.32: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{1}{12.0\text{ cm}}-\frac{1}{-17.0\text{ cm}}\Rightarrow s=7.0\text{ cm}.$

$m=-\frac{s'}{s}=-\frac{(-17.0)}{7.2}=+2.4\Rightarrow y=\frac{y'}{m}=\frac{0.800\text{ cm}}{+2.4}=+0.34\text{ cm}$, so the object is 0.34 cm tall, erect, same side.



34.33: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{s}=\frac{1}{-48.0\text{ cm}}+\frac{1}{17.0\text{ cm}}\Rightarrow s=+26.3\text{ cm}.$

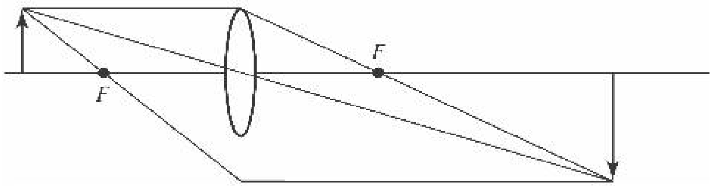
$m=-\frac{s'}{s}=-\frac{-17.0}{+26.3}=0.646\Rightarrow y=\frac{y'}{m}=\frac{0.800\text{ cm}}{0.646}=1.24\text{ cm}$ tall, erect, same side.



34.34: a) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{f}=\frac{1}{16.0\text{ cm}}+\frac{1}{36.0\text{ cm}}\Rightarrow f=11.1\text{ cm}$, converging.

b) $y = y'\left(-\frac{s'}{s}\right) = (0.80\text{ cm})\left(-\frac{36}{16}\right) = -1.8\text{ cm}$, so the image is inverted.

c)



34.35: a) $|m| = \left|\frac{y'}{y}\right| = \left|\frac{s'}{s}\right| \Rightarrow s' = s\left|\frac{y'}{y}\right| = 600\text{ m}\left(\frac{0.024\text{ m}}{240\text{ m}}\right) = 0.0600\text{ m} = 60\text{ mm}.$

$\Rightarrow \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{6.0\times10^5\text{ mm}} + \frac{1}{60\text{ mm}} \Rightarrow f = 60\text{ mm}.$ So one should use the 85-mm lens.

b) $s' = s\left|\frac{y'}{y}\right| = 40.0\text{ m}\left(\frac{0.036\text{ m}}{9.6\text{ m}}\right) = 0.15\text{ m} = 150\text{ mm}.$

$\Rightarrow \frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{40\times10^3\text{ mm}} + \frac{1}{150\text{ mm}} \Rightarrow f = 149\text{ mm}.$ So one should use the 135-mm lens.

34.36: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{3.90\text{ m}} + \frac{1}{s'} = \frac{1}{0.085\text{ m}} \Rightarrow s' = 0.0869\text{ m}.$

$y' = -\frac{s'}{s}y = -\frac{0.0869}{3.90}1750\text{ mm} = 39.0\text{ mm}$, so it will not fit on the 24-mm× 36-mm film.

34.37: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{20.4\text{ cm}} = \frac{1}{20.0\text{ cm}} \Rightarrow s = 1020\text{ cm}.$

34.38: $y' = -\frac{s'}{s}y \cong -\frac{f}{s}y = \frac{5.00\text{ m}}{9.50\times10^3\text{ m}}(70.7\text{ m}) = 0.0372\text{ m} = 37.2\text{ mm}.$

34.39: a) $|m| = \frac{s'}{s} \approx \frac{f}{s} \Rightarrow |m| = \frac{28\text{ mm}}{200{,}000\text{ mm}} = 1.4\times10^{-4}.$

b) $|m| = \frac{s'}{s} \approx \frac{f}{s} \Rightarrow |m| = \frac{105\text{ mm}}{200{,}000\text{ mm}} = 5.3\times10^{-4}.$

c) $|m| = \dfrac{s'}{s} \approx \dfrac{f}{s} \Rightarrow |m| = \dfrac{300 \text{ mm}}{200{,}000 \text{ mm}} = 1.5 \times 10^{-3}.$

34.40: a) $s_1 = \infty \Rightarrow s_1' = f_1 = 12 \text{ cm}.$

b) $s_2 = 4.0 \text{ cm} - 12 \text{ cm} = -8 \text{ cm}.$

c) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{-8 \text{ cm}} + \dfrac{1}{s_2'} = \dfrac{1}{-12 \text{ cm}} \Rightarrow s_2' = 24 \text{ cm}$, to the right.

d) $s_1 = \infty \Rightarrow s_1' = f_1 = 12 \text{ cm}.$

$s_2 = 8.0 \text{ cm} - 12 \text{ cm} = -4 \text{ cm}.$

$\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{-4 \text{ cm}} + \dfrac{1}{s_2'} = \dfrac{1}{-12 \text{ cm}} \Rightarrow s_2' = 6 \text{ cm}.$

34.41: a) $f/4 \Rightarrow 4 = \dfrac{f}{D} \Rightarrow D = \dfrac{f}{4} = \dfrac{300 \text{ mm}}{4} = 75 \text{ mm}.$

b) $f/8 \Rightarrow D = \dfrac{f}{8}$, so the diameter is 0.5 times smaller, and the area is 0.25 times smaller. Therefore only a quarter of the light entered the aperture, and the film must be exposed four times as long for the correct exposure.

34.42: The square of the aperture diameter (~ the area) is proportional to the length of the exposure time required. $\left(\dfrac{1}{30}s\right)\left(\dfrac{8 \text{ mm}}{23.1 \text{ mm}}\right)^2 \cong \left(\dfrac{1}{250}s\right).$

34.43: a) A real image is formed at the film, so the lens must be convex.

b) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f}$ so $\dfrac{1}{s'} = \dfrac{s-f}{sf}$ and $s' = \dfrac{sf}{s-f}$, with $f = +50.0.0 \text{ mm}$

For $s = 45 \text{ cm} = 450 \text{ mm}, s' = 56 \text{ mm}.$

For $s = \infty, s' = f = 50 \text{ mm}.$

The range of distances between the lens and film is 50 mm to 56 mm.

34.44: a) $\dfrac{1}{s} + \dfrac{1}{s'} = \dfrac{1}{f} \Rightarrow \dfrac{1}{s} + \dfrac{1}{9.00 \text{ m}} = \dfrac{1}{0.150 \text{ m}} \Rightarrow s = 0.153 \text{ m} = 15.3 \text{ cm}.$

b) $|m| = \dfrac{s'}{s} = \dfrac{9.00}{0.153} = 58.8 \Rightarrow$ dimensions are $(24 \text{ mm} \times 36 \text{ mm})m = (1.41 \text{ m} \times 2.12 \text{ m}).$

34.45: a) $f=\dfrac{1}{\text{power}}=\dfrac{1}{2.75\ \text{m}^{-1}}=0.364\ \text{m}=36.4\ \text{cm}.$ The near-point is normally at 25 cm: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{25\ \text{cm}}+\dfrac{1}{s'}=\dfrac{1}{36.4\ \text{cm}}\Rightarrow s'=-80\ \text{cm},$ in front of the eye.

b) $f=\dfrac{1}{\text{power}}=\dfrac{1}{-1.30\ \text{m}^{-1}}=-0.769\ \text{m}=-76.9\ \text{cm}.$ The far point is ideally at infinity, so: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{\infty}+\dfrac{1}{s'}=\dfrac{1}{-76.9\ \text{cm}}\Rightarrow s'=-76.9\ \text{cm}.$

34.46: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=\dfrac{n_b-n_a}{R}\Rightarrow\dfrac{1}{40.0\ \text{cm}}+\dfrac{1.40}{2.60\ \text{cm}}=\dfrac{0.40}{R}\Rightarrow R=0.710\ \text{cm}.$

34.47: a) $\dfrac{1}{f}=\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{0.25\ \text{m}}+\dfrac{1}{-0.600\ \text{m}}\Rightarrow\text{power}=\dfrac{1}{f}=+2.33\ \text{diopters}.$

b) $\dfrac{1}{f}=\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{\infty}+\dfrac{1}{-0.600\ \text{m}}\Rightarrow\text{power}=\dfrac{1}{f}=-1.67\ \text{diopters}.$

34.48: a) Angular magnification $M=\dfrac{25.0\ \text{cm}}{f}=\dfrac{25.0\ \text{cm}}{6.00\ \text{cm}}=4.17.$

b) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s}+\dfrac{1}{-25.0\ \text{cm}}=\dfrac{1}{6.00\ \text{cm}}\Rightarrow s=4.84\ \text{cm}.$

34.49: a) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s}+\dfrac{1}{-25.0\ \text{m}}=\dfrac{1}{8.00\ \text{cm}}\Rightarrow s=6.06\ \text{cm}.$

b) $|m|=\dfrac{s'}{s}=\dfrac{25.0\ \text{cm}}{6.06\ \text{cm}}=4.13\Rightarrow y'=ym=(1.00\ \text{mm})(4.13)=4.13\ \text{mm}$

34.50: $\theta=\dfrac{y}{f}\Rightarrow f=\dfrac{y}{\theta}=\dfrac{2.00\ \text{mm}}{0.025\ \text{rad}}=80.0\ \text{mm}=8.00\ \text{cm}.$

34.51: $m=-\dfrac{s'}{s}=6.50\Rightarrow s'=-6.50s\Rightarrow\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{s}+\dfrac{1}{-6.50s}=\dfrac{1}{4.00\ \text{cm}}$

$\Rightarrow\dfrac{1}{s}\left(1-\dfrac{1}{6.50}\right)=\dfrac{1}{4.00}\Rightarrow s=3.38\ \text{cm},\ s'=-6.50s=-22.0\ \text{cm}.$

34.52: a) $M = \frac{(250\text{ mm})s_1'}{f_1 f_2} = \frac{(250\text{ mm})(160\text{ mm} + 5.0\text{ mm})}{(5.00\text{ mm})(26.0\text{ mm})} = 317.$

b) $m = \frac{y'}{y} \Rightarrow y = \frac{y'}{m} = \frac{0.10\text{ mm}}{317} = 3.15 \times 10^{-4}\text{ mm}.$

34.53: a) The image from the objective is at the focal point of the eyepiece, so $s_1' = d_{oe} - f_2 = 19.7\text{ cm} - 1.80\text{ cm} = 17.9\text{ cm}$

$$\Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{s} + \frac{1}{17.9\text{ cm}} = \frac{1}{0.800\text{ cm}} \Rightarrow s = 0.837\text{ cm}.$$

b) $|m_1| = \frac{s'}{s} = \frac{17.9\text{ cm}}{0.837\text{ cm}} = 21.4.$

c) The overall magnification is $|M| = |m_1|\frac{25.0\text{ cm}}{f_2} = (21.4)\frac{25.0\text{ cm}}{1.80\text{ cm}} = 297.$

34.54: Using the approximation $s_1 \approx f$, and then $|m_1| = \frac{s_1'}{f_1}$, we have:

$f = 16\text{ mm}: s' = 120\text{ mm} + 16\text{ mm} = 136\text{ mm};\ s = 16\text{ mm}$

$\Rightarrow |m_1| = \frac{s'}{s} = \frac{136\text{ mm}}{16\text{ mm}} = 8.5.$

$f = 4\text{ mm}: s' = 120\text{ mm} + 4\text{ mm} = 124\text{ mm};\ s = 4\text{ mm} \Rightarrow |m_1| = \frac{s'}{s} = \frac{124\text{ mm}}{4\text{ mm}} = 31.$

$f = 1.9\text{ mm}: s' = 120\text{ mm} + 1.9\text{ mm} = 122\text{ mm};\ s = 1.9\text{ mm}$

$\Rightarrow |m_1| = \frac{s'}{s} = \frac{122\text{ mm}}{1.9\text{ mm}} = 64.$

The eyepiece magnifies by either 5 or 10, so:

a) The maximum magnification occurs for the 1.9-mm objective and 10x eyepiece:

$\Rightarrow M = |m_1|m_e = (64)(10) = 640.$

b) The minimum magnification occurs for the 16-mm objective and 5x eyepiece:

$\Rightarrow M = |m_1|m_e = (8.5)(5) = 43.$

34.55: a) $M = -\frac{f_1}{f_2} = -\frac{95.0\text{ cm}}{15.0\text{ cm}} = -6.33.$

b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{3000\text{ m}}+\frac{1}{s'}=\frac{1}{0.950\text{ m}} \Rightarrow s'=0.950\text{ m},$ so the height of an image of a building is $|y'|=\frac{s'}{s}y=\frac{0.950}{3000}(60.0\text{ m})=0.019\text{ m}.$

c) $\theta'=M\theta=6.33\arctan(60.0/3000)\approx 6.33(60.0)/(3000)=0.127\text{ rad}.$

34.56: $f_1+f_2=d_{ss'} \Rightarrow f_1=d_{ss'}-f_2=1.80\text{ m}-0.0900\text{ m}=1.71\text{ m}$

$\Rightarrow M=-\frac{f_1}{f_2}=-\frac{171}{9.00}=-19.0.$

34.57: $\frac{f}{D}=19.0 \Rightarrow f=(19.0)D=(19.0)(1.02\text{ m})=19.4\text{ m}.$

34.58: $|y'|=y\frac{s'}{s}=y\frac{f}{s}=\theta f=(0.014°)\left(\frac{\pi}{180°}\right)(18\text{ m})=4.40\times10^{-3}\text{ m}.$

34.59: a) $f_1=\frac{R}{2}=0.650\text{ m} \Rightarrow d=f_1+f_2=0.661\text{ m}.$

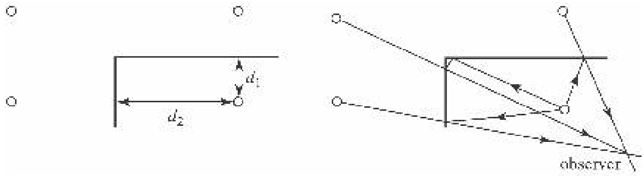
b) $|M|=\frac{f_1}{f_2}=\frac{0.650\text{ m}}{0.011\text{ m}}=59.1.$

34.60: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{0.75\text{ m}-1.3\text{ m}}+\frac{1}{0.75\text{ m}+0.12\text{ m}}=\frac{1}{f} \Rightarrow f=-1.50\text{ m} \Rightarrow R=$ $2f=-3.0\text{ m}.$

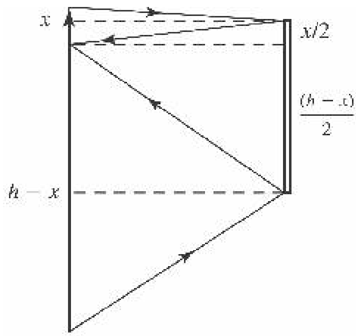
So the smaller mirror must be convex (negative focal length) and have a radius of curvature equal to 3.0 m.

34.61: If you move away from the mirror at 2.40 m/s, then your image moves away from the mirror at the same speed, but in the opposite direction. Therefore you see the image receding at 4.80 m/s, the sum of your speed and that of the image in the mirror.

34.62: a) There are three images formed.



34.63: The minimum length mirror for a woman to see her full height h, is $h/2$, as shown in the figure below.



34.64: $|m| = 2.25 = \frac{s'}{s} = \frac{s + 4.00\text{ m}}{s} \Rightarrow 1.25s = 4.00\text{ m} \Rightarrow s = 3.2\text{ m}.$ So the mirror is 7.20 m from the wall. Also:

$$\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{2}{R} = \frac{1}{3.2\text{ m}} + \frac{1}{7.20\text{ m}} \Rightarrow R = 4.43\text{ m}.$$

34.65: a) $|m| = \frac{y'}{y} = \frac{360}{6.00} = 60.0 = \frac{s'}{s} \Rightarrow s = \frac{8.00\text{ m}}{60.0} = 0.133\text{ m}$ is where the filament should be placed.

b) $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{2}{R} = \frac{1}{0.133\text{ m}} + \frac{1}{8.00\text{ m}} \Rightarrow R = 0.261\text{ m}.$

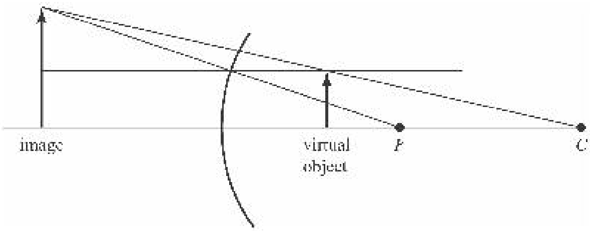
34.66: $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{13.0\text{ m}} + \frac{1}{s'} = \frac{2}{-0.180\text{ m}} \Rightarrow s' = -0.0894\text{ m}.$

$$y' = y\left(-\frac{s'}{s}\right) = (1.50\text{ m})\left(-\frac{-0.0894}{13.0}\right) = 0.0103\text{ m}.$$

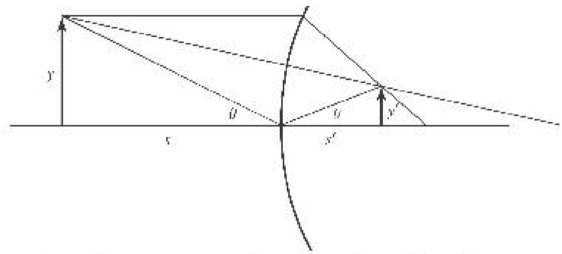
b) The height of the image is less then 1% of the true height of the car, and is less than the image would appear in a plane mirror at the same location. This gives the illusion that the car is further away then "expected."

34.67: a) $R < 0$ and $s < 0$, so a real image $(s' > 0)$ is produced for virtual object positions between the focal point and vertex of the mirror. So for a 24.0 cm radius mirror, the virtual object positions must be between the vertex and 12.0 cm to the right of the mirror. b) The image orientation is erect, since $m = -\frac{s'}{s} = -\frac{s'}{-|s|} > 0.$

c)



34.68: The derivations of Eqs. (34.6) and (34.7) are identical for convex mirrors, as long as one recalls that R and s' are negative. Consider the diagram below:



We have: $\frac{1}{\infty} + \frac{1}{s'} = \frac{2}{R} \Rightarrow s' = f = \frac{R}{2} \Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{2}{R} = \frac{1}{f}$ and $m = \frac{y'}{y} = -\frac{s'}{s}$, since s' is not on the outgoing side of the mirror.

34.69: a) $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{8.0\text{ cm}} + \frac{1}{s'} = \frac{2}{19.4\text{ cm}} \Rightarrow s' = -46\text{ cm},$ so the image is virtual.

b) $m = -\frac{s'}{s} = -\frac{-46}{8.0} = 5.8,$ so the image is erect, and its height is:
$y' = (5.8)y = (5.8)(5.0\text{ mm}) = 29\text{ mm}.$

c) When the filament is 8 cm from the mirror, there is no place where a real image can be formed.

34.70: $m = \frac{5}{2} = \frac{-s'}{s} \Rightarrow s = \frac{-2}{5}s' \Rightarrow$ since $m > 0$, $s' < 0$, $\frac{5}{2s'} - \frac{1}{|s'|} = \frac{2}{R} \Rightarrow \frac{3}{4}R = |s'|$

$\Rightarrow s = \frac{3}{10}R$ and $s' = -\frac{3}{4}R.$

34.71: a) $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ and taking its derivative with respect to s we have

$$0 = \frac{d}{ds}\left(\frac{1}{s} + \frac{1}{s'} - \frac{1}{f}\right) = -\frac{1}{s^2} - \frac{1}{s'^2}\frac{ds'}{ds} \Rightarrow \frac{ds'}{ds} = -\frac{s'^2}{s^2} = -m^2. \text{ But } \frac{ds'}{ds} = m' \Rightarrow m' =$$

$-m^2$. Images are always inverted longitudinally.

b) (i) Front face: $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{200.000 \text{ cm}} + \frac{1}{s'} = \frac{2}{150.000 \text{ cm}} \Rightarrow s' = 120.000 \text{ cm}.$

Rear face: $\frac{1}{s} + \frac{1}{s'} = \frac{2}{R} \Rightarrow \frac{1}{200.100 \text{ cm}} + \frac{1}{s'} = \frac{2}{150.000 \text{ cm}} \Rightarrow s' = 119.964 \text{ cm}.$

(ii) Front face: $m = -\frac{s'}{s} = -\frac{120.000}{200.000} = -0.600000, m' = -m^2 = -(-0.600000)^2 =$

$-0.360.$

Rear face: $m = -\frac{s'}{s} = -\frac{119.964}{200.100} = -0.599520, m' = -m^2 = -(-0.599520)^2 =$

$-0.359425.$

(iii) So the front legs are magnified by 0.600000, the back legs by 0.599520, and the side legs by 0.359425, the average of the front and back longitudinal magnifications.

34.72: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R}$ and taking its derivative with respect to s we have:

$$0 = \frac{d}{ds}\left(\frac{n_a}{s} + \frac{n_b}{s'} - \frac{n_b - n_a}{R}\right) = -\frac{n_a}{s^2} - \frac{n_b}{s'^2}\frac{ds'}{ds}$$

$$\Rightarrow \frac{ds'}{ds} = -\frac{s'^2}{s^2}\frac{n_a}{n_b} = -\left(\frac{s'^2}{s^2}\frac{n_a^2}{n_b^2}\right)\frac{n_b}{n_a} = -m^2\frac{n_b}{n_a}.$$

But $\frac{ds'}{ds} = m' \Rightarrow m' = -m^2\frac{n_b}{n_a}.$

34.73: a) $R < 0$ for convex so $\frac{1}{s} + \frac{1}{s'} = \frac{2}{-R} \Rightarrow s' = \frac{-sR}{2s+R} \Rightarrow v' = \frac{ds'}{dt} = \frac{ds'}{ds}\frac{ds}{dt} =$

$$v\left(\frac{-R}{2s+R}+\frac{2sR}{(2s+R)^2}\right)$$

$$\Rightarrow v' = -v\frac{R^2}{(2s+R)^2} = -(-2.50\ \text{m/s})\frac{(1.25\ \text{m})^2}{(2(10.0\ \text{m})+1.25\ \text{m})^2} = +8.65\times10^{-3}\ \text{m/s}.$$

b) $$v' = -v\frac{R^2}{(2s+R)^2} = -(-2.50\ \text{m/s})\frac{(1.25\ \text{m})^2}{(2(2.0\ \text{m})+1.25\ \text{m})^2} = +0.142\ \text{m/s}.$$

Note: The signs are somewhat confusing. If a *real object* is moving with $v > 0$, this implies it is moving *away* from the mirror. However, if a *virtual image* is moving with $v > 0$, this implies it is moving from "behind" the mirror *toward* the vertex.

34.74: In this context, the microscope just takes an image and makes it visible. The real optics are at the glass surfaces.

$$\frac{n_a}{s}+\frac{n_b}{s'} = 0 \Rightarrow \frac{n}{s}+\frac{1}{s'} = 0 \Rightarrow n = -\frac{s}{s'} = -\frac{2.50\ \text{mm}+0.780\ \text{mm}}{-2.50\ \text{mm}} = 1.31.$$

Note that the object and image are measured from the front surface of the second plate, making the image virtual.

34.75: a) Reflection from the front face of the glass means that the image is just h below the glass surface, like a normal mirror.

b) The reflection from the mirrored surface behind the glass will not be affected because of the intervening glass. The light travels through a distance $2d$ of glass, so the path through the glass appears to be $\frac{2d}{n}$, and the image appears to be $h+\frac{2d}{n}$ behind the front surface of the glass.

c) The distance between the two images is just $\frac{2d}{n}$.

34.76: a) The image from the left end acts as the object for the right end of the rod.

b) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{23.0\text{ cm}}+\frac{1.60}{s'}=\frac{0.60}{6.0\text{ cm}} \Rightarrow s'=28.3\text{ cm}.$

So the second object distance is $s_2 = 40.0\text{ cm} - 28.3\text{ cm} = 11.7\text{ cm}.$

Also: $m_1 = -\frac{n_a s'}{n_b s} = -\frac{28.3}{(1.60)(23.0)} = -0.769.$

c) The object is real and inverted.

d) $\frac{n_a}{s_2}+\frac{n_b}{s_2'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1.60}{11.7\text{ cm}}+\frac{1}{s_2'}=\frac{-0.60}{-12.0\text{ cm}} \Rightarrow s'=-11.5\text{ cm}.$

Also: $m_2 = -\frac{n_a s'}{n_b s} = -\frac{(1.60)(-11.5)}{11.7} = 1.57 \Rightarrow m = m_1 m_2 = (-0.769)(1.57) = -1.21.$

e) So the final image is virtual, and inverted.

f) $y' = (1.50\text{ mm})(-1.21) = -1.82\text{ mm}.$

34.77: a) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{23.0\text{ cm}}+\frac{1.60}{s'}=\frac{0.60}{6.0\text{ cm}} \Rightarrow s'=28.3\text{ cm}.$

So the second object distance is $s_2 = 25.0\text{ cm} - 28.3\text{ cm} = -3.3\text{ cm}.$

Also: $m_1 = -\frac{n_a s'}{n_b s} = -\frac{28.3}{(1.60)(23.0)} = -0.769.$

b) The object is virtual.

c) $\frac{n_a}{s_2}+\frac{n_b}{s_2'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1.60}{-3.3\text{ cm}}+\frac{1}{s_2'}=\frac{-0.60}{-12.0\text{ cm}} \Rightarrow s_2'=1.87\text{ cm}.$

Also:

$$m_2 = -\frac{n_a s_2'}{n_b s_2} = -\frac{(1.60)(1.87)}{-3.3} = 0.901 \Rightarrow m = m_1 m_2 = (-0.769)(0.901) = -0.693.$$

d) So the final image is real and inverted.

e) $y' = ym = (1.50\text{ mm})(-0.693) = -1.04\text{ mm}.$

34.78: For the water-benzene interface to get the apparent water depth:

$$\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{1.33}{6.50\text{ cm}}+\frac{1.50}{s'}=0 \Rightarrow s'=-7.33\text{ cm}.$$

For the benzene-air interface, to get the total apparent distance to the bottom:

$$\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{1.50}{(7.33\text{ cm}+2.60\text{ cm})}+\frac{1}{s'}=0 \Rightarrow s'=-6.62\text{ cm}.$$

34.79: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}$, but $s=\infty, s'=2R \Rightarrow \frac{1}{\infty}+\frac{n}{2R}=\frac{n-1}{R} \Rightarrow \frac{n}{2}=1 \Rightarrow n=2.00.$

34.80: a) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{12.0\text{ cm}}+\frac{1.60}{s'}=\frac{0.60}{15.0\text{ cm}} \Rightarrow s'=-36.9\text{ cm}.$ So the object distance for the far end of the rod is $50.0\text{ cm}-(-36.9\text{ cm})=86.9\text{ cm}.$

$$\Rightarrow \frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1.60}{86.9\text{ cm}}+\frac{1}{s'}=0 \Rightarrow s'=-54.3\text{ cm}.$$

b) The magnification is the product of the two magnifications:

$$m_1=-\frac{n_a s'}{n_b s}=-\frac{-36.9}{(1.60)(12.0)}=1.92,\ m_2=1.00 \Rightarrow m=m_1 m_2=1.92.$$

34.81: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1}{\infty}+\frac{1.80}{s'}=\frac{0.80}{4.00\text{ cm}} \Rightarrow s'=9.00\text{ cm}.$ So the object distance for the far side of the ball is $8.00\text{ cm}-9.00\text{ cm}=-1.00\text{ cm}.$

$$\Rightarrow \frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{1.80}{-1.00\text{ cm}}+\frac{1}{s'}=\frac{-0.80}{-4.00\text{ cm}} \Rightarrow s'=0.50\text{ cm},$$ which is 4.50 cm from the center of the sphere.

34.82: $\frac{n_a}{s}+\frac{n_b}{s'}=0 \Rightarrow \frac{n}{15.0\text{ cm}}+\frac{1}{-9.50\text{ cm}}=0 \Rightarrow n=\frac{15.0}{9.50}=1.58.$

When viewed from the curved end of the rod:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R} \Rightarrow \frac{n}{s}+\frac{1}{s'}=\frac{1-n}{\text{R}} \Rightarrow \frac{1.58}{15.0\text{ cm}}+\frac{1}{s'}=\frac{-0.58}{-10.0\text{ cm}} \Rightarrow s'=-21.1\text{ cm},$$

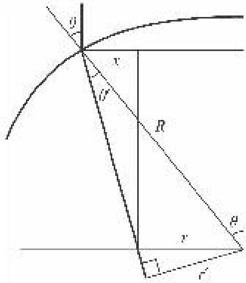
so the image is 21.1 cm within the rod from the curved end.

34.83: a) From the diagram:

$$\sin\theta = \frac{0.190}{R} = 1.50\sin\theta'. \text{ But } \sin\theta' = \frac{r'}{R} \approx \frac{r}{R} = \frac{0.190}{R(1.50)}$$

$$\Rightarrow r = \frac{0.190\text{ cm}}{1.50} = 0.127\text{ cm}.$$

So the diameter of the light hitting the surface is $2r = 0.254$ cm.



b) There is no dependence on the radius of the glass sphere in the calculation above.

34.84: a) Treating each of the goblet surfaces as spherical surfaces, we have to pass, from left to right, through four interfaces. For the empty goblet:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{\infty} + \frac{1.50}{s_1'} = \frac{0.50}{4.00\text{ cm}} \Rightarrow s_1' = 12\text{ cm}$$

$$\Rightarrow s_2 = 0.60\text{ cm} - 12\text{ cm} = -11.4\text{ cm} \Rightarrow \frac{1.50}{-11.4\text{ cm}} + \frac{1}{s_2'} = \frac{-0.50}{3.40\text{ cm}} \Rightarrow s_2' = -64.6\text{ cm}.$$

$$\Rightarrow s_3 = 64.6\text{ cm} + 6.80\text{ cm} = 71.4\text{ cm} \Rightarrow \frac{1}{71.4\text{ cm}} + \frac{1.50}{s_3'} = \frac{0.50}{-3.40\text{ cm}} \Rightarrow s_3' = -9.31\text{ cm}.$$

$$\Rightarrow s_4 = 9.31\text{ cm} + 0.60\text{ cm} = 9.91\text{ cm} \Rightarrow \frac{1.50}{9.91\text{ cm}} + \frac{1}{s_4'} = \frac{-0.50}{-4.00\text{ cm}} \Rightarrow s_4' = -37.9\text{ cm}.$$

So the image is $37.9\text{ cm} - 2(4.0\text{ cm}) = 29.9\text{ cm}$ to the left of the goblet.

b) For the wine-filled goblet:

$$\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{\infty} + \frac{1.50}{s_1'} = \frac{0.50}{4.00\text{ cm}} \Rightarrow s_1' = 12\text{ cm}$$

$$\Rightarrow s_2 = 0.60\text{ cm} - 12\text{ cm} = -11.4\text{ cm} \Rightarrow \frac{1.50}{-11.4\text{ cm}} + \frac{1.37}{s_2'} = \frac{-0.13}{3.40\text{ cm}} \Rightarrow s_2' = 14.7\text{ cm}.$$

$$\Rightarrow s_3 = 6.80\text{ cm} - 14.7\text{ cm} = -7.9\text{ cm} \Rightarrow \frac{1.37}{-7.9\text{ cm}} + \frac{1.50}{s_3'} = \frac{0.13}{-3.40\text{ cm}} \Rightarrow s_3' = 11.1\text{ cm}.$$

$$\Rightarrow s_4 = 0.60\text{ cm} - 11.1\text{ cm} = -10.5\text{ cm} \Rightarrow \frac{1.50}{-10.5\text{ cm}} + \frac{1}{s_4'} = \frac{-0.50}{-4.00\text{ cm}} \Rightarrow s_4' = 3.73\text{ cm},$$

to the right of the goblet.

34.85: Entering the sphere: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{3R}+\frac{4}{3s'}=\frac{1}{3R}\Rightarrow s'=\infty.$

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.33}{\infty}+\frac{1}{s'}=\frac{1}{3R}\Rightarrow s'=3R.$$

So the final image is a distance $3R$ from the right-hand side of the sphere, or $4R$ to the right of the center of the globe.

34.86: a) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{n_a}{f}+\frac{n_b}{\infty}=\frac{n_b-n_a}{R}$ and $\frac{n_a}{\infty}+\frac{n_b}{f'}=\frac{n_b-n_a}{R}.$

$$\Rightarrow\frac{n_a}{f}=\frac{n_b-n_a}{R}\text{ and }\frac{n_b}{f'}=\frac{n_b-n_a}{R}\Rightarrow\frac{n_a}{f}=\frac{n_b}{f'}\Rightarrow n_a=n_b\frac{f}{f'}.$$

b) $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{n_b f}{sf'}+\frac{n_b}{s'}=\frac{n_b(1-f/f')}{R}\Rightarrow\frac{f}{s}+\frac{f'}{s'}=\frac{f'(1-f/f')}{R}=$

$\frac{f'-f}{R}=1.$

Note that the first two equations on the second line can be rewritten as

$\frac{n_a}{n_b-n_a}=\frac{f}{R}$ and $\frac{n_b}{n_b-n_a}=\frac{f'}{R}$ so we can write $\frac{f'-f}{R}=1.$

34.87: Below, x is the distance from object to the screen's original position.

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow\frac{1}{x-30\text{ cm}}+\frac{1}{30\text{ cm}}=\frac{1}{f}\text{ and }\frac{1}{x-26\text{ cm}}+\frac{1}{22\text{ cm}}=\frac{1}{f}$$

$\Rightarrow x^2-56x+450\text{ cm}^2=0\Rightarrow x=46.3\text{ cm}, 9.72\text{ cm}.$ But the object must be to the left of the lens, so $s=46.3\text{ cm}-30\text{ cm}=16.3\text{ cm}$. The corresponding focal length is 10.56 cm.

34.88: We have images formed from both ends. From the first:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{25.0\text{ cm}}+\frac{1.55}{s'}=\frac{0.55}{6.00\text{ cm}}\Rightarrow s'=30.0\text{ cm}.$$

This image becomes the object for the second end:

$$\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.55}{d-30.0\text{ cm}}+\frac{1}{65.0\text{ cm}}=\frac{-0.55}{-6.00\text{ cm}}$$

$$\Rightarrow d-30.0\text{ cm}=20.3\text{ cm}\Rightarrow d=50.3\text{ cm}.$$

34.89: a) For the first lens: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{20.0\text{ cm}}+\frac{1}{s'}=\frac{1}{12.0\text{ cm}}\Rightarrow s'=30.0\text{ cm}.$

So $m_1 = -\dfrac{30.0}{20.0} = -1.50.$

For the second lens: $s = 9.00\text{ cm} - 30.0\text{ cm} = -21.0\text{ cm}.$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{-21.0\text{ cm}}+\frac{1}{s'}=-\frac{1}{12.0\text{ cm}} \Rightarrow s'=-28.0\text{ cm},\ m_2=-\frac{-28.0}{-21.0}=-1.33.$$

So the image is 28.0 cm to the left of the second lens, and is therefore 19.0 cm to the left of the first lens.

b) The final image is virtual.

c) Since the magnification is $m = m_1 m_2 = (-1.50)(-1.33) = 2.00,$ the final image is erect and has a height $y' = (2.00)(2.50\text{ mm}) = 5.00\text{ mm}.$

34.90: a) $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right) = (0.60)\left(\dfrac{1}{12.0\text{ cm}}-\dfrac{1}{28.0\text{ cm}}\right) \Rightarrow f = 35.0\text{ cm}.$

$$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{45\text{ cm}}+\frac{1}{s'}=\frac{1}{35\text{ cm}} \Rightarrow s'=158\text{ cm, and}$$

$$y' = y\left(-\frac{s'}{s}\right) = (0.50\text{ cm})\left(-\frac{158}{45}\right) = -1.76\text{ cm}.$$

b) Adding a second identical lens 315 cm to the right of the first means that the first lens's image becomes an object for the second, a distance of 157 cm from that second lens.

$$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{157\text{ cm}}+\frac{1}{s'}=\frac{1}{35\text{ cm}}$$

$$\Rightarrow s' = 45.0\text{ cm},\ y' = (-1.76\text{ cm})\left(-\frac{45}{157}\right) = 0.5\text{ cm},$$

and the image is erect.

c) Putting an identical lens just 45 cm from the first means that the first lens's image becomes an object for the second, a distance of 113 cm to the right of the second lens.

$$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{-113\text{ cm}}+\frac{1}{s'}=\frac{1}{35\text{ cm}} \Rightarrow s' = 26.7\text{ cm, and } y' = (-1.76\text{ cm})\times\left(\frac{26.7}{113}\right) = -0.41$$

and the image is inverted.

34.91: $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f} \Rightarrow \dfrac{1}{80.0\text{ cm}}+\dfrac{1}{s'}=\dfrac{1}{40.0\text{ cm}} \Rightarrow s' = 80.0\text{ cm}.$

So the object distance for the second lens is 52.0 cm – (8.00 cm) = – 28.0 cm.

$$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{-28.0\text{ cm}}+\frac{1}{s'}=\frac{1}{40.0\text{ cm}} \Rightarrow s'=16.47\text{ cm}.$$

So the object distance for the third lens in 52.0 cm – (16.47 cm) = 35.53 cm.

$\Rightarrow \frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{35.53\text{ cm}}+\frac{1}{s'}=\frac{1}{40.0\text{ cm}} \Rightarrow s'=-318\text{ cm},$ so the final image is virtual and 318 cm to the left of the third mirror, or equivalently 214 cm to the left of the first mirror.

34.92: a) $s+s'=18.0\text{ cm}$ and $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \Rightarrow \frac{1}{18.0\text{ cm}-s'}+\frac{1}{s'}=\frac{1}{3.00\text{ cm}}$

$\Rightarrow (s')^2-(18.0\text{ cm})s'+54.0\text{ cm}^2=0 \Rightarrow s'=14.2\text{ cm},\ 3.80\text{ cm}=s.$ So the screen must either be 3.80 cm or 14.2 cm from the object.

b) $s=3.80\text{ cm}: m=-\frac{s'}{s}=-\frac{3.80}{14.2}=-0.268.$

$s=14.2\text{ cm}: m=-\frac{s'}{s}=-\frac{14.2}{3.80}=-3.74.$

34.93: a) Bouncing first off the convex mirror, then the concave mirror:

$$\frac{1}{s}+\frac{1}{s'}=\frac{2}{R} \Rightarrow \frac{1}{0.600\text{ m}-x}+\frac{1}{s'}=\frac{2}{-0.360\text{ m}} \Rightarrow \frac{1}{s'}=-5.56\text{ m}^{-1}-\frac{1}{x-0.600\text{ m}}$$

$$\Rightarrow s'=\frac{x-0.600\text{ m}}{-5.56\text{ m}^{-1}x+4.33}.$$

But the object distance for the concave mirror is just

$$s=0.600\text{ m}-s'=\frac{4.33x+3.20\text{ m}}{5.56\text{ m}^{-1}x-4.33}.$$

So for the concave mirror: $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R} \Rightarrow \frac{5.56\text{ m}^{-1}x-4.33}{4.33x+3.20\text{ m}}+\frac{1}{x}=\frac{2}{0.360}$

$\Rightarrow 18.5x^2-17.8x+3.20=0 \Rightarrow x=0.72\text{ m},\ 0.24\text{ m}.$

But the object position must be between the mirrors, so the distance must be the smaller of the two above, 0.24 m, from the concave mirror.

b) Now having the light bounce first from the concave mirror, and then the convex mirror, we have:

$$\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{1}{x}+\frac{1}{s'}=\frac{2}{0.360}\Rightarrow\frac{1}{s'}=5.56\ \text{m}^{-1}-\frac{1}{x}\Rightarrow s'=\frac{x}{5.56\ \text{m}^{-1}x-1.00}.$$

But the object distance for the convex mirror is just

$$s=0.600\ \text{m}-s'=\frac{2.33x-0.600\ \text{m}}{5.56\ \text{m}^{-1}x-1}.$$

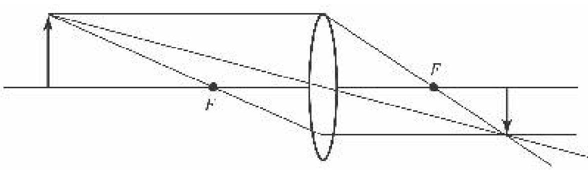
So for the convex mirror: $\frac{1}{s}+\frac{1}{s'}=\frac{2}{R}\Rightarrow\frac{5.56\ \text{m}^{-1}x-1}{2.33x-0.600\ \text{m}}+\frac{1}{x}=\frac{2}{-0.360}$

$\Rightarrow 18.5x^2-2.00x-0.600=0\Rightarrow x=-0.13\ \text{m},\ 0.24\text{m}.$

But the object position must be between the mirrors, so the distance must be 0.24 m from the concave mirror.

34.94: Light passing straight through the lens:

a)



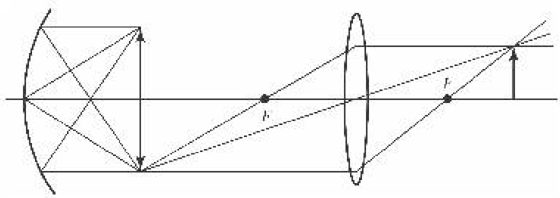
b) $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{85.0\ \text{cm}}+\frac{1}{s'}=\frac{1}{32.0\ \text{cm}}\Rightarrow s'=51.3\ \text{cm},$ to the right of the lens.

c) The image is real.

d) The image is inverted.

For light reflecting off the mirror, and then passing through the lens:

a)



$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{20.0\ \text{cm}}+\frac{1}{s'}=\frac{1}{10.0\ \text{cm}}\Rightarrow s'=20.0\ \text{cm},$ so the image from the mirror, which becomes the new object for the lens, is at the same location as the object. So the final image position is 51.3 cm to the right of the lens, as in the first case above.

c) The image is real

d) The image is erect.

34.95: Parallel light coming in from the left is focused 12.0 cm from the left lens, which is 8.00 cm to right of the second lens. Therefore:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow \frac{1}{-8.00\text{ cm}}+\frac{1}{s'}=\frac{1}{12.0\text{ cm}}\Rightarrow s'=4.80\text{ cm, to the right of the second lens,}$$

and this is where the first focal point of the eyepiece is located. The second focal point is obtained by sending in parallel light from the right, and the symmetry of the lens set-up enables us to immediately state that the second focal point is 4.80 cm to the left of the first lens.

34.96: a) With two lenses of different focal length in contact, the image distance from the first lens becomes exactly minus the object distance for the second lens. So we have:

$$\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow \frac{1}{s_1'}=\frac{1}{f_1}-\frac{1}{s_1} \text{ and } \frac{1}{s_2}+\frac{1}{s_2'}$$

$$=\frac{1}{-s_1'}+\frac{1}{s_2'}=\left(\frac{1}{s_1}-\frac{1}{f_1}\right)+\frac{1}{s_2'}=\frac{1}{f_2}.$$

But overall for the lens system, $\frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f}\Rightarrow \frac{1}{f}=\frac{1}{f_2}+\frac{1}{f_1}.$

b) With carbon tetrachloride sitting in a meniscus lens, we have two lenses in contact. All we need in order to calculate the system's focal length is calculate the individual focal lengths, and then use the formula from part (a).

For the meniscus: $\frac{1}{f_m}=(n_b-n_a)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=$

$$(0.55)\left(\frac{1}{4.50\text{ cm}}-\frac{1}{9.00\text{ cm}}\right)=0.061.$$

For the CCl_4 : $\frac{1}{f_w}=(n_b-n_a)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(0.46)\left(\frac{1}{9.00\text{ cm}}-\frac{1}{\infty}\right)=0.051.$

$$\Rightarrow \frac{1}{f}=\frac{1}{f_2}+\frac{1}{f_1}=0.112\Rightarrow f=8.93\text{ cm.}$$

34.97: At the first surface, $\frac{n_a}{s}+\frac{n_b}{s'}=0\Rightarrow s'=-\frac{n_b}{n_a}s=-\frac{1.60}{1.00}(-14.4\text{ cm})=23.04\text{ cm.}$

At the second surface,

$$s'=14.7\text{ cm}-t=-\frac{n_b}{n_a}s=+\frac{1.00}{1.60}(23.0\text{ cm}-t)\Rightarrow 23.52-1.60t=23.04-t$$

$$\Rightarrow 0.60t=0.48\Rightarrow t=0.80\text{ cm.}$$

(Note, as many significant figures as possible should be kept during the calculation, since numbers comparable in size are subtracted.)

34.98: a) Starting with the two equations:

$$\frac{n_a}{s_1}+\frac{n_b}{s_1'}=\frac{n_b-n_a}{R_1} \text{ and } \frac{n_b}{s_2}+\frac{n_c}{s_2'}=\frac{n_c-n_b}{R_2}, \text{ and using } n_a=n_{\text{liq}}=n_c, n_b=n, \text{ and}$$

$$s_1'=-s_2, \text{ we get}: \frac{n_{\text{liq}}}{s_1}+\frac{n}{s_1'}=\frac{n-n_{\text{liq}}}{R_1} \text{ and } \frac{n}{-s_1'}+\frac{n_{\text{liq}}}{s_2'}=\frac{n_{\text{liq}}-n}{R_2}.$$

$$\Rightarrow \frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{f'}=(n/n_{\text{liq}}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

b) Comparing the equations for focal length in and out of air we have:

$$f(n-1)=f'(n/n_{\text{liq}}-1)=f'\left(\frac{n-n_{\text{liq}}}{n_{\text{liq}}}\right)\Rightarrow f'=\left[\frac{n_{\text{liq}}(n-1)}{n-n_{\text{liq}}}\right]f.$$

34.99: The image formed by the converging lens is 30.0 cm from the converging lens, and becomes a virtual object for the diverging lens at a position 15.0 cm to the right of the diverging lens. The final image is projected $15+19.2=34.2$ cm from the diverging lens.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{-15.0\text{ cm}}+\frac{1}{34.2\text{ cm}}=\frac{1}{f}\Rightarrow f=-26.7\text{ cm}.$$

34.100: The first image formed by the spherical mirror is the one where the light immediately strikes its surface, without bouncing from the plane mirror.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{10.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}}\Rightarrow s'=-7.06\text{ cm}, \text{and the image height:}$$

$$y'=-\frac{s'}{s}y=-\frac{-7.06}{10.0}(0.250\text{ cm})=0.177\text{ cm}.$$

The second image is of the plane mirror image, located $(20.0\text{ cm}+10.0\text{ cm})$ from the vertex of the spherical mirror. So:

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{30.0\text{ cm}}+\frac{1}{s'}=\frac{1}{-24.0\text{ cm}}\Rightarrow s'=-13.3\text{ cm} \text{ and the image height.}$$

$$y'=-\frac{s'}{s}y=-\frac{-13.3}{30.0}(0.250\text{ cm})=0.111\text{ cm}.$$

34.101: $\dfrac{n_a}{s}+\dfrac{n_b}{s'}=0\Rightarrow\dfrac{1}{6.00\text{ cm}}+\dfrac{1.55}{s_1'}=0\Rightarrow s_1'=-9.30\text{ cm}$

$$\Rightarrow s_2=3.50\text{ cm}+9.30\text{ cm}=12.80\text{ cm}\Rightarrow\frac{1.55}{12.8\text{ cm}}+\frac{1}{s_1'}=0\Rightarrow s_1'=-8.26\text{ cm}.$$

So the image is 8.26 cm below the top glass surface, or 1.24 cm above the page.

34.102: a) $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{40\text{ cm}} = 0.52\left(\frac{2}{R}\right) \Rightarrow R = 41.6\text{ cm}.$

At the air–lens interface: $\frac{n_a}{s} + \frac{n_b}{s'} = \frac{n_b - n_a}{R} \Rightarrow \frac{1}{70.0\text{ cm}} + \frac{1.52}{s_1'} = \frac{0.52}{41.6\text{ cm}}$

$\Rightarrow s_1' = -851\text{ cm} = s_2.$

At the lens–water interface: $\Rightarrow \frac{1.52}{851\text{ cm}} + \frac{1.33}{s_2'} = \frac{-0.187}{-41.6\text{ cm}} \Rightarrow s_2' = 491\text{ cm}.$

The mirror reflects the image back (since there is just 90 cm between the lens and mirror.) So, the position of the image is 401 cm to the left of the mirror, or 311 cm to the left of the lens. So:

At the water–lens interface: $\Rightarrow \frac{1.33}{-311\text{ cm}} + \frac{1.52}{s_3'} = \frac{0.187}{41.6\text{ cm}} \Rightarrow s_3' = +173\text{ cm}.$

At the lens–air interface: $\Rightarrow \frac{1.52}{-173\text{ cm}} + \frac{1}{s_4'} = \frac{-0.52}{-41.6\text{ cm}} \Rightarrow s_4' = +47.0\text{ cm},$ to the left of lens.

$$m = m_1 m_2 m_3 m_4 = \left(\frac{n_{a1}s_1'}{n_{b1}s_1}\right)\left(\frac{n_{a2}s_2'}{n_{b2}s_2}\right)\left(\frac{n_{a3}s_3'}{n_{b3}s_3}\right)\left(\frac{n_{a4}s_4'}{n_{b4}s_4}\right)$$

$$= \left(\frac{-851}{70}\right)\left(\frac{491}{-851}\right)\left(\frac{+173}{-311}\right)\left(\frac{+47.0}{-173}\right) = -1.06.$$

(Note all the indices of refraction cancel out.)

b) The image is real.

c) The image is inverted.

d) The final height is $y' = my = (1.06)(4.00\text{ mm}) = 4.24\text{ mm}.$

34.103: a) $m = -\frac{s'}{s} = \frac{y'}{y} = \frac{3}{4}\frac{(0.0360\text{ m})}{(22.7\text{ m})} \Rightarrow s' = (1.19\times10^{-3})\,s$

$$\Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{1}{(1.19\times10^{-3})s} = \frac{1}{s}\left(1 + \frac{1}{1.19\times10^{-3}}\right) = \frac{1}{f} = \frac{1}{0.035\text{ m}} \Rightarrow s = 29.4\text{ m}.$$

b) To just fill the frame, the magnification must be 1.59×10^{-3} so:

$$\Rightarrow \frac{1}{s}\left(1 + \frac{1}{1.59\times10^{-3}}\right) = \frac{1}{f} = \frac{1}{0.035\text{ m}} \Rightarrow s = 22.0\text{ m}.$$

34.104: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{25{,}000\text{ mm}} + \frac{1}{s'} = \frac{1}{35.0\text{ mm}} \Rightarrow s' = 35.05\text{ mm}.$

The resolution of 120 lines per millimeter means that the image line width is 0.0083 mm between lines. That is $y' = 0.0083\text{ mm}.$ But:

$\left|\frac{y'}{y}\right| = \left|\frac{s'}{s}\right| \Rightarrow y = |y'| \cdot \frac{s}{s'} = (0.0083 \text{ mm})\frac{25{,}000 \text{ mm}}{35.05 \text{ mm}} = 5.92 \text{ mm},$ which is the minimum separation between two lines 25.0 m away from the camera.

34.105: a) From the diagram below, we see that $|m| = \frac{d}{W} = \frac{s'}{s} \Rightarrow \frac{1}{s'} = \frac{W}{sd}$.

$\Rightarrow \frac{1}{s} + \frac{1}{s'} = \frac{1}{s} + \frac{W}{sd} = \frac{1}{s}\left(1 + \frac{W}{d}\right) = \frac{d+W}{sd} = \frac{1}{f} \Rightarrow f = \frac{sd}{d+W}$. But when the object is much larger than the image we have the approximation:

$s' \approx f$ and $d + W \approx W \Rightarrow m = \frac{d}{W} \approx \frac{f}{s} \Rightarrow \tan\frac{\theta}{2} = \frac{W}{2} \cdot \frac{1}{s} = \frac{d}{2f} \Rightarrow \theta =$

$2\arctan\left(\frac{d}{2f}\right)$.

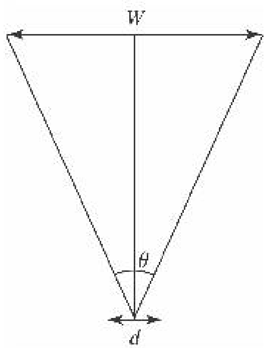
b) The film is $24 \text{ mm} \times 36 \text{ mm}$, so the diagonal length is just:

$d = \left(\sqrt{24^2 + 36^2}\right)\text{mm} = 43.3 \text{ mm. So:}$

$f = 28 \text{ mm}: \theta = 2\arctan\left(\frac{43.3 \text{ mm}}{2(28 \text{ mm})}\right) = 75°.$

$f = 105 \text{ mm}: \theta = 2\arctan\left(\frac{43.3 \text{ mm}}{2(105 \text{ mm})}\right) = 23°.$

$f = 300 \text{ mm}: \theta = 2\arctan\left(\frac{43.3 \text{ mm}}{2(300 \text{ mm})}\right) = 8.2°.$



34.106: $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{1300 \text{ mm}} + \frac{1}{s'} = \frac{1}{90 \text{ mm}} \Rightarrow s' = 96.7 \text{ mm}.$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{6500\text{ mm}}+\frac{1}{s'}=\frac{1}{90\text{ mm}}\Rightarrow s'=91.3\text{ mm}.$$
$$\Rightarrow\Delta s'=96.7\text{ mm}-91.3\text{ mm}=5.4\text{ mm toward the film}$$

34.107: a) At age 10 : $f_n=7\text{ cm}: M=2.0=\dfrac{7\text{ cm}}{f}\Rightarrow f=3.5\text{ cm}.$

b) At age 30 : $f_n=14\text{ cm}: M=2.0=\dfrac{14\text{ cm}}{f}\Rightarrow f=7.0\text{ cm}.$

c) At age 60 : $f_n=200\text{ cm}: M=2.0=\dfrac{200\text{ cm}}{f}\Rightarrow f=100\text{ cm}.$

d) If the 2.8 cm focal length lens is used by the 60-year old, then
$$M=\frac{200\text{ cm}}{f}=\frac{200\text{ cm}}{3.5\text{ cm}}=57.1.$$

e) This does not mean that the older viewer sees a more magnified image. The object is over 28 times further away from the 60-year old, which is exactly the ratio needed to result in the magnification of 2.0 as seen by the 10-year old.

34.108: a) $\dfrac{1}{s}+\dfrac{1}{s'}=\dfrac{1}{f}\Rightarrow\dfrac{1}{s}+\dfrac{1}{-25\text{ cm}}=\dfrac{1}{f}\Rightarrow s=\dfrac{f(25\text{ cm})}{f+25\text{ cm}}.$

b) Height $=y\Rightarrow\theta'=\arctan\left(\dfrac{y}{s}\right)=\arctan\left(\dfrac{y(f+25\text{ cm})}{f(25\text{ cm})}\right)\approx\dfrac{y(f+25\text{cm})}{f(25\text{ cm})}.$

c) $\text{M}=\dfrac{\theta'}{\theta}=\dfrac{y(f+25\text{ cm})}{f(25\text{ cm})}.\dfrac{1}{y/25\text{ cm}}=\dfrac{f+25\text{ cm}}{f}.$

d) If $f=10\text{ cm}\Rightarrow M=\dfrac{10\text{ cm}+25\text{ cm}}{10\text{ cm}}=3.5.$ This is 1.4 times greater than the magnification obtained if the image if formed at infinity $\left(M_\infty=\dfrac{25\text{ cm}}{f}=2.5\right).$

e) Having the first image form just within the focal length puts one in the situation described above, where it acts as a source that yields an enlarged virtual image. If the first image fell just outside the second focal point, then the image would be real and diminished.

34.109: The near point is at infinity, so that is where the image must be found for any objects that are close. So:
$$P=\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}=\frac{1}{24\text{ cm}}+\frac{1}{-\infty}=\frac{1}{0.24\text{ m}}=4.17\text{ diopters}.$$

34.110: $\frac{n_a}{s}+\frac{n_b}{s'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1}{36.0\text{ cm}}+\frac{1.40}{s'}=\frac{0.40}{0.75\text{ cm}}\Rightarrow s'=2.77\text{ cm}.$ This distance is greater than the normal eye, which has a cornea vertex to retina distance of about 2.6 cm.

34.111: a) From Figure 34.56, we define

$$x=r_0-r_0'\Rightarrow r_0'=r_0-x=r_0-r_0\left(\frac{d}{f_1}\right)=\frac{r_0(f_1-d)}{f_1}.$$

b) $s_2=d-f_1\Rightarrow\frac{1}{d-f_1}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow\frac{1}{s_2'}=\frac{d-f_1-f_2}{f_2(d-f_1)}=\frac{|f_2|-f_1+d}{|f_2|(f_1-d)}$

$$\Rightarrow s_2'=\frac{|f_2|(f_1-d)}{|f_2|-f_1+d}$$

. c) $\frac{r_0'}{s_2'}=\frac{r_0}{f}\Rightarrow f=\frac{r_0}{r_0'}s_2'=\frac{f_1}{f_1-d}\cdot\frac{|f_2|(f_1-d)}{|f_2|-f_1+d}\Rightarrow f=\frac{f_1|f_2|}{|f_2|-f_1+d}.$

d) $f_{\max}=\frac{f_1|f_2|}{|f_2|-f_1+d}=\frac{(12.0\text{ cm})(18.0\text{ cm})}{(18.0\text{ cm}-12.0\text{ cm}+0.0\text{ cm})}=36\text{ cm}.$

$$f_{\min}=\frac{f_1|f_2|}{|f_2|-f_1+d}=\frac{(12.0\text{ cm})(18.0\text{ cm})}{(18.0\text{ cm}-12.0\text{ cm}+4.0\text{ cm})}=21.6\text{ cm}.$$

If the effective focal length is 30 cm, then the separation can be calculated:

$$f=\frac{f_1|f_2|}{|f_2|-f_1+d}\Rightarrow 30\text{ cm}=\frac{(12.0\text{ cm})(18.0\text{ cm})}{(18.0\text{ cm}-12.0\text{ cm}+d)}$$

$$\Rightarrow 18.0\text{ cm}-12.0\text{ cm}+d=7.2\text{ cm}\Rightarrow d=1.2\text{ cm}.$$

34.112: First recall that $|M|=\frac{\theta'}{\theta}$, and that $\theta=\left|\frac{y_1'}{f_1}\right|$ and $\theta'=\left|\frac{y_2'}{s_2'}\right|\Rightarrow|M|=\left|\frac{y_2'}{s_2'}\cdot\frac{f_1}{y_1'}\right|.$

But since the image formed by the objective is used as the object for the eyepiece, $y_1'=y_2$. So $|M|=\left|\frac{y_2'}{s_2'}\cdot\frac{f_1}{y_2}\right|=\left|\frac{y_2'}{y_2}\cdot\frac{f_1}{s_2'}\right|=\left|\frac{s_2'}{s_2}\cdot\frac{f_1}{s_2'}\right|=\left|\frac{f_1}{s_2}\right|.$

Therefore, $s_2=\frac{f_1}{|M|}=\frac{48.0\text{ cm}}{36}=1.33\text{ cm},$ and this is just outside the eyepiece focal point.

Now the distance from the mirror vertex to the lens is $f_1+s_2=49.3$ cm, and so $\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow s_2'=\left(\frac{1}{1.20\text{ cm}}-\frac{1}{1.33\text{ cm}}\right)^{-1}=12.3\text{ cm}.$ Thus we have a final image

which is real and 12.3 cm from the eyepiece. (Take care to carry plenty of figures in the calculation because two close numbers are subtracted.)

34.113: a) $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}\Rightarrow\frac{1}{s_1}+\frac{1}{18.0\text{ cm}}=\frac{1}{0.800\text{cm}}\Rightarrow s_1=0.837\text{ cm}.$

$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}\Rightarrow\frac{1}{s_2}+\frac{1}{200\text{cm}}=\frac{1}{7.50\text{cm}}\Rightarrow s_2=7.79\text{cm}.$

Also $m_1=-\frac{s_1'}{s_1}=-\frac{18.0\text{cm}}{0.837\text{ cm}}=-21.5$ and $m_2=-\frac{s_2'}{s_2}=-\frac{200\text{ cm}}{7.79\text{ cm}}=-25.7.$

$\Rightarrow m_{\text{total}}=m_1m_2=(-21.5)(-25.7)=553.$

b) $d=s_1'+s_2=18.0\text{ cm}+7.79\text{ cm}=25.8\text{ cm}.$

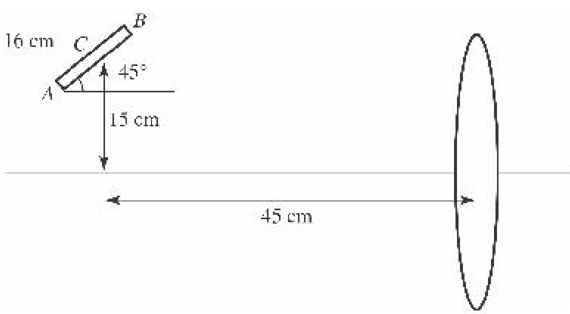
34.114: a) From the figure, $u=\frac{y}{f_1}$ and $u'=\frac{y}{|f_2|}=-\frac{y}{f_2}.$

So the angular magnification is: $M=\frac{u'}{u}=-\frac{f_1}{f_2}.$

b) $M=-\frac{f_1}{f_2}\Rightarrow f_2=-\frac{f_1}{M}=-\frac{95.0\text{ cm}}{6.33}=-15.0\text{ cm}.$

c) The length of the telescope is $95.0\text{ cm}-15.0\text{ cm}=80.0\text{ cm}$, compared to the length of 110 cm for the telescope in Ex. 34.55.

34.115:



a) For point C: $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}\Rightarrow\frac{1}{45.0\text{ cm}}+\frac{1}{s'}=\frac{1}{20.0\text{ cm}}\Rightarrow s'=36.0\text{ cm}.$

$y' = -\frac{s'}{s}y = -\frac{36.0}{45.0}(15.0\text{ cm}) = -12.0\text{ cm}$, so the image of point C is 36.0 cm to the right of the lens, and 12.0 cm below the axis.

For point A: $s = 45.0\text{ cm} + 8.00\text{ cm}(\cos 45°) = 50.7\text{ cm}$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{50.7\text{ cm}} + \frac{1}{s'} = \frac{1}{20.0\text{ cm}} \Rightarrow s' = 33.0\text{ cm}.$$

$y' = -\frac{s'}{s}y = -\frac{33.0}{45.0}(15.0\text{ cm} - 8.00\text{ cm}(\sin 45°)) = -6.10\text{ cm}$, so the image of point A is 33.0 cm to the right of the lens, and 6.10 cm below the axis. For point B: $s = 45.0\text{ cm} - 8.00\text{ cm}(\cos 45°) = 39.3\text{ cm}$

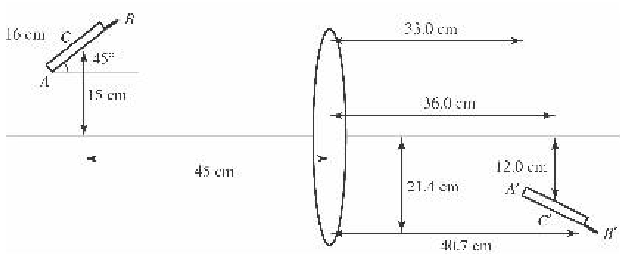
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \Rightarrow \frac{1}{39.3\text{ cm}} + \frac{1}{s'} = \frac{1}{20.0\text{ cm}} \Rightarrow s' = 40.7\text{ cm}.$$

$y' = -\frac{s'}{s}y = -\frac{40.7}{39.3}(15.0\text{ cm} + 8.00\text{ cm}(\sin 45°)) = -21.4\text{ cm}$, so the image of point B is 40.7 cm to the right of the lens, and 21.4 cm below the axis.

b) The length of the pencil is the distance from point A to B:

$$L = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{(33.0\text{ cm} - 40.7\text{ cm})^2 + (6.10\text{ cm} - 21.4\text{ cm})^2}$$

$\Rightarrow L = 17.1\text{ cm}.$



34.116: a) Using the diagram and law of sines

$$\frac{\sin\theta}{(R-f)} = \frac{\sin\alpha}{g} \text{ but } \sin\theta = \frac{h}{R} = \sin\alpha \text{ (Reflection Rule).}$$

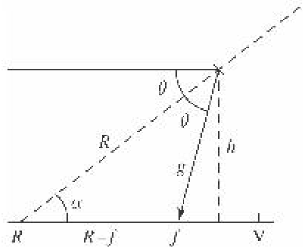
So $g = (R - f)$.

Bisecting the triangle: $\cos\theta = \frac{R/2}{(R-f)} \Rightarrow R\cos\theta - f\cos\theta = \frac{R}{2}$

$$\Rightarrow f = \frac{R}{2}\left[2 - \frac{1}{\cos\theta}\right] = f_0\left[2 - \frac{1}{\cos\theta}\right] = f_0 = \frac{R}{2} \;(\theta \text{ near } 0).$$

b) $\frac{f - f_0}{f_0} = -0.02 \Rightarrow \frac{f}{f_0} = 0.98$ so $2 - \frac{1}{\cos\theta} = 0.98 \Rightarrow \cos\theta = \frac{1}{2 - 0.98} = 0.98$

$\Rightarrow \theta = 11.4°.$



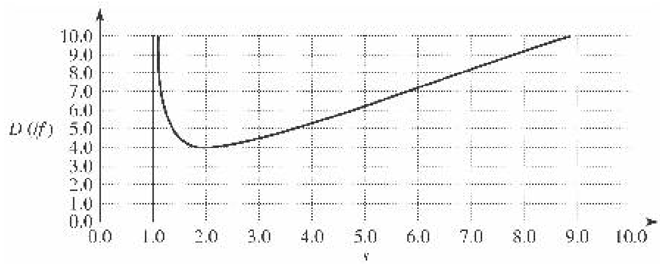
34.117: a) The distance between image and object can be calculated by taking the derivative of the separation distance and minimizing it.

$$D = s + s' \text{ but } s' = \frac{sf}{s-f} \Rightarrow D = s + \frac{sf}{s-f} = \frac{s^2}{s-f}$$

$$\Rightarrow \frac{dD}{ds} = \frac{d}{ds}\left(\frac{s^2}{s-f}\right) = \frac{2s}{s-f} - \frac{s^2}{(s-f)^2} = \frac{s^2 - 2sf}{(s-f)^2} = 0$$

$\Rightarrow s^2 - 2sf = 0 \Rightarrow s(s-2f) = 0 \Rightarrow s = 0,\ 2f = s'$, so for a real image, the minimum separation between object and image is $4f$.

b)

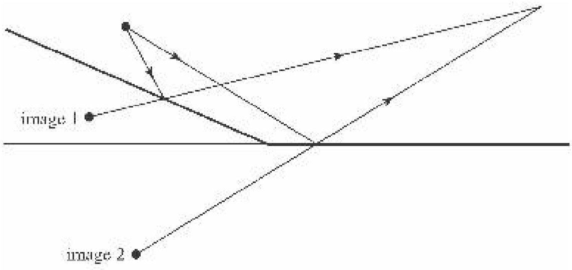


Note that the minimum does occur for D=4f.

34.118: a) By the symmetry of image production, any image must be the same distance D as the object from the mirror intersection point. But if the images and the object are equal distances from the mirror intersection, they lie on a circle with radius equal to D.

b) The center of the circle lies at the mirror intersection as discussed above.

c)



34.119: a) People with normal vision cannot focus on distant objects under water because the image is unable to be focused in a short enough distance to form on the retina. Equivalently, the radius of curvature of the normal eye is about five or six times too great for focusing at the retina to occur.

b) When introducing glasses, let's first consider what happens at the eye:

$$\frac{n_a}{s_2}+\frac{n_b}{s_2'}=\frac{n_b-n_a}{R}\Rightarrow\frac{1.33}{s_2}+\frac{1.40}{2.6\text{ cm}}=\frac{0.07}{0.74\text{ cm}}\Rightarrow s_2=-3.00\text{ cm}.$$ That is, the object for the cornea must be 3.00 cm behind the cornea. Now, assume the glasses are 2.00 cm in front of the eye, then:

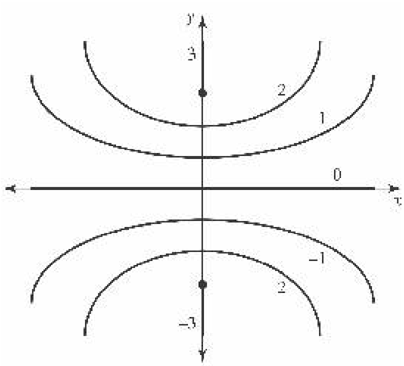
$$s_1'=2.00\text{ cm}+s_2=5.00\text{ cm}\Rightarrow\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1'}\Rightarrow\frac{1}{\infty}+\frac{1}{5.00\text{ cm}}=\frac{1}{f_1'}\Rightarrow f_1'=5.00\text{ cm}.$$

This is the focal length in water, but to get it in air, we use the formula from Problem 34.98: $f_1=f_1'\left[\frac{n-n_{\text{liq}}}{n_{\text{liq}}(n-1)}\right]=(5.00\text{ cm})\left[\frac{1.52-1.333}{1.333(1.52-1)}\right]=1.34\text{ cm}$.

35.1: Measuring with a ruler from both S_1 and S_2 to there different points in the antinodal line labeled $m = 3$, we find that the difference in path length is three times the wavelength of the wave, as measured from one crest to the next on the diagram.

35.2: a) At $S_1, r_2 - r_1 = 4\lambda$, and this path difference stays the same all along the y-axis, so $m = +4$. At $S_2, r_2 - r_1 = -4\lambda$, and the path difference below this point, along the negative y-axis, stays the same, so $m = -4$.

b)



c) The maximum and minimum m-values are determined by the largest integer less than or equal to $\frac{d}{\lambda}$.

d) If $d = 7\frac{1}{2}\lambda \Rightarrow -7 \leq m \leq +7,$ so there will be a total of 15 antinodes between the sources. (Another antinode cannot be squeezed in until the separation becomes six times the wavelength.)

35.3: a) For constructive interference the path diference is $m\lambda, n = 0, \pm 1, \pm 2, \ldots$ The separation between sources is 5.00 m, so for points between the sources the largest possible path difference is 5.00 m. Thus only the path difference of zero is possible. This occurs midway between the two sources, 2.50 m from A.

b) For destructive interference the path difference is $(m + \frac{1}{2})\lambda, m = 0, \pm 1, \pm 2, \ldots$ A path difference of $\pm \lambda/2 = 3.00$ m is possible but a path difference as large as $3\lambda/2 = 9.00\,\text{m}$ is not possible. For a point a distance x from A and $5.00 - x$ from B the path difference is

$x - (5.00\text{ m} - x).$

$x - (5.00\text{ m} - x) = +3.00\text{ m gives } x = 4.00\text{ m}$

$x - (5.00\text{ m} - x) = -3.00\text{ m gives } x = 1.00\text{ m}$

35.4: a) The path difference is 120 m, so for destructive interference:

$\frac{\lambda}{2} = 120 \text{ m} \Rightarrow \lambda = 240 \text{ m}.$

b) The longest wavelength for constructive interference is $\lambda = 120 \text{ m}.$

35.5: For constructive interference, we need $r_2 - r_1 = m\lambda \Rightarrow (9.00 \text{ m} - x) - x = m\lambda$

$$\Rightarrow x = 4.5 \text{ m} - \frac{m\lambda}{2} = 4.5\text{m} - \frac{mc}{2f} = 4.5\text{m} - \frac{m(3.00\times 10^8 \text{ m/s})}{2(120\times 10^6 \text{ Hz})} = 4.5 \text{ m} - m(1.25 \text{ m}).$$

$\Rightarrow x = 0.75$ m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m, 8.25 m. For $m = 3, 2, 1, 0, -1, -2, -3$. (Don't confuse this m with the unit meters, also represented by an "m").

35.6: a) The brightest wavelengths are when constructive interference occurs:

$$d = m\lambda \Rightarrow \lambda = \frac{d}{m} \Rightarrow \lambda_3 = \frac{2040 \text{ nm}}{3} = 680 \text{ nm}, \lambda_4 = \frac{2040 \text{ nm}}{4} = 510 \text{ nm and}$$

$$\lambda_5 = \frac{2040 \text{ nm}}{5} = 408 \text{ nm}.$$

b) The path-length difference is the same, so the wavelengths are the same as part (a).

35.7: Destructive interference occurs for:

$$\lambda = \frac{d}{m + 1/2} \Rightarrow \lambda_3 = \frac{2040 \text{ nm}}{3.5} = 583 \text{ nm and } \lambda_4 = \frac{2040 \text{ nm}}{4.5} = 453 \text{ nm}.$$

35.8: a) For the number of antinodes we have:

$$\sin\theta = \frac{m\lambda}{d} = \frac{mc}{df} = \frac{m(3.00\times 10^8 \text{ m/s})}{(12.0 \text{ m})(1.079\times 10^8 \text{ Hz})} = 0.2317\, m, \text{ so, setting } \theta = 90°,$$

the maximum integer value is four. The angles are $\pm 13.4°$, $\pm 27.6°$, $\pm 44.0°$, and $\pm 67.9°$ for $m = 0, \pm 1, \pm 2, \pm 3, \pm 4$.

b) The nodes are given by $\sin\theta = \frac{(m + 1/2)\lambda}{d} = 0.2317\,(m + 1/2)$. So the angles are $\pm 6.65°$, $\pm 20.3°$, $\pm 35.4°$, $54.2°$ for $m = 0, \pm 1, \pm 2, \pm 3$.

35.9: $\Delta y = \frac{R\lambda}{d} \Rightarrow \lambda = \frac{d\Delta y}{R} = \frac{(4.60\times 10^{-4} \text{ m})(2.82\times 10^{-3} \text{ m})}{2.20 \text{ m}} = 5.90\times 10^{-7} \text{ m}.$

35.10: For bright fringes:

$$d = \frac{Rm\lambda}{y_m} = \frac{(1.20 \text{ m})(20)(5.02\times 10^{-7} \text{ m})}{0.0106 \text{ m}} = 1.14\times 10^{-3} \text{ m} = 1.14 \text{ mm}.$$

35.11: Recall $y_m = \frac{Rm\lambda}{d} \Rightarrow \Delta y_{23} = y_3 - y_2 = \frac{R\lambda(3-2)}{d} = \frac{(0.750 \text{ m})\ (5.00\times 10^{-7} \text{ m})}{4.50\times 10^{-4} \text{ m}}$

$\Rightarrow \Delta y_{23} = 8.33 \times 10^{-4}\ \text{m} = 0.833\ \text{mm}.$

35.12: The width of a bright fringe can be defined to be the distance between its two adjacent destructive minima. Assuming the small angle formula for destructive interference

$$y_m = R\frac{(m+\frac{1}{2})\lambda}{d},$$

the distance between any two successive minima is

$$y_{n+1} - y_n = R\frac{\lambda}{d} = (4.00\ \text{m})\frac{(400\times 10^{-9}\ \text{m})}{(0.200\times 10^{-3}\ \text{m})} = 8.00\ \text{mm}.$$

Thus, the answer to both part (a) and part (b) is that the width is 8.00 mm.

35.13: Use the information given about the bright fringe to find the distance d between the two slits: $y_1 = \dfrac{R\lambda_1}{d}$ (Eq.35.6), so $d = \dfrac{R\lambda_1}{y_1} = \dfrac{(3.00\ \text{m})(600\times 10^{-9}\ \text{m})}{4.84\times 10^{-3}\ \text{m}} = 3.72\times 10^{-4}\ \text{m}.$

(R is much greater than d, so Eq.35.6 is valid.)

The dark fringes are located by $d\sin\theta = (m+\frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots$ The first order dark fringe is located by $\sin\theta = \lambda_2/2d$, where λ_2 is the wavelength we are seeking.

$$y = R\tan\theta \approx R\sin\theta = \frac{\lambda_2 R}{2d}$$

We want λ_2 such that $y = y_1$. This gives $\dfrac{R\lambda_1}{d} = \dfrac{R\lambda_2}{2d}$ and $\lambda_2 = 2\lambda_1 = 1200\ \text{nm}.$

35.14: Using Eq.35.6 for small angles,

$$y_m = R\frac{m\lambda}{d},$$

we see that the distance between corresponding bright fringes is

$$\Delta_y = \frac{Rm}{d}\Delta\lambda = \frac{(5.00\ \text{m})(1)}{(0.300\times 10^{-3}\ \text{m})}(660-470)\times(10^{-9}\ \text{m}) = 3.17\ \text{mm}.$$

35.15: We need to find the positions of the first and second dark lines:

$$\theta_1 = \arcsin\left(\frac{\lambda}{2d}\right) = \arcsin\left(\frac{5.50\times 10^{-7}\ \text{m}}{2(1.80\times 10^{-6}\ \text{m})}\right) = 8.79°$$

$$\Rightarrow y_1 = R\tan\theta_1 = (0.350\ \text{m})\tan(8.79°) = 0.0541\ \text{m}.$$

Also $\theta_2 = \arcsin\left(\dfrac{3\lambda}{2d}\right) = \arcsin\left(\dfrac{3(5.50\times 10^{-7}\ \text{m})}{2(1.80\times 10^{-6}\ \text{m})}\right) = 27.3°$

$$\Rightarrow y_2 R\tan\theta_2 = (0.350\ \text{m})\tan(27.30) = 0.1805\ \text{m}.$$

The fringe separation is then $\Delta y = y_2 - y_1 = 0.1805\ \text{m} - 0.0541\ \text{m} = 0.1264\ \text{m}.$

35.16: (a) Dark fringe implies destructive interference.

$$d \sin\theta = \frac{1}{2}\lambda$$

$$d = \frac{\lambda}{2\sin\theta} = \frac{624\times10^{-9}\,\mathrm{m}}{2\sin 11.0°} = 1.64\ \times 10^{-6}\,\mathrm{m}$$

(b) Bright fringes: $d \sin\theta_{max} = m_{max}\lambda$

The largest that θ can be is $90°$, so $m_{max} = d/\lambda = \frac{1.64\times10^{-6}\,\mathrm{m}}{624\times10^{-9}\,\mathrm{m}} = 2.6$ Since m is an integer, its maximum value is 2. There are 5 bright fringes, the central spot and 2 on each side of it. Dark fringes: $d \sin\theta = \left(m+\frac{1}{2}\right)\lambda$. This equation has solutions for $\theta = \pm11.0°; \pm34.9°;$ and $\pm72.6°$. Therefore, there are 6 dark fringes.

35.17: Bright fringes for wavelength λ are located by $d \sin\theta = m\lambda$. First-order $(m=1)$ is closest to the central bright line, so $\sin\theta = \lambda/d$.

$\lambda = 400$ nm gives $\sin\theta = (400\times10^{-9}\ \mathrm{m})/(0.100\times10^{-3}\ \mathrm{m})$ and $\theta = 0.229°$

$\lambda = 700$ nm gives $\sin\theta = (700\times10^{-9}\ \mathrm{m})/(0.100\times10^{-3}\ \mathrm{m})$ and $\theta = 0.401°$

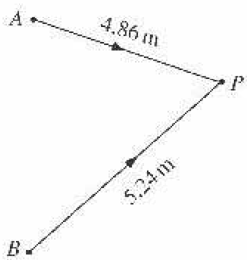
The angular width of the visible spectrum is thus $0.401° - 0.229° = 0.172°$.

35.18: $y = \dfrac{R\lambda}{d} \Rightarrow d = \dfrac{R\lambda}{y} = \dfrac{(1.80\ \mathrm{m})\,(4.50\times10^{-7}\ \mathrm{m})}{4.20\times10^{-3}\ \mathrm{m}} - 1.93\times10^{-4}\,\mathrm{m} = 0.193\ \mathrm{m}.$

35.19: The phase difference ϕ is given by $\phi = (2\pi d/\lambda)\sin\theta$(Eq.35.13)

$\phi = [2\pi(0.340\times10^{-3}\mathrm{m})/(500\times10^{-9}\ \mathrm{m})]\sin 23.0° = 1670$ rad

35.20:



$$\frac{\Delta\phi}{2\pi} = \frac{\text{Path difference}}{\lambda}$$

$$\Delta\phi = 2\pi\left(\frac{524\mathrm{cm} - 486\ \mathrm{cm}}{2\ \mathrm{cm}}\right) = 119\ \text{radians}$$

35.21: a) Eq.(35.10): $I = I_0\cos^2(\phi/2) = I_0(\cos 30.0°)^2 = 0.750 I_0$

b) $60.0° = (\pi/3)$ rad

Eq. (35.11) : $\phi = (2\pi/\lambda)(r_2 - r_1)$, so

$(r_2 - r_1) = (\phi/2\pi)\lambda = [(\pi/3)/2\pi]\lambda = \lambda/6 = 80$ nm

35.22: a) The source separation is 9.00 m, and the wavelength of the wave is $\lambda = \frac{c}{f} = \frac{3.00\times10^8\ \text{m/s}}{1.50\times10^7\ \text{Hz}} = 20.0\ \text{m}$. So there is only one antinode between the sources $(m = 0)$, and it is a perpendicular bisector of the line connecting the sources.

b) $$I = I_0 \cos^2\left(\frac{\phi}{2}\right) = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = I_0 \cos^2\left(\frac{\pi(9.00\ \text{m})}{(20.0\ \text{m})}\sin\theta\right)$$
$$= I_0 \cos^2((1.41)\sin\theta)$$

So, for $\theta = 30°, I = 0.580\ I_0; \theta = 45°, I = 0.295\ I_0;$
$\theta = 60°, I = 0.117\ I_0; \theta = 90°, I = 0.026\ I_0.$

35.23: a) The distance from the central maximum to the first minimum is half the distance to the first maximum, so:

$$y = \frac{R\lambda}{2d} = \frac{(0.700\ \text{m})(6.60\times10^{-7}\ \text{m})}{2(2.60\times10^{-4}\ \text{m})} = 8.88\times10^{-4}\ \text{m}.$$

b) The intensity is half that of the maximum intensity when you are halfway to the first minimum, which is 4.44×10^{-4} m. Remember, all angles are *small*.

35.24: a) $\lambda = \frac{c}{f} = \frac{3.00\times10^8\ \text{m/s}}{1.20\times10^8\ \text{Hz}} = 2.50\ \text{m}$, and we have:

$$\phi = \frac{2\pi}{\lambda}(r_1 - r_2) = \frac{2\pi}{2.50\ \text{m}}(1.8\ \text{m}) = 4.52\ \text{rad}.$$

b) $$I = I_0 \cos^2\left(\frac{\phi}{2}\right) = I_0 \cos^2\left(\frac{4.52\ \text{rad}}{2}\right) = 0.404 I_0.$$

35.25: a) To the first maximum: $y_1 = \frac{R\lambda}{d} = \frac{(0.900\ \text{m})(5.50\times10^{-7}\ \text{m})}{1.30\times10^{-4}\ \text{m}} = 3.81\times10^{-3}\ \text{m}.$

So the distance to the first minimum is one half this, 1.91 mm.

b) The first maximum and minimum are where the waves have phase differences of zero and pi, respectively. Halfway between these points, the phase difference between the waves is $\frac{\pi}{2}$. So:

$$I = I_0 \cos^2\left(\frac{\phi}{2}\right) = I_0 \cos^2\left(\frac{\pi}{4}\right) = \frac{I_0}{2} = 2.00\times10^{-6}\ \text{W/m}^2.$$

35.26: From Eq. (35.14), $I = I_0 \cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)$. So the intensity goes to zero when the cosine's argument becomes an odd integer of $\frac{\pi}{2}$. That is : $\frac{\pi d}{\lambda}\sin\theta = (m + 1/2)\pi \Rightarrow$ $d\sin\theta = \lambda(m+1/2)$, which is Eq. (35.5).

35.27: By placing the paper between the pieces of glass, the space forms a cavity whose height varies along the length. If *twice* the height at any given point is one wavelength (recall it has to make a return trip), constructive interference occurs. The distance between the maxima (i.e., the # of meters per fringe) will be

$$\Delta x = \frac{\lambda l}{2h} = \frac{\lambda}{2\tan\theta} \Rightarrow \theta = \arctan\left(\frac{\lambda}{2\Delta x}\right) = \arctan\left(\frac{5.46\times10^{-7}\ \text{m}}{2((1/1500)\ \text{m})}\right) = 4.095\times10^{-4}\ \text{rad} = 0.0235°.$$

35.28: The distance between maxima is

$$\Delta x = \frac{\lambda l}{2h} = \frac{(6.56\times10^{-7}\ \text{m})\ (9.00\ \ \text{cm})}{2(8.00\times10^{-5}\ \text{m})} = 0.0369\,\text{cm}.$$

So the number of fringes per centimeter is $\frac{1}{\Delta x} = 27.1$ fringes/cm.

35.29: Both parts of the light undergo half-cycle phase shifts when they reflect, so for destructive interference $t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{6.50\times10^{-7}\ \text{m}}{4(1.42)} = 1.14\times10^{-7}\ \text{m} = 114\ \text{nm}.$

35.30: There is a half-cycle phase shift at both interfaces, so for destructive interference:

$$t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{480\ \text{nm}}{4(1.49)} = 80.5\ \text{nm}.$$

35.31: Destructive interference for $\lambda_1 = 800\,\text{nm}$ incident light. Let n be the refractive index of the oil. There is a $\lambda/2$ phase shift for the reflection at the air-oil interface but no phase shift for the reflection at the oil-water interface. Therefore, there is a net $\lambda/2$ phase difference due to the reflections, and the condition for destructive interference is $2t = m(\lambda/n)$. Smallest nonzero thickness means $m=1$, so $2tn = \lambda_1$.

The condition for constructive interference with incident wavelength λ is $2t = (m+\frac{1}{2})(\lambda/n)$ and $2tn = (m+\frac{1}{2})\lambda$.

But $2tn = \lambda_1$, so $\lambda = \lambda_1/(m+\frac{1}{2})$, where $\lambda_1 = 800$ nm.

for $m=0, \lambda = 1600$ nm

for $m=1, \lambda = 533$ nm

for $m=2, \lambda = 320$ nm, and so on

The visible wavelength for which there is constructive interference is 533 nm.

35.32: a) The number of wavelengths is given by the total extra distance traveled, divided by the wavelength, so the number is

$$\frac{x}{\lambda} = \frac{2tn}{\lambda_0} = \frac{2(8.76\times10^{-6}\text{ m})(1.35)}{6.48\times10^{-7}\text{m}} = 36.5.$$

b) The phase difference for the two parts of the light is zero because the path difference is a half-integer multiple of the wavelength and the top surface reflection has a half-cycle phase shift, while the bottom surface does not.

35.33: Both rays, the one reflected from the pit and the one reflected from the flat region between the pits, undergo the same phase change due to reflection. The condition for destructive interference is $2t = (m+\frac{1}{2})(\lambda/n)$, where n is the refractive index of the plastic substrate. The minimum thickness is for $m = 0$, and equals $t = \lambda/(4n) = 790\text{ nm}/[(4)(1.8)] = 110\text{ nm} = 0.11\mu\text{m}.$

35.34: A half-cycle phase change occurs, so for destructive interference

$$t = \frac{\lambda}{2} = \frac{\lambda_0}{2n} = \frac{480\text{nm}}{2(1.33)} = 180\text{ nm}.$$

35.35: a) To have a strong reflection, constructive interference is desired. One part of the light undergoes a half-cycle phase shift, so:

$$2d = \left(m+\frac{1}{2}\right)\frac{\lambda}{n} \Rightarrow \lambda = \frac{2dn}{\left(m+\frac{1}{2}\right)} = \frac{2(290\text{ nm})(1.33)}{\left(m+\frac{1}{2}\right)} = \frac{771\text{ nm}}{\left(m+\frac{1}{2}\right)}.$$ For an integer value of zero, the wavelength is not visible (infrared) but for $m = 1$, the wavelength is 514 nm, which is green.

b) When the wall thickness is 340 nm, the first visible constructive interference occurs again for $m = 1$ and yields $\lambda = \dfrac{904\text{ nm}}{\left(m+\frac{1}{2}\right)} = 603$ nm, which is orange.

35.36: a) Since there is a half-cycle phase shift at just one of the interfaces, the minimum thickness for constructive interference is:

$$t = \frac{\lambda}{4} = \frac{\lambda_0}{4n} = \frac{550\text{ nm}}{4(1.85)} = 74.3\text{ nm}.$$

b) The next smallest thickness for constructive interference is with another half wavelength thickness added: $t = \dfrac{3\lambda}{4} = \dfrac{3\lambda_0}{4n} = \dfrac{3(550\text{ nm})}{4(1.85)} = 223$ nm.

35.37: $x = \dfrac{m\lambda}{2} = \dfrac{1800(6.33\times10^{-7}\ \text{m})}{2} = 5.70\times10^{-4}\ \text{m} = 0.570\ \text{mm}.$

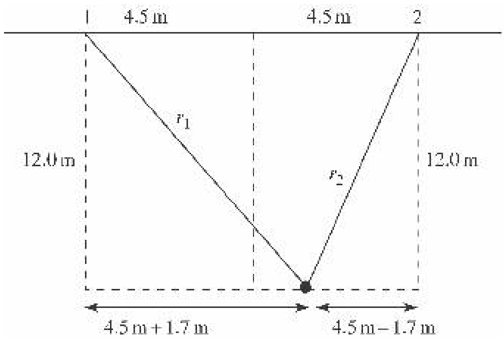
35.38: a) For Jan, the total shift was $\Delta x_1 = \dfrac{m\lambda 1}{2} = \dfrac{818(6.06\times10^{-7}\ \text{m})}{2} = 2.48\times10^{-4}\text{m}.$

For Linda, the total shift was $\Delta x_2 = \dfrac{m\lambda^2}{2} = \dfrac{818(5.02\times10^{-7}\ \text{m})}{2} = 2.05\times10^{-4}\ \text{m}.$

b) The net displacement of the mirror is the difference of the above values:
$\Delta x = \Delta x_1 = \Delta x_2 = 0.248\ \text{mm} - 0.205\ \text{mm} = 0.043\ \text{mm}.$

35.39: Immersion in water just changes the wavelength of the light from Exercise 35.11, so: $y = \dfrac{R\lambda}{dn} = \dfrac{y_{\text{vacuum}}}{n} = \dfrac{0.833\ \text{mm}}{1.33} = 0.626\ \text{mm}$, using the solution from Exercise 35.11.

35.40: Destructive interference occurs 1.7 m from the centerline.



$r_1 = \sqrt{(12.0\ \text{m})^2 + (6.2\ \text{m})^2} = 13.51\ \text{m}$

$r_2 = \sqrt{(12.0\ \text{m})^2 + (2.8\ \text{m})^2} = 12.32\ \text{m}$

For destructive interference, $r_1 - r_2 = \lambda/2 = 1.19\ \text{m}$ and $\lambda = 2.4\ \text{m}$. The wavelength we have calculated is the distance between the wave crests.

Note: The distance of the person from the gaps is not large compared to the separation of the gaps, so the path length is not accurately given by $d\sin\theta$.

35.41: a) Hearing minimum intensity sound means that the path lengths from the individual speakers to you differ by a half-cycle, and are hence out of phase by 180° at that position.

b) By moving the speakers toward you by 0.398 m, a maximum is heard, which means that you moved the speakers one-half wavelength from the min and the signals are back

in phase. Therefore the wavelength of the signals is 0.796 m, and the frequency is
$f=\frac{v}{\lambda}=\frac{340\text{ m/s}}{0.796\text{ m}}$=427 Hz.

c) To reach the next maximum, one must move an additional distance of one wavelength, a distance of 0.796 m.

35.42: To find destructive interference, $d=r_2-r_1=\sqrt{(200\ m)^2+x^2}-x=\left(m+\frac{1}{2}\right)\lambda$

$$\Rightarrow (200\text{ m})^2+x^2=x^2+\left[\left(m+\frac{1}{2}\right)\lambda\right]^2+2x\left(m+\frac{1}{2}\right)\lambda$$

$$\Rightarrow x=\frac{20{,}000\text{ m}^2}{\left(m+\frac{1}{2}\right)\lambda}-\frac{1}{2}\left(m+\frac{1}{2}\right)\lambda.$$

The wavelength is calculated by $\lambda=\frac{c}{f}=\frac{3.00\times10^8\text{ m/s}}{5.80\times10^6\text{ Hz}}=51.7$ m.

$\Rightarrow m=0:x=761$ m, and $m=1:x=219$ m, and $m=2:x=90.1$ m, and $m=3;\ x=20.0$ m.

35.43: At points on the same side of the centerline as point A, the path from B is longer than the path from A, and the path difference $d\ \sin\theta$ puts speaker A ahead of speaker B in phase. Constructive interference occurs when

$$d\sin\theta-\lambda/6=\left(m+\frac{1}{2}\right)\lambda,\ m=0,1,2,\ldots$$

$$\sin\theta=\left(m+\frac{2}{3}\right)(\lambda/d)=\left(m+\frac{2}{3}\right)(0.2381),\ m=0,1,2,\ldots$$

$m=0, 9.13°;\ m=1, 23.4°;\ m=2, 39.4°;\ m=3, 60.8°;\ m=4,$ no solution

At points on the other side of the centerline, the path from A is longer than the path from B, and the path difference $d\ \sin\theta$ puts speaker A behind speaker B in phase. Constructive interference occurs when

$$d\sin\theta+\lambda/6=\left(m+\frac{1}{2}\right)\lambda,\ m=0,1,2,\ldots$$

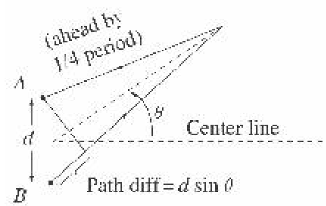
$$\sin\theta=\left(m+\frac{2}{3}\right)(\lambda/d)=\left(m+\frac{1}{3}\right)(0.2381),\ m=0,1,2,\ldots$$

$m=0, 4.55°;\ m=1, 18.5°;\ m=2, 33.7°;\ m=3, 52.5°;\ m=4,$ no solution

35.44: First find out what fraction the 0.159 ms time lag is of the period.

$$\Delta t = \frac{0.159\times10^{-3}s}{T} = (0.159\times10^{-3}\,\text{s})f = (0.159\times10^{-3}\text{s})\;(1570\,\text{Hz})$$

$\Delta t = 0.250$, so the speakers are $1/4$ period out of phase. Let A be ahead of B in phase.



$$\lambda = v/f = \frac{330\text{ m/s}}{1570\text{ Hz}} = 0.210\text{ m}$$

On A's side of centerline: Since A is ahead by $1/4$ period, the path difference must retard B's phase enough so the waves are in phase.

$$d\sin\theta = \frac{3}{4}\lambda, \frac{7}{4}\lambda, \ldots$$

$$\sin\theta_1 = \frac{3}{4}\left(\frac{0.210\text{ m}}{0.422\text{ m}}\right) \rightarrow \theta_1 = 21.9°$$

$$\sin\theta_2 = \frac{7}{4}\left(\frac{0.210\text{ m}}{0.422\text{ m}}\right) \rightarrow \theta_2 = 60.6°$$

On B's side of centerline: The path difference must now retard A's sound by $\frac{1}{4}\lambda, \frac{5}{4}\lambda, \ldots$

$$-d\sin\theta = \frac{1}{4}\lambda, \frac{5}{4}\lambda, \ldots \text{ gives } -7.2°, -38.5°$$

35.45: a) If the two sources are out of phase by one half-cycle, we must add an extra half a wavelength to the path difference equations Eq. (35.1) and Eq. (35.2).

This exactly changes one for the other, for $m \rightarrow m+\frac{1}{2}$ and $m+\frac{1}{2} \rightarrow m$, since m in any integer.

b) If one source leads the other by a phase angle ϕ, the fraction of a cycle difference is $\frac{\phi}{2\pi}$. Thus the path length difference for the two sources must be adjusted for both destructive and constructive interference, by this amount. So for constructive inference: $r_1 - r_2 = (m + \phi/2\pi)\lambda$, and for destructive interference, $r_1 - r_2 = (m + 1/2 + \phi/2\pi)\lambda$.

35.46: a) The electric field is the sum of the two wave functions, and can be written: $E_p(t) = E_2(t) + E_1(t) = E\cos(\omega t) + E\cos(\omega t + \phi) \Rightarrow E_p(t) = 2E\cos(\phi/2)\cos(\omega t + \phi/2)$.

b) $E_p(t) = A\cos(\omega t + \phi/2)$, so comparing with part (a), we see that the amplitude of the wave (which is always positive) must be $A = 2E\,|\cos(\phi/2)|$.

c) To have an interference maximum, $\dfrac{\phi}{2} = 2\pi m$. So, for example, using $m = 1$, the relative phases are $E_2: \varphi = 0$; $E_1: \varphi = \phi = 4\pi$; $E_p: \varphi = \dfrac{\phi}{2} = 2\pi$, and all waves are in phase.

d) To have an interference minimum, $\dfrac{\phi}{2} = \pi\left(m + \dfrac{1}{2}\right)$. So, for example using $m = 0$, relative phases are $E_2: \varphi = 0$; $E_1: \varphi = \phi = \pi$; $E_p: \varphi = \phi/2 = \pi/2$, and the resulting wave is out of phase by a quarter of a cycle from both of the original waves.

e) The instantaneous magnitude of the Poynting vector is:

$$|\vec{S}| = \varepsilon_0 c E_p{}^2(t) = \varepsilon_0 c(4E^2\cos^2(\phi/2)\cos^2(\omega t + \phi/2)).$$

For a time average, $\cos^2(\omega t + \phi/2) = \dfrac{1}{2}$, so $|S_{av}| = 2\varepsilon_0 c E^2\cos^2(\phi/2)$.

35.47: a)

$$\Delta r = m\lambda$$

$$r_1 = \sqrt{x^2 + (y-d)^2}.$$

$$r_2 = \sqrt{x^2 + (y+d)^2}.$$

$$\text{So } \Delta r = \sqrt{x^2 + (y+d)^2} - \sqrt{x^2 + (y-d)^2} = m\lambda.$$

b) The definition of hyperbola is the locus of points such that the difference between P to S_2 and P to S_1 is a constant. So, for a given m and λ we get a hyperbola. Or, in the case of all m for a given λ, a family of hyperbola.

c) $\sqrt{x^2 + (y+d)^2} - \sqrt{x^2 + (y-d)^2} = (m + \frac{1}{2})\lambda.$

35.48: a) $E_p^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(\pi - \phi)$

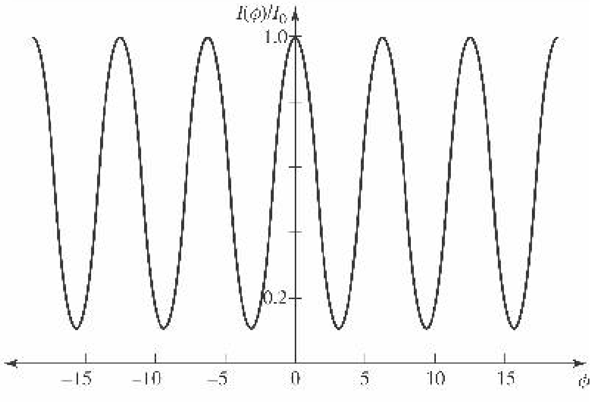
$$= E^2 + 4E^2 + 4E^2\cos\phi = 5E^2 + 4E^2\cos\phi.$$

$$I = \frac{1}{2}\varepsilon_0 c E_p{}^2 = \varepsilon_0 c\left[\left(\frac{5}{2}E^2\right) + \left(\frac{4}{2}E^2\right)\cos\phi\right].$$

$$\phi = 0 \Rightarrow I_0 = \frac{9}{2}\varepsilon_0 c E^2.$$

$$\text{So } I = I_0\left[\frac{5}{9} + \frac{4}{9}\cos\phi\right].$$

b) $I_{\min} = \frac{1}{9} I_0$ which occurs when $\phi = n\pi (n \text{ odd})$.



35.49: For this film on this glass, there is a net $\lambda/2$ phase change due to reflection and the condition for destructive interference is $2t = m(\lambda/n)$, where $n = 1.750$.
Smallest nonzero thickness is given by $t = \lambda/2n$.
At 20.0°C, $t_0 - (582.4 \text{ nm})/[(2)(1.750)] = 166.4 \text{ nm}$.
At 170°C, $t_0 = (588.5 \text{ nm})/[(2)(1.750)] = 168.1 \text{ nm}$.
$t = t_0(1 + \alpha \Delta T)$ so
$\alpha = (t - t_0)/(t_0 \Delta T) = (1.7 \text{ nm})/[(166.4 \text{ nm})(150\text{C}^\circ)] = 6.8 \times 10^{-5} \ (\text{C}^\circ)^{-1}$

35.50: For constructive interference: $d \sin\theta = m\lambda_1 \Rightarrow d \sin\theta = 3(700 \text{ nm}) = 2100 \text{ nm}$.

For destructive interference: $d \sin\theta = \left(m + \frac{1}{2}\right)\lambda_2 \Rightarrow \lambda_2 = \frac{d \sin\theta}{m + \frac{1}{2}} = \frac{2100 \text{ nm}}{m + \frac{1}{2}}$.

So the possible wavelengths are $\lambda_2 = 600$ nm, for $m = 3$, and $\lambda_2 = 467$ nm, for $m = 4$.

Both d and θ drop out of the calculation since their combination is just the path difference, which is the same for both types of light.

35.51: First we need to find the angles at which the intensity drops by one-half from the value of the m th bright fringe.

$$I = I_0 \cos^2\left(\frac{\pi d}{\lambda} \sin\theta\right) = \frac{I_0}{2} \Rightarrow \frac{\pi d}{\lambda} \sin\theta \approx \frac{\pi d \theta_m}{\lambda} = (m + 1/2)\frac{\pi}{2}.$$

$$\Rightarrow m = 0 : \theta = \theta_m^- = \frac{\lambda}{4d}; m = 1 : \theta = \theta_m^+ = \frac{3\lambda}{4d} \Rightarrow \Delta\theta_m = \frac{\lambda}{2d'}$$

so there is no dependence on the m - value of the fringe.

35.52: There is just one half-cycle phase change upon reflection, so for constructive interference: $2t=(m_1+\frac{1}{2})\lambda_1=(m_2+\frac{1}{2})\lambda_2$. But the two different wavelengths differ by just one m - value, $m_2=m_1-1$.

$$\Rightarrow\left(m_1+\frac{1}{2}\right)\lambda_1=\left(m_1-\frac{1}{2}\right)\lambda_2\Rightarrow m_1(\lambda_2-\lambda_1)=\frac{\lambda_1+\lambda_2}{2}\Rightarrow m_1=\frac{\lambda_1+\lambda_2}{2(\lambda_2-\lambda_1)}$$

$$\Rightarrow m_1=\frac{477.0\text{ nm}+540.6\text{ nm}}{2(540.6\text{ nm}-477.0\text{ nm})}=8.$$

$$\Rightarrow 2t=\left(8+\frac{1}{2}\right)\frac{\lambda_1}{n}\Rightarrow t=\frac{17(477.0\text{ nm})}{4(1.52)}=1334\text{ nm}.$$

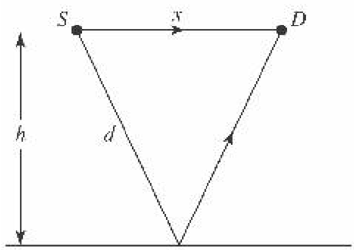
35.53: a) There is a half-cycle phase change at the glass, so for constructive interference:

$$2d-x=2\sqrt{h^2+\left(\frac{x}{2}\right)^2}-x=\left(m+\frac{1}{2}\right)\lambda$$

$$\Rightarrow\sqrt{x^2+4h^2}-x=\left(m+\frac{1}{2}\right)\lambda.$$

Similarly for destructive interference:

$$\sqrt{x^2+4h^2}-x=m\lambda.$$



b) The longest wavelength for constructive interference is when $m=0$:

$$\lambda=\frac{\sqrt{x^2+4h^2}-x}{m+\frac{1}{2}}=\frac{\sqrt{(14\text{ cm})^2+4(24\text{ cm})^2}-14\text{ cm}}{1/2}=72\text{ cm}.$$

35.54: a) At the water (or cytoplasm) to guanine interface, is a half-cycle phase shift for the reflected light, but there is not one at the guanine to cytoplasm interface. Therefore there will always be one half-cycle phase difference between two neighboring reflected beams. For the guanine layers:

$$2t_g=(m+\frac{1}{2})\frac{\lambda}{n_g}\Rightarrow\lambda=\frac{2t_gn_g}{(m+\frac{1}{2})}=\frac{2(74\text{ nm})(1.80)}{(m+\frac{1}{2})}=\frac{266\text{ nm}}{(m+\frac{1}{2})}\Rightarrow\lambda=533\text{ nm}(m=0).$$

For the cytoplasm layers:

$$2t_c = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_c} \Rightarrow \lambda = \frac{2t_c n_c}{(m+\frac{1}{2})} = \frac{2(100\text{ nm})(1.333)}{(m+\frac{1}{2})} = \frac{267\text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533\text{ nm}(m=0).$$

b) By having many layers the reflection is strengthened, because at each interface some more of the transmitted light gets reflected back, increasing the total percentage reflected.

c) At different angles, the path length in the layers change (always to a larger value than the normal incidence case). If the path length changes, then so do the wavelengths that will interfere constructively upon reflection.

35.55: a) Intensified reflected light means we have constructive interference. There is one half-cycle phase shift, so:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \Rightarrow \lambda = \frac{2tn}{(m+\frac{1}{2})} = \frac{2(485\text{ nm})(1.53)}{(m+\frac{1}{2})} = \frac{1484\text{ nm}}{(m+\frac{1}{2})}.$$

$\Rightarrow \lambda = 593\text{ nm}(m=2)$, and $\lambda = 424\text{ nm }(m=3)$.

b) Intensified transmitted light means we have destructive interference at the upper surface. There is still a one half-cycle phase shift, so:

$$2t = \frac{m\lambda}{n} \Rightarrow \lambda = \frac{2tn}{m} = \frac{2(485\text{ nm})(1.53)}{m} = \frac{1484\text{ nm}}{m}.$$

$$\Rightarrow \lambda = 495\text{ nm }(m=3)$$

is the only wavelength of visible light that is intensified. We could also think of this as the result of internal reflections interfering with the outgoing ray *without* any extra phase shifts.

35.56: a) There is one half-cycle phase shift, so for constructive interference:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda_0}{n} \Rightarrow \lambda = \frac{2tn}{(m+\frac{1}{2})} = \frac{2(380\text{ nm})(1.45)}{(m+\frac{1}{2})} = \frac{1102\text{ nm}}{(m+\frac{1}{2})}.$$

Therefore, we have constructive interference at $\lambda = 441\text{ nm }(m=2)$, which corresponds to blue-violet.

b) Beneath the water, looking for maximum intensity means that the reflected part of the wave at the wavelength must be weak, or have interfered destructively. So:

$$2t = \frac{m\lambda_0}{n} \Rightarrow \lambda_0 = \frac{2tn}{m} = \frac{2(380\text{ nm})(1.45)}{m} = \frac{1102\text{ nm}}{m}.$$

Therefore, the strongest transmitted wavelength (as measured in air) is $\lambda = 551\text{ nm }(m=2)$, which corresponds to green.

35.57: For maximum intensity, with a half-cycle phase shift,

$2t = \left(m + \frac{1}{2}\right)\lambda$ and $t = R - \sqrt{R^2 - r^2} \Rightarrow \frac{(2m+1)\lambda}{4} = R - \sqrt{R^2 - r^2}$

$$\Rightarrow \sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4} \Rightarrow R^2 - r^2 = R^2 + \left[\frac{(2m+1)\lambda}{4}\right]^2 - \frac{(2m+1)\lambda R}{2}$$

$$\Rightarrow r = \sqrt{\frac{(2m+1)\lambda R}{2} - \left[\frac{(2m+1)\lambda}{4}\right]^2} \Rightarrow r \approx \sqrt{\frac{(2m+1)\lambda R}{2}}, \text{ for } R >> \lambda.$$

The second bright ring is when $m = 1$:

$$r \approx \sqrt{\frac{(2(1)+1)\,(5.80 \times 10^{-7}\text{ m})\,(0.952\text{ m})}{2}} = 9.10 \times 10^{-4}\text{ m} = 0.910\text{ mm}.$$

So the diameter of the third bright ring is 1.82 mm.

35.58: As found in Problem (35.51), the radius of the mth bright ring is in general:

$$r \approx \sqrt{\frac{(2m+1)\lambda R}{2}},$$

for $R >> \lambda$. Introducing a liquid between the lens and the plate just changes the wavelength from $\lambda \to \frac{\lambda}{n}$.

So:

$$r(n) \approx \sqrt{\frac{(2m+1)\lambda R}{2n}} = \frac{r}{\sqrt{n}} = \frac{0.850\text{ mm}}{\sqrt{1.33}} = 0.737\text{ mm}.$$

35.59: a) Adding glass over the top slit increases the effective path length from that slit to the screen. The interference pattern will therefore change, with the central maximum shifting downwards.

b) Normally the phase shift is $\phi = \frac{2\pi d}{\lambda}\sin\theta$, but now there is an added shift from the glass, so the total phase shift is now

$$\phi = \frac{2\pi d}{\lambda}\sin\theta + \left(\frac{2\pi L n}{\lambda} - \frac{2\pi L}{\lambda}\right) = \frac{2\pi d}{\lambda}\sin\theta + \frac{2\pi L(n-1)}{\lambda} = \frac{2\pi}{\lambda}(d\sin\theta + L(n-1)).$$

So the intensity becomes $I = I_0\cos^2\frac{\phi}{2} = I_0\cos^2\left(\frac{\pi}{\lambda}(d\sin\theta + L(n-1))\right)$.

c) The maxima occur at $\frac{\pi}{\lambda}(d\sin\theta + L(n-1)) = m\pi \Rightarrow d\sin\theta = m\lambda - L(n-1)$

35.60: The passage of fringes indicates an effective change in path length, since the wavelength of the light is getting shorter as more gas enters the tube.

$$\Delta m = \frac{2L}{\lambda/n} - \frac{2L}{\lambda} = \frac{2L}{\lambda}(n-1) \Rightarrow (n-1) = \frac{\Delta m\lambda}{2L}.$$

So here:

$$(n-1)=\frac{48(5.46\times10^{-7}\text{ m})}{2(0.0500\text{ m})}=2.62\times10^{-4}.$$

35.61: There are two effects to be considered: first, the expansion of the rod, and second, the change in the rod's refractive index. The extra length of rod replaces a little of the air so that the change in the number of wavelengths due to this is given by:

$$\Delta N_1=\frac{2n_{\text{glass}}\Delta L}{\lambda_0}-\frac{2n_{\text{air}}\Delta L}{\lambda_0}=\frac{2(n_{\text{glass}}-1)L_0\alpha\Delta T}{\lambda_0}$$

$$\Rightarrow \Delta N_1=\frac{2(1.48-1)\,(0.030\text{ m})\,(5.00\times10^{-6}/\text{C}°)\,(5.00\text{ C}°)}{5.89\times10^{-7}\text{ m}}=1.22.$$

The change in the number of wavelengths due to the change in refractive index of the rod is:

$$\Delta N_2=\frac{2\Delta n_{\text{glass}}L_0}{\lambda_0}=\frac{2(2.50\times10^{-5}/\text{C}°)\,(5.00\text{ C}°/\text{min})\,(1.00\text{ min})\,(0.0300\text{ m})}{5.89\times10^{-7}\text{m}}=12.73.$$

So the total change in the number of wavelengths as the rod expands is $\Delta N=12.73+1.22=14.0$ fringes/minute.

35.62: a) Since we can approximate the angles of incidence on the prism as being small, Snell's Law tells us that an incident angle of θ on the flat side of the prism enters the prism at an angle of θ/n, where n is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n-A$ from the normal, and the outgoing, relative to the prism, is $n(\theta/n-A)$. So the beam leaving the prism is at an angle of $\theta'=n(\theta/n-A)+A$ from the optical axis. So $\theta-\theta'=(n-1)A$.

At the plane of the source S_0, we can calculate the height of one image above the source: $\frac{d}{2}=\tan(\theta-\theta')a\approx(\theta-\theta')a=(n-1)Aa\Rightarrow d=2aA(n-1)$.

b) To find the spacing of fringes on a screen, we use:

$$\Delta y=\frac{R\lambda}{d}=\frac{R\lambda}{2aA(n-1)}=\frac{(2.00\text{ m}+0.200\text{ m})\,(5.00\times10^{-7}\text{ m})}{2(0.200\text{ m})\,(3.50\times10^{-3}\text{ rad})\,(1.50-1.00)}=1.57\times10^{-3}\text{ m}.$$

36.1: $y_1=\frac{x\lambda}{a}\Rightarrow\lambda=\frac{y_1a}{x}=\frac{(1.35\times10^{-3}\text{ m})\,(7.50\times10^{-4}\text{ m})}{2.00\text{ m}}=5.06\times10^{-7}\text{ m}.$

36.2: $y_1=\frac{x\lambda}{a}\Rightarrow a\frac{x\lambda}{y_1}=\frac{(0.600\text{ m})\,(5.46\times10^{-7}\text{ m})}{10.2\times10^{-3}\text{m}}=3.21\times10^{-5}\text{m}.$

36.3: The angle to the first dark fringe is simply:

$$\theta=\arctan\left(\frac{\lambda}{a}\right)=\arctan\left(\frac{633\times10^{-9}\text{ m}}{0.24\times10^{-3}\text{ m}}\right)=0.15°.$$

36.4: $D = 2y_1 = \dfrac{2x\lambda}{a} = \dfrac{2(3.50\text{ m})(6.33\times10^{-7}\text{ m})}{7.50\times10^{-4}\text{ m}} = 5.91\times10^{-3}\text{ m}.$

36.5: The angle to the first minimum is $\theta = \arcsin\left(\dfrac{\lambda}{a}\right) = \arcsin\left(\dfrac{9.00\text{ cm}}{12.00\text{ cm}}\right) = 48.6°.$

So the distance from the central maximum to the first minimum is just $y_1 = x\tan\theta =$ $(40.0\text{ cm})\tan(48.6°) = \pm45.4\text{ cm}.$

36.6: a) According to Eq. 36.2

$$\sin(\theta) = \frac{m\lambda}{a} = \sin(90.0°) = 1 = \frac{m\lambda}{a} = \frac{\lambda}{a}$$

Thus, $a = \lambda = 580\text{ nm} = 5.80\times10^{-4}\text{ mm}.$

b) According to Eq. 36.7

$$\frac{I}{I_0} = \left\{\frac{\sin[\pi a(\sin\theta)/\lambda]}{\pi a(\sin\theta)/\lambda}\right\}^2 = \left\{\frac{\sin[\pi(\sin\frac{\pi}{4})]}{\pi(\sin\frac{\pi}{4})}\right\}^2 = 0.128.$$

36.7: The diffraction minima are located by $\sin\theta = m\lambda/a$, $m = \pm1, \pm2, \ldots$

$\lambda = v/f = (344\text{ m/s})/(1250\text{ Hz}) = 0.2752\text{ m};\ a = 1.00\text{ m}$

$m = \pm1,\ \theta = \pm16.0°;\ m = \pm2,\ \theta = \pm33.4°;\ m = \pm3,\ \theta = \pm55.6°;$ no solution for larger m

36.8: a) $E = E_{max}\sin(kx - \omega t)$

$$k = \frac{2\pi}{\lambda} \to \lambda = \frac{2\pi}{k} = \frac{2\pi}{1.20\times10^{7}\text{ m}^{-1}} = 5.24\times10^{-7}\text{ m}$$

$$f\lambda = c \to f = \frac{c}{\lambda} = \frac{3.0\times10^{8}\text{ m/s}}{5.24\times10^{-7}\text{ m}} = 5.73\times10^{14}\text{ Hz}$$

b) $a\sin\theta = \lambda$

$$a = \frac{\lambda}{\sin\theta} = \frac{5.24\times10^{-7}\text{ m}}{\sin 28.6°} = 1.09\times10^{-6}\text{ m}$$

c) $a\sin\theta = m\lambda\ (m = 1,2,3,\ldots)$

$$\sin\theta_2 = \pm2\tfrac{\lambda}{D} = \pm2\tfrac{5.24\times10^{-7}\text{ m}}{1.09\times10^{-6}\text{ m}}$$

$$\theta_2 = \pm74°$$

36.9: $\sin\theta = \lambda/a$ locates the first minimum

$y = x\tan\theta,\ \tan\theta = y/x = (36.5\text{ cm})/(40.0\text{ cm})$ and $\theta = 42.38°$

$a = \lambda/\sin\theta = (620\times10^{-9}\text{ m})/(\sin 42.38°) = 0.920\ \mu\text{m}$

36.10: a) $y_1 = \dfrac{x\lambda}{a} \Rightarrow a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})\ (5.00\times10^{-7}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 4.17\times10^{-4}\ \text{m}.$

b) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times10^{-5}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 4.17\times10^{-2}\ \text{m} = 4.2\ \text{cm}.$

c) $a = \dfrac{x\lambda}{y_1} = \dfrac{(2.50\ \text{m})(5.00\times10^{-10}\ \text{m})}{3.00\times10^{-3}\ \text{m}} = 4.17\times10^{-7}\ \text{m}.$

36.11: a) $y_1 = \dfrac{x\lambda}{a} = \dfrac{(3.00\ \text{m})(6.33\times10^{-7}\ \text{m})}{3.50\times10^{-4}\ \text{m}} = 5.43\times10^{-3}\ \text{m}.$ So the width of the brightest fringe is twice this distance to the first minimum, 0.0109 m.

b) The next dark fringe is at $y_2 = \dfrac{2x\lambda}{a} = \dfrac{2(3.00\ \text{m})(6.33\times10^{-7}\ \text{m})}{3.50\times10^{-4}\ \text{m}} = 0.0109\ \text{m}.$
So the width of the first bright fringe on the side of the central maximum is the distance from y_2 to y_1, which is $5.43\times10^{-3}\ \text{m}$.

36.12: $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \approx \dfrac{2\pi a}{\lambda}\cdot\dfrac{y}{x} = \dfrac{2\pi(4.50\times10^{-4}\ \text{m})}{(6.20\times10^{-7}\ \text{m})(3.00\ \text{m})}y = (1520\ \text{m}^{-1})y.$

a) $y = 1.00\times10^{-3}\ \text{m}: \dfrac{\beta}{2} = \dfrac{(1520\ \text{m}^{-1})\ (1.00\times10^{-3}\ \text{m})}{2} = 0.760.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(0.760)}{0.760}\right)^2 = 0.822I_0$$

b) $y = 3.00\times10^{-3}\ \text{m}: \dfrac{\beta}{2} = \dfrac{(1520\ \text{m}^{-1})(3.00\times10^{-3}\ \text{m})}{2} = 2.28.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(2.28)}{2.28}\right)^2 = 0.111I_0.$$

c) $y = 5.00\times10^{-3}\ \text{m}: \dfrac{\beta}{2} = \dfrac{(1520\ \text{m}^{-1})\ (5.00\times10^{-3}\ \text{m})}{2} = 3.80.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\frac{\sin(3.80)}{3.80}\right)^2 = 0.0259I_0.$$

36.13: a) $y_1 = \dfrac{x\lambda}{a} = \dfrac{(3.00\ \text{m})(5.40\times10^{-7}\ \text{m})}{2.40\times10^{-4}\ \text{m}} = 6.75\times10^{-3}\ \text{m}.$

b) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \approx \dfrac{2\pi a}{\lambda}\cdot\dfrac{y_1/2}{x} = \dfrac{2\pi a}{\lambda}\cdot\dfrac{x\lambda}{2ax} = \pi.$

$$\Rightarrow I = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = (6.00\times10^{-6}\ \text{W/m}^2)\left(\frac{\sin\ (\pi/2)}{\pi/2}\right)^2 = 2.43\times10^{-6}\ \text{W/m}^2.$$

36.14: a) $\theta = 0 : \beta = \dfrac{2\pi a}{\lambda}\sin 0° = 0.$

b) At the second minimum from the center $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\cdot\dfrac{2\lambda}{a} = 4\pi.$

c) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi(1.50\times 10^{-4}\ \text{m})}{6.00\times 10^{-7}\text{m}}\sin\ 7.0° = 191\ \text{rad.}$

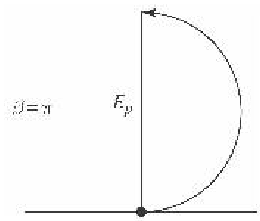
36.15: $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \Rightarrow \lambda = \dfrac{2\pi a}{\beta}\sin\theta = \dfrac{2\pi(3.20\times 10^{-4}\text{m})}{\pi/2}\sin\ 0.24° = 5.36\times 10^{-6}\text{m}.$

36.16: The total intensity is given by drawing an arc of a circle that has length E_0 and finding the length of the cord which connects the starting and ending points of the curve. So graphically we can find the electric field at a point by examining the geometry as shown below for three cases.

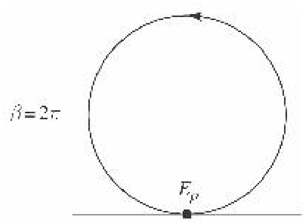
a) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\cdot\dfrac{\lambda}{2a} = \pi.$ From the diagram, $\pi\dfrac{E_p}{2} = E_0 \Rightarrow E_p = \dfrac{2}{\pi}E_0.$

So the intensity is just: $I = \left(\dfrac{2}{\pi}\right)^2 I_0 = \dfrac{4I_0}{\pi^2}.$

This agrees with Eq. (36.5).



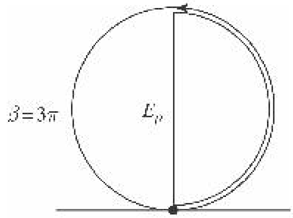
b) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\cdot\dfrac{\lambda}{a} = 2\pi.$ From the diagram, it is clear that the total amplitude is zero, as is the intensity. This also agrees with Eq. (36.5).



c) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\cdot\dfrac{3\lambda}{2a} = 3\pi.$ From the diagram, $3\pi\dfrac{E_p}{2} = E_0 \Rightarrow E_p = \dfrac{2}{3\pi}E_0.$ So the intensity is just:

$$I = \left(\frac{2}{3\pi}\right)^2 I_0 = \frac{4}{9\pi^2}I_0.$$

This agrees with Eq. (36.5).



36.17: a) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta \Rightarrow \lambda = \dfrac{2\pi a}{\beta}\sin\theta = \dfrac{2\pi(1.05\times10^{-4}\text{ m})}{56.0\text{ rad}}\sin 3.25° =$

6.68×10^{-7} m.

b) $I = I_0\left(\dfrac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\left(\dfrac{\sin(56.0/2)}{56.0/2}\right)^2 = (9.36\times10^{-5})\,I_0.$

36.18: a) Ignoring diffraction, the first five maxima will occur as given by:

$d\sin\theta = m\lambda \Rightarrow \theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{m\lambda}{4a}\right)$, for $m = 1, 2, 3, 4, 5$.

b) $\beta = \dfrac{2\pi a}{\lambda}\sin\theta = \dfrac{2\pi a}{\lambda}\cdot\dfrac{m\lambda}{d} = \dfrac{m\pi}{2}$, and $\phi = \dfrac{2\pi d}{\lambda}\sin\theta = \dfrac{2\pi d}{\lambda}\cdot\dfrac{m\lambda}{d} = 2\pi m.$

So including diffraction, the intensity:

$$I = I_0\cos^2\frac{\phi}{2}\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2 = I_0\cos^2\frac{2\pi m}{2}\left(\frac{\sin(m\pi/4)}{m\pi/4}\right)^2 = I_0\left(\frac{\sin(m\pi/4)}{m\pi/4}\right)^2.$$

So for

$$m=1: I_1 = \left(\frac{\sin(\pi/4)}{\pi/4}\right)^2 I_0 = 0.811 I_0;\ m=2: I_2 = \left(\frac{\sin(\pi/2)}{\pi/2}\right)^2 I_0 = 0.405 I_0;$$

$$m=3: I_3 = \left(\frac{\sin(3\pi/4)}{3\pi/4}\right)^2 I_0 = 0.0901 I_0;\ m=4: I_4 = \left(\frac{\sin(\pi)}{\pi}\right)^2 I_0 = 0$$

$$m=5: I_5 = \left(\frac{\sin(5\pi/4)}{5\pi/4}\right)^2 I_0 = 0.0324 I_0.$$

36.19: a) If $\dfrac{d}{a} = 3$, then there are five fringes: $m = 0, \pm1, \pm2$.

b) The $m = 6$ interference fringe coincides with the second diffraction minimum, so there are two fringes $(m = +4, m = +5)$ within the first diffraction maximum on one side of the central maximum.

36.20: By examining the diagram, we see that every fourth slit cancels each other.

36.21: a) If the slits are very narrow, then the first maximum is at

$$\frac{d}{\lambda}\sin\theta_1 = 1.$$

$$\Rightarrow \theta_1 = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{5.80\times10^{-7}\ \text{m}}{5.30\times10^{-4}\ \text{m}}\right) = \pm 0.0627°.$$

Also, the second maximum is at

$$\frac{d}{\lambda}\sin\theta_2 = 2$$

$$\Rightarrow \theta_2 = \arcsin\left(\frac{2\lambda}{d}\right) = \arcsin\left(\frac{2(5.80\times10^{-7}\ \text{m})}{5.30\times10^{-4}\text{m}}\right) = \pm\, 0.125°.$$

b) $I = I_0\cos^2\frac{\phi}{2}\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$ but $\cos\frac{\phi}{2} = 1$, since we are at the 2 slit maximum. So

$$I_1 = I_0\left(\frac{\sin(\pi a\sin\theta_1/\lambda)}{\pi a\sin\theta_1/\lambda}\right)^2 = I_0\left(\frac{\sin(\pi a/d)}{\pi a/d}\right)^2$$

$$\Rightarrow I_1 = I_0\left(\frac{\sin(\pi(3.20\times10^{-4}\ \text{m})/(5.30\times10^{-4}\ \text{m}))}{\pi(3.20\times10^{-4}\ \text{m})/(5.30\times10^{-4}\ \text{m})}\right)^2 = 0.249 I_0.$$

And

$$I_2 = I_0\left(\frac{\sin(\pi a\sin\theta_2/\lambda)}{\pi a\sin\theta_2/\lambda}\right)^2 = I_0\left(\frac{\sin(2\pi\, a/d)}{2\pi a/d}\right)^2$$

$$\Rightarrow I_2 = I_0\left(\frac{\sin\,(2\pi(3.20\times10^{-4}\ \text{m})/(5.30\times10^{-4}\ \text{m}))}{2\pi(3.20\times10^{-4}\ \text{m})\ /(5.30\times10^{-4}\ \text{m})}\right)^2 = 0.0256 I_0.$$

36.22: We will use $I = I_0\cos^2\frac{\phi}{2}\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$, and must calculate the phases ϕ and β.

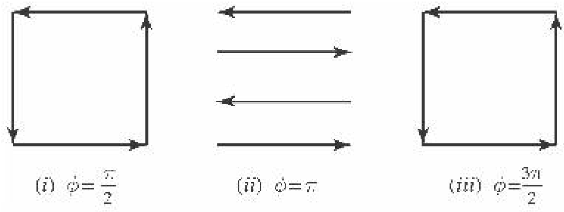
Using $\beta/2 = \frac{\pi a}{\lambda}\sin\theta$, and $\phi = \frac{2\pi d}{\lambda}\sin\theta$, we have:

a) $\theta = 1.25\times10^{-4}$ rad : $\beta/2 = 0.177(10.1°)$, and $\phi = 1.01(57.9°) \Rightarrow I = 0.757 I_0$.

b) $\theta = 2.50\times10^{-4}$ rad : $\beta/2 = 0.355(20.3°)$, and $\phi = 2.03(116.3°) \Rightarrow I = 0.268 I_0$.

c) $\theta = 3.00\times10^{-4}$ rad : $\beta/2 = 0.426(24.1°)$, and $\phi = 2.43(139.2°) \Rightarrow I = (0.114) I_0$.

36.23: With four slits there must be four vectors in each phasor diagram, with the orientation of each successive one determined by the relative phase shifts. So:



We see that destructive interference occurs from adjacent slits in case (ii) and from alternate slits in cases (i) and (iii).

36.24: Diffraction dark fringes occur for $\sin\theta = \dfrac{m_d\lambda}{a}$, and interference maxima occur for $\sin\theta = \dfrac{m_i\lambda}{d}$. Setting them equal to each other yields a missing bright spot whenever the destructive interference matches the bright spots. That is:

$\dfrac{m_i\lambda}{d} = \dfrac{m_d\lambda}{a} \Rightarrow m_i = \dfrac{d}{a}m_d = 3m_d$. That is, the missing parts of the pattern occur for $m_i = 3, 6, 9\ldots = 3m$, for m = integers.

36.25: a) Interference maxima: and Diffraction minima:

$$d \sin\theta_i = m\lambda \qquad \text{and} \qquad a \sin\theta_d = n\lambda.$$

If the mth interference maximum corresponds to the nth diffraction minimum then $\theta_i = \theta_d$.

or
$$\frac{d}{a} = \frac{m}{n}$$
so
$$a = \frac{n}{m}d = \frac{1}{3}(0.840\text{ mm}) = 0.280\text{ mm}.$$

b) The diffraction minima will squelch the interference maxima for all $\dfrac{m}{n} = 3$ up to the highest seen order. For $\lambda = 630$ nm, the largest value of m will be when $\theta = 90°$.

$$m_{\max} = \frac{d}{\lambda} = \frac{8.40\times10^{-4}\text{ m}}{6.30\times10^{-7}\text{ m}} = 1333.$$

$$n_{\max} = \frac{a}{\lambda} = \frac{8.40\times10^{-4}\text{ m}}{3(6.30\times10^{-7}\text{ m})} = 444.$$

So after $m = 3$, $m = 6, 9,\ldots, 1332$ for $n = 1, 2, 3,\ldots, 444$ will also be missing.

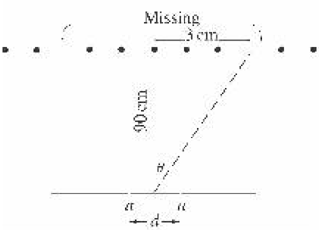
c) By changing λ we only change the highest order seen.

$$m_{\max} = \frac{d}{\lambda} = \frac{8.40\times10^{-4}\text{ m}}{4.20\times10^{-7}\text{ m}} = 2000.$$

$$n_{\max} = \frac{a}{\lambda} = \frac{2000}{3} = 666.$$

So $m = 3, 6, 9, \ldots, 1999$ for $n = 1, 2, 3, \ldots, 666$.

36.26: The third bright band is missing because the first order single slit minimum occurs at the same angle as the third order double slit maximum.



$$\tan\theta = \frac{3\text{ cm}}{90\text{ cm}}$$
$$\theta = 1.91°$$

Single-slit dark spot: $a \sin\theta = \lambda$

$$a = \frac{\lambda}{\sin\theta} = \frac{500\text{ nm}}{\sin 1.91°} = 1.50\times 10^4\text{ nm(width)}$$

Double-slit bright fringe:

$$d\sin\theta = 3\lambda$$
$$d = \frac{3\lambda}{\sin\theta} = \frac{3(500\text{ nm})}{\sin 1.91°} = 4.50\times 10^4\text{ nm(separation)}$$

36.27: a) Find $d : d\sin\theta = m\lambda$

$\theta = 78.4°$ for $m = 3$ and $\lambda = 681$ nm, so $d = m\lambda/\sin\theta = 2.086\times 10^{-4}$ cm

The number of slits per cm is $1/d = 4790$ slits/cm

b) 1st order: $m = 1$, so $\sin\theta = \lambda/d = (681\times 10^{-9}\text{ m})/(2.086\times 10^{-6}\text{ m})$ and $\theta = 19.1°$

2nd order: $m = 2$, so $\sin\theta = 2\lambda/d$ and $\theta = 40.8°$

c) For $m = 4$, $\sin\theta = 4/d$ is greater than 1.00, so there is no 4th-order bright band.

36.28: First-order: $d\sin\theta_1 = \lambda$

Fourth-order: $d\sin\theta_1 = 4\lambda$

$$\frac{d\sin\theta_4}{d\sin\theta_1} = \frac{4\lambda}{\lambda}$$
$$\sin\theta_4 = 4\sin\theta_1 = 4\sin 8.94°$$
$$\theta_4 = 38.4°$$

36.29: a) $d = \left(\frac{1}{900}\right)\text{cm} = 1.111\times 10^{-5}$ m

For $\lambda = 700$ nm, $\lambda/d = 6.3\times 10^{-4}$. The first-order lines are located at

$\sin\theta = \lambda/d$; $\sin\theta$ is small enough for $\sin\theta \approx \theta$ to be an excellent approximation.

b) $y = x\lambda/d$, where $x = 2.50$ m.

The distance on the screen between 1st order bright bands for two different wavelengths is $\Delta y = x(\Delta y)/d$, so $\Delta\lambda = d(\Delta y)/x$

$$= (1.111\times10^{-5}\ \text{m})\,(3.00\times10^{-3}\ \text{m})/(2.50\ \text{m}) = 13.3\ \text{nm}$$

36.30: a) $R = \dfrac{\lambda}{\Delta\lambda} = Nm \Rightarrow N = \dfrac{\lambda}{m\Delta\lambda} = \dfrac{6.5645\times10^{-7}\,\text{m}}{2(6.5645\times10^{-7}\ \text{m} - 6.5627\times10^{-7}\ \text{m})} = 1820$ slits.

b) $d = (500\ \text{slits/mm})^{-1} = (500{,}000\ \text{slits/m})^{-1} \cdot \theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right) \Rightarrow$

$$\theta_1 = \sin^{-1}((2)(6.5645\times10^{-7}\ \text{m})\cdot 500{,}000) = 41.0297°$$

$$\theta_2 = \sin^{-1}((2)(6.5627\times10^{-7}\ \text{m})\cdot 500{,}000) = 41.0160°$$

$$\Rightarrow \Delta\theta = 0.0137°.$$

36.31: $\theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{\text{m}(6.328\times10^{-7}\ \text{m})}{1.60\times10^{-6}\ \text{m}}\right) = \arcsin((0.396)m)$

$\Rightarrow m = 1 : \theta_1 = 23.3°$; $m = 2 : \theta = 52.3°$. All other m-values lead to angles greater than 90°.

36.32: 5000 slits/cm $\Rightarrow d = \dfrac{1}{5.00\times10^{5}\ \text{m}^{-1}} = 2.00\times10^{-6}$ m.

a) $d\sin\theta = m\lambda \Rightarrow \lambda = \dfrac{d\sin\theta}{m} = \dfrac{(2.00\times10^{-6}\ \text{m})\sin 13.5°}{1} = 4.67\times10^{-7}$ m.

b) $m = 2 : \theta = \arcsin\left(\dfrac{m\lambda}{d}\right) = \arcsin\left(\dfrac{2(4.67\times10^{-7}\ \text{m})}{2.00\times10^{-6}\ \text{m}}\right) = 27.8°$.

36.33: 350 slits/mm $\Rightarrow d = \dfrac{1}{3.50\times10^{5}\ \text{m}^{-1}} = 2.86\times10^{-6}$ m. Then :

$$d\sin\theta = m\lambda \Rightarrow \theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(5.20\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = \arcsin((0.182)m)$$

$$\Rightarrow m = 1 : \theta = 10.5°;\quad m = 2 : \theta = 21.3°;\quad m = 3 : \theta = 33.1°.$$

36.34: 350 slits/mm $\Rightarrow d = \dfrac{1}{3.50\times10^{5}\ \text{m}^{-1}} = 2.86\times10^{-6}$ m., and $d\sin\theta = m\lambda$.

$$\Rightarrow m = 1: \theta_{400} = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{4.00\times10^{-7}\ \text{m}}{2.86\times10^{-6}\ \text{m}}\right) = 8.05^\circ.$$

$$\theta_{700} = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{7.00\times10^{-7}\ \text{m}}{2.86\times10^{-6}\ \text{m}}\right) = 14.18^\circ.$$

$$\Rightarrow \Delta\theta_1 = 14.18^\circ - 8.05^\circ = 6.13^\circ.$$

$$\Rightarrow m = 3: \theta_{400} = \arcsin\left(\frac{3\lambda}{d}\right) = \arcsin\left(\frac{3(4.00\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = 24.8^\circ.$$

$$\theta_{700} = \arcsin\left(\frac{3\lambda}{d}\right) = \arcsin\left(\frac{3(7.00\times10^{-7}\ \text{m})}{2.86\times10^{-6}\ \text{m}}\right) = 47.3^\circ.$$

$$\Rightarrow \Delta\theta_1 = 47.3^\circ - 24.8^\circ = 22.5^\circ.$$

36.35: 4000 slits/cm $\Rightarrow d = \dfrac{1}{4.00\times10^{5}\ \text{m}^{-1}} = 2.50\times10^{-6}$ m. So for the α-hydrogen line, we have:

$$\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(6.56\times10^{-7}\ \text{m})}{2.50\times10^{-6}\ \text{m}}\right) = \arcsin\ ((0.262)m).$$

$$\Rightarrow m = 1: \theta_1 = 15.2^\circ;\ m = 2: \theta = 31.6^\circ.$$

And for the β-hydrogen line, the angle is given by:

$$\theta = \arcsin\left(\frac{m\lambda}{d}\right) = \arcsin\left(\frac{m(4.86\times10^{-7}\ \text{m})}{2.50\times10^{-6}\ \text{m}}\right) = \arcsin\ ((0.194)m).$$

$$\Rightarrow m = 1: \theta_1 = 11.2^\circ;\ m = 2: \theta = 22.9^\circ;\ \text{so, a)}\ \Delta\theta_1 = 4.00^\circ,\ \text{b)}\ \Delta\theta_2 = 8.77^\circ.$$

36.36: $\dfrac{\Delta}{\Delta\lambda} = Nm \Rightarrow N = \dfrac{\lambda}{m\Delta\lambda} = \dfrac{587.8002\ \text{nm}}{(587.9782\ \text{nm} - 587.8002\ \text{nm})} = \dfrac{587.8002}{0.178}$

$$\Rightarrow N = 3302\ \text{slits}.$$

$$\frac{N}{1.20\ \text{cm}} = \frac{3302}{1.20\ \text{cm}} = 2752\ \frac{\text{slits}}{\text{cm}}.$$

36.37: For x-ray diffraction,

$$2d\ \sin\theta = m\lambda \Rightarrow d = \frac{m\lambda}{2\sin\theta} \Rightarrow d = \frac{2(8.50\times10^{-11}\ \text{m})}{2\sin 21.5^\circ} = 2.32\times10^{-10}\ \text{m}.$$

36.38: For the first order maximum in Bragg reflection:

$$2d\ \sin\theta = m\lambda \Rightarrow \lambda = \frac{2d\ \sin\theta}{m} = \frac{2(4.40\times10^{-10}\ \text{m})\sin 39.4^\circ}{1} = 5.59\times10^{-10}\ \text{m}.$$

36.39: The best resolution is 0.3 arcseconds, which is about $8.33\times10^{-5\circ}$.

a) $D=\dfrac{1.22\lambda}{\sin\theta_1}=\dfrac{1.22(5.5\times10^{-7}\text{ m})}{\sin(8.33\times10^{-5\circ})}=0.46\text{ m}\approx0.5\text{ m}.$

b) The Keck telescopes are able to gather more light than the Hale telescope, and hence they can detect fainter objects. However, their larger size does not allow them to have greater resolution—atmospheric conditions limit the resolution.

36.40: $D=\dfrac{1.22\lambda}{\sin\theta_1}=\dfrac{1.22(5.5\times10^{-7}\text{ m})}{\sin(1/60)^\circ}=2.31\times10^{-3}\text{ m}=2.3\text{ mm}.$

36.41: $\sin\theta_1=1.22\dfrac{\lambda}{D}\Rightarrow D=\dfrac{1.22\,\lambda}{\sin\theta_1}=1.22\lambda\dfrac{h}{W}=1.22(0.036\text{ m})\dfrac{1.2\times10^6\text{ m}}{2.8\times10^4\text{ m}}$

$\Rightarrow D=1.88\text{ m}.$

36.42: $\sin\theta_1=1.22\dfrac{\lambda}{D}\Rightarrow\lambda=\dfrac{D\sin\theta_1}{1.22}\approx\dfrac{D\theta_1}{1.22}=\dfrac{(8.00\times10^6\text{ m})(1.00\times10^{-8})}{1.22}$

$\Rightarrow\lambda=0.0656\text{ m}=6.56\text{ cm}.$

36.43: $\sin\theta_1=1.22\dfrac{\lambda}{D}=1.22\dfrac{6.20\times10^{-7}\text{ m}}{7.4\times10^{-6}\text{ m}}=0.102.$ The screen is 4.5 m away, so the diameter of the Airy ring is given by trigonometry:

$D=2y=2x\tan\theta\approx2x\sin\theta=2(4.5\text{ m})(0.102)=91.8\text{ cm}.$

36.44: The image is 25.0 cm from the lens, and from the diagram and Rayleigh's criteria, the diameter of the circles is twice the "height" as given by:

$$D=2\,|\,y'\,|=\frac{2s'}{s}y=\frac{2fy}{s}=\frac{2(0.180\text{ m})(8.00\times10^{-3}\text{ m})}{25.0\text{ m}}=1.15\times10^{-4}\text{m}=0.115\text{ mm}.$$

36.45: $\sin\theta_1=1.22\dfrac{\lambda}{D}\Rightarrow D=\dfrac{1.22\lambda}{\sin\theta_1}\approx1.22\lambda\dfrac{R}{W}$

$=1.22(5.0\times10^{-7}\text{ m})\dfrac{5.93\times10^{11}\text{ m}}{2.50\times10^5\text{ m}}=1.45\text{ m}.$

36.46: $\sin\theta_1=1.22\dfrac{\lambda}{D}\cong\dfrac{y}{s}\Rightarrow s=\dfrac{yD}{1.22\lambda}=\dfrac{(4.00\times10^{-3}\text{m})(0.0720\text{ m})}{1.22(5.50\times10^{-7}\text{ m})}=429\text{ m}.$

36.47: Let y be the separation between the two points being resolved and let s be their distance from the telescope. Then the limit of resolution corresponds to $1.22\dfrac{\lambda}{D}=\dfrac{y}{s}$

a) Let the two points being resolved be the opposite edges of the crater, so y is the diameter of the crater. For the moon, $s = 3.8 \times 10^8$ m.

$y = 1.22\lambda s/D$

Hubble: $D = 2.4$ m and $\lambda = 400$ nm gives the maximum resolution, so $y = 77$ m

Arecibo: $D = 305$ m and $\lambda = 0.75$ m; $y = 1.1 \times 10^6$ m

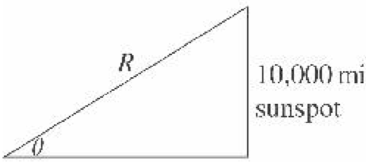
b) $s = \dfrac{yD}{1.22\lambda}$

Let $y \approx 0.30$ (the size of a license plate)

$s = (0.30 \text{ m})(2.4 \text{ m})/[(1.22)(400 \times 10^{-9} \text{ m})] = 1500 \text{ km}$

36.48: Smallest resolving angle is for short-wavelength light(400 nm)

$$\theta \approx 1.22\frac{\lambda}{D} = (1.22)\frac{400\times10^{-9}\ \text{m}}{5.08\ \text{m}} = 9.61\times10^{-8}\ \text{rad}$$



$$\theta = \frac{10{,}000\ \text{mi}}{R}$$

$$R = \frac{10{,}000\ \text{mi}}{\theta} = \frac{16{,}000\ \text{km}}{9.6\times10^{-8}\,\text{rad}} = 1.7\times10^{11}\ \text{km}$$

This is less than a light year, so there are no stars this close.

36.49: Let y be the separation between the two points being resolved and let s be their distance from the telescope. The limit of resolution corresponds to $1.22\lambda/D = y/s$

$s = 4.28\ \text{ly} = 4.05\times10^{16}\ \text{m}$

Assume visible light, with $\lambda = 400\ \text{m}$

$y = 1.22\lambda s/D = 1.22(400\times10^{-9}\ \text{m})(4.05\times10^{16}\ \text{m}/(10.0\ \text{m}) = 2.0\times10^{9}\,\text{m}$

The diameter of Jupiter is 1.38×10^{8} m, so the resolution is insufficient, by about one order of magnitude.

36.50: a) For dark spots, $a\sin\theta = m\lambda$, so $\sin\theta \propto 1/a$. Heating the sheet causes the slit width to increase due to thermal expansion, so $\sin\theta$ and hence θ will decrease. Therefore the bright region gets *narrower*.

b) *At the lower temperature:*

$$a_1 = \frac{\lambda}{\sin\theta_1}\ \text{where}\ \tan\theta_1 = \frac{5\ \text{cm}}{800\ \text{cm}} \rightarrow \theta_1 = 0.35809^\circ$$

$$a_1 = \frac{500\ \text{nm}}{\sin 0.35809^\circ} = 80{,}002\ \text{nm}$$

At the higher temperature:

$$\tan\theta_2 = \frac{5\ \text{cm} - 0.001\,\text{cm}}{800\,\text{cm}} \rightarrow \theta_2 = 0.35802^\circ$$

$$a_2 = \frac{\lambda}{\sin\theta_1} = a_1 = \frac{500\ \text{nm}}{\sin\ 0.35802^\circ} = 80{,}018\ \text{nm}$$

Thermal expansion: $\Delta a = \alpha a_1 \Delta T$

$$\alpha = \frac{\Delta a}{a_1 \Delta T} = \frac{80,018 \text{ nm} - 80,002 \text{ nm}}{(80,002 \text{ nm})(80°\text{C})}$$
$$= 2.5 \times 10^{-6} \,°C^{-1}$$

36.51: a) $I = I_0/2 \Rightarrow \frac{1}{\sqrt{2}} = \frac{\sin(\pi\ a \sin\theta/\lambda)}{\pi\ a \sin\theta/\lambda} \equiv \frac{\sin x}{x} = 0.7071$. Solving for x through trial and error, and remembering to use radians throughout, one finds $x = 1.39$ rad and $\beta = 2x = 2.78$ rad. Also, $\Delta\theta = |\theta_+ - \theta_-| = 2\theta_+$, and

$$\beta = \frac{2\pi a}{\lambda}\sin\theta \Rightarrow \sin\theta_+ \frac{\lambda\beta}{2\pi a} = \frac{\lambda}{a}\left(\frac{2.78 \text{ rad}}{2\pi\ \text{ rad}}\right) = 0.442\frac{\lambda}{a}.$$

i) $\frac{a}{\lambda} = 2 \Rightarrow \sin\theta_+ = 0.221 \Rightarrow \theta_+ = 0.223 \text{ rad} \Rightarrow \Delta\theta = 0.446 \text{ rad} = 25.3°.$

ii) $\frac{a}{\lambda} = 5 \Rightarrow \sin\theta_+ = 0.0885 \Rightarrow \theta_+ = 0.0886 \text{ rad} \Rightarrow \Delta\theta = 0.177 \text{ rad} = 10.1°.$

iii) $\frac{a}{\lambda} = 10 \Rightarrow \sin\theta_+ = 0.0442 \Rightarrow \theta_+ = 0.0443 \text{ rad} \Rightarrow \Delta\theta = 0.885 \text{ rad} = 5.07°.$

b) For the first minimum, $\sin\theta_0 = \frac{\lambda}{a}$.

i) $\frac{a}{\lambda} = 2 \Rightarrow \theta_0 = \arcsin\left(\frac{1}{2}\right) = 0.524 \text{ rad} \Rightarrow 2\theta_0 = 1.05 \text{ rad} = 60.2°.$

ii) $\frac{a}{\lambda} = 5 \Rightarrow \theta_0 = \arcsin\left(\frac{1}{5}\right) = 0.201 \text{ rad} \Rightarrow 2\theta_0 = 0.402 \text{ rad} = 23.0°.$

iii) $\frac{a}{\lambda} = 10 \Rightarrow \theta_0 = \arcsin\left(\frac{1}{10}\right) = 0.100 \text{ rad} \Rightarrow 2\theta_0 = 0.200 \text{ rad} = 11.5°.$

Both methods show the central width getting smaller as the slit width a is increased.

36.52: If the apparatus of Exercise 36.4 is placed in water, then all that changes is the wavelength $\lambda \to \lambda' = \frac{\lambda}{n}$. So : $D' = 2y_1' = \frac{2x\lambda'}{a} = \frac{2x\lambda}{an} = \frac{D}{n} = \frac{5.91 \times 10^{-3}\text{m}}{1.33} =$ $4.44 \times 10^{-3}\text{m} = 4.44$ mm.

36.53: $\sin\theta = \lambda/a$ locates the first dark band

$$\sin\theta_{\text{air}} = \frac{\lambda_{\text{air}}}{a}; \sin\theta_{\text{liquid}} = \frac{\lambda_{\text{liquid}}}{a}$$

$$\lambda_{\text{liquid}} = \lambda_{\text{air}}\left(\frac{\sin\theta_{\text{liquid}}}{\sin\theta_{\text{air}}}\right) = 0.4836$$

$\lambda = \lambda_{\text{air}}/n$ (Eq.33.5), so $n = \lambda_{\text{air}}/\lambda_{\text{liquid}} = 1/0.4836 = 2.07$

36.54: For bright spots, $\frac{1}{N}\sin\theta = \lambda$

Red: $\frac{1}{N}\sin\theta_{\rm R} = 700\ \text{nm}$

Violet: $\frac{1}{N}\sin\theta_{\rm V} = 400\ \text{nm}$

$$\frac{\sin\theta_R}{\sin\theta_V} = \frac{7}{4}$$

$$\theta_{\rm R} - \theta_{\rm V} = 15° \rightarrow \theta_{\rm R} = \theta_V + 15°$$

$$\frac{\sin(\theta_V + 15°)}{\sin\theta_{\rm V}} = \frac{7}{4}$$

Expand: $\dfrac{\sin\theta_{\rm V}\ \cos 15° + \cos\theta_{\rm V}\ \sin 15°}{\sin\theta_{\rm V}} = 7/4$

$$\cos 15° + \cot\theta_{\rm V}\sin 15° = 7/4$$

$$\tan\theta_{\rm V} = 0.330 \rightarrow \theta_{\rm V} = 18.3°$$

$$\theta_{\rm R} = \theta_{\rm V} + 15° = 18.3° + 15° = 33.3°$$

Line density: $\frac{1}{N}\sin\theta_{\rm R} = 700\text{nm}$

$$N = \frac{\sin\theta_{\rm R}}{700\ \text{nm}} = \frac{\sin 33.3°}{700\times10^{-9}\ \text{m}} = 7.84\times10^5\ \text{lines/m}$$

$$= 7840\ \text{lines/cm}$$

The spectrum begins at $18.3°$ and ends at $33.3°$

36.55: a) $y_1 = \dfrac{x\lambda}{a} = \dfrac{(1.20\ \text{m})(5.40\times10^{-7}\ \text{m})}{3.60\times10^{-4}\ \text{m}} = 1.80\times10^{-3}\,\text{m}.$

b) $\dfrac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda} = \dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{\pi a\sin\theta}{\lambda} = 1.39$

$$\Rightarrow \sin\theta = \frac{(1.39)(5.40\times10^{-7}\ \text{m})}{\pi(3.60\times10^{-4}\text{m})} = 6.64\times10^{-4}$$

$$\Rightarrow y = x\tan\theta \approx x\sin\theta = (1.20\ \text{m})(6.64\times10^{-4}) = 7.97\times10^{-4}\,\text{m} = 0.797\ \text{mm}.$$

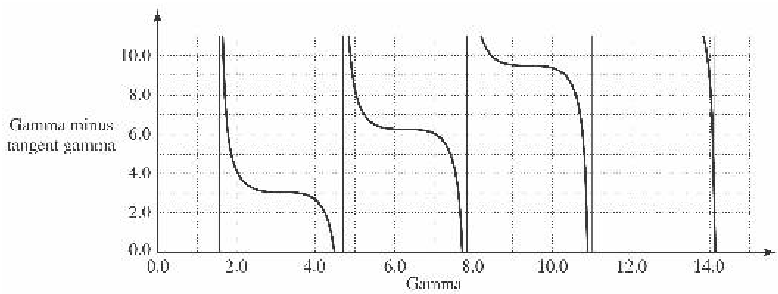
36.56: a) $I = I_0\left(\dfrac{\sin\gamma}{\gamma}\right)^2$. The maximum intensity occurs when the derivative of the intensity function with respect to γ is zero.

$$\frac{dI}{d\gamma} = I_0 \frac{d}{d\gamma}\left(\frac{\sin\gamma}{\gamma}\right)^2 = 2\left(\frac{\sin\gamma}{\gamma}\right)\left(\frac{\cos\gamma}{\gamma} - \frac{\sin\gamma}{\gamma^2}\right) = 0$$

$$\Rightarrow \frac{\cos\gamma}{\gamma} - \frac{\sin\gamma}{\gamma^2} \Rightarrow \gamma\cos\gamma = \sin\gamma$$

$$\Rightarrow \gamma = \tan\gamma.$$

b)



The graph above is a plot of f(γ) = $\gamma - \tan\gamma$. So when it equals zero, one has an intensity maximum. Getting estimates from the graph, and then using trial and error to narrow in on the value, we find that the three smallest γ-values are γ = 4.49 rad 7.73 rad, and 10.9 rad.

36.57: The phase shift for adjacent slits is $\phi = \frac{2\pi d}{\lambda}\sin\theta \approx \frac{2\pi d\theta}{\lambda} \Rightarrow \theta = \frac{\phi\lambda}{2\pi d}.$

So, with the principal maxima at phase shift values of $\phi = 2\pi m,$ and $(N-1)$ minima between the maxima, the phase shift between the minima adjacent to the maximum, and the maximum itself, must be $\pm\frac{2\pi}{N}.$

Therefore total phase shifts of these minima are $2\pi m \pm \frac{2\pi}{N}.$

Hence the angle at which they are found, and the angular width, will be:

$$\theta_{\pm} = \frac{\lambda}{2\pi d}\left(2\pi m \pm \frac{2\pi}{N}\right) = \frac{m\lambda}{d} \pm \frac{\lambda}{dN} \Rightarrow \Delta\theta_{\pm} = \frac{2\lambda}{dN}.$$

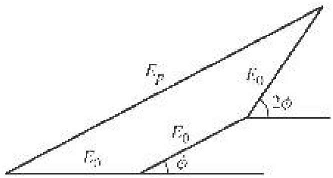
36.58: a) $E_p^2 = E_{p_x}^2 + E_{p_y}^2$. So, from the diagram at right, we have:

$$\frac{E_p^2}{E_0^2} = (1+\cos\phi+\cos 2\phi)^2 + (\sin\phi + \sin 2\phi)^2$$

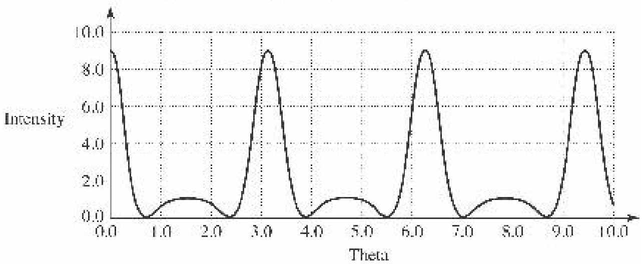
$$= (2\cos^2\phi + \cos\phi)^2 + (\sin\phi + 2\sin\phi\cos\phi)^2$$

$$= (\cos^2\phi + \sin^2\phi)(1+2\cos\phi)^2$$

$$\Rightarrow \frac{E_p^2}{E_0^2} = (1+2\cos\phi)^2 \Rightarrow E_p = E_0(1+2\cos\phi).$$



b) $\phi = \frac{2\pi d}{\lambda}\sin\theta \Rightarrow I_p = I_0\left(1+2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right)^2$. This is graphed below:



c) (i) At $\theta = 0, I_p = I_0(1+2\cos(0^0))^2 = 9I_0$.

(ii) The principal maximum is when $I_0 \frac{2\pi d\sin\theta}{\lambda} = 2\pi m \Rightarrow d\sin\theta = m\lambda$

(iii) & (iv) The minima occur at $2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right) = -1 \Rightarrow \frac{2\pi d\sin\theta}{\lambda} = \frac{2\pi m}{3}$

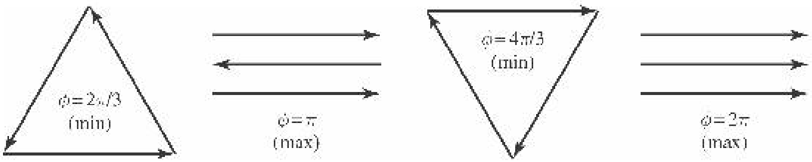
$\Rightarrow d\sin\theta = \frac{m\lambda}{3}$, with m not divisible by 3. Thus there are two minima between every principal maximum.

(v) The secondary maxima occur when $\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right) = -1 \Rightarrow I_p = I_0 = \frac{I_{max}}{9}$.

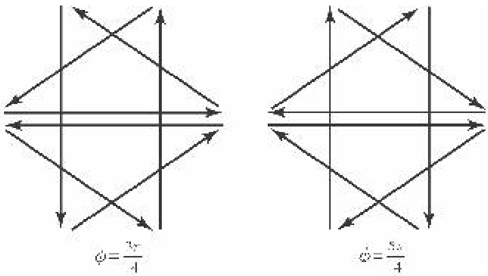
Also $\dfrac{2\pi d \sin\theta}{\lambda} = m\pi \Rightarrow d \sin\theta = \dfrac{m\lambda}{2}$.

All of these findings agree with the N-slit statements in Section 35.5.

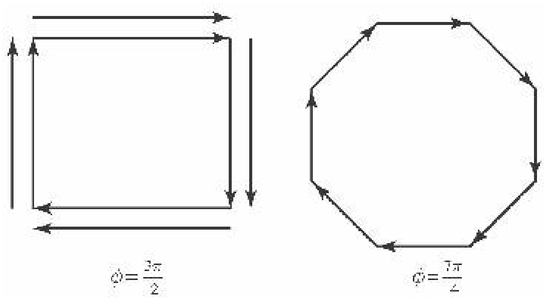
d) Below are phasor diagrams for specific phase shifts.



36.59: a) For eight slits, the phasor diagrams must have eight vectors:



b) For $\phi = \dfrac{3\pi}{4}$, $\phi = \dfrac{5\pi}{4}$, and $\phi = \dfrac{7\pi}{4}$, totally destructive interference occurs between slits four apart. For $\phi = \dfrac{3\pi}{2}$, totally destructive interference occurs with every second slit.

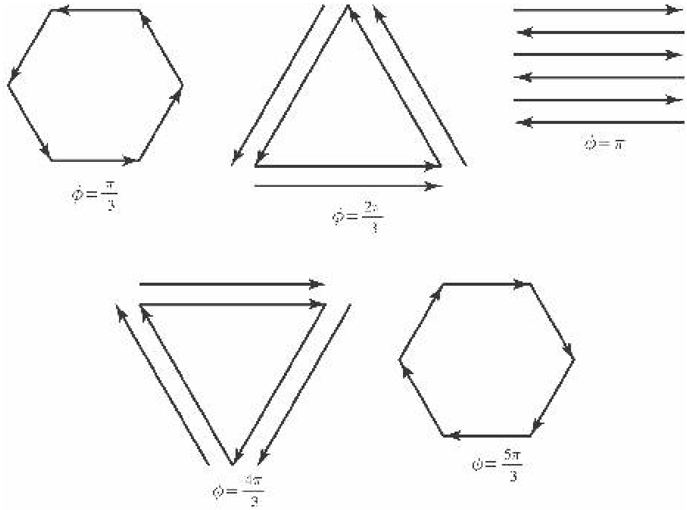


36.60: For six slits, the phasor diagrams must have six vectors.

a) Zero phase difference between adjacent slits means that the total amplitude is $6E$, and the intensity is $36I$.

b) If the phase difference is 2π, then we have the same phasor diagram as above, and equal amplitude, $6E$, and intensity, $36I$.

c) There is an interference minimum whenever the phasor diagrams close on themselves, such as in the five cases below.

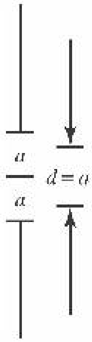


36.61: a) For the maxima to occur for N slits, the sum of all the phase differences between the slits must add to zero (the phasor diagram closes on itself). This requires that, adding up all the relative phase shifts, $N\phi = 2\pi m$, for some integer m. Therefore $\phi = \dfrac{2\pi m}{N}$, for m not an integer multiple of N, which would give a maximum.

b) The sum of N phase shifts $\phi = \dfrac{2\pi m}{N}$ brings you full circle back to the maximum, so only the $N-1$ previous phases yield minima between each pair of principal maxima.

36.62: As shown below, a pair of slits whose width and separation are equal is the same as having a single slit, of twice the width.

$$\phi = \frac{2\pi d}{\lambda}\sin\theta, \text{ so } \beta = \frac{2\pi a}{\lambda}\sin\theta = \phi.$$



So then the intensity is

$$I = I_0 \cos^2(\beta/2)\left(\frac{\sin^2(\beta/2)}{(\beta/2)^2}\right) = I_0 \frac{(2\sin(\beta/2)\cos(\beta/2))^2}{\beta^2}$$

$$\Rightarrow I = I_0 \frac{\sin^2 \beta}{\beta^2} = I_0 \frac{\sin^2(\beta'/2)}{(\beta'/2)^2},$$

where $\beta' = \dfrac{2\pi(2a)}{\lambda} \sin\theta,$ which is Eq. (35.5) with double the slit width.

36.63: For 6500 slits/cm $\Rightarrow d = \dfrac{1}{6.50 \times 10^5 \text{ m}^{-1}} = 1.54 \times 10^{-6}$ m. When $\theta = 90°$, $m = 3$ so one barely gets the 3rd order.

$$d \sin\theta = m\lambda \Rightarrow \lambda = \frac{d \sin\theta}{m} \Rightarrow \lambda_{3\max} = \frac{d}{3} = \frac{1.54 \times 10^{-6} \text{ m}}{3} = 5.13 \times 10^{-7} \text{ m.}$$

36.64: a) As the rays first reach the slits there is already a phase difference between adjacent slits of $\dfrac{2\pi d \sin\theta'}{\lambda}$.

This, added to the usual phase difference introduced after passing through the slits, yields the condition for an intensity maximum:

$$\frac{2\pi d \sin\theta}{\lambda} + \frac{2\pi d \sin\theta'}{\lambda} = 2\pi m \Rightarrow d(\sin\theta + \sin\theta') = m\lambda$$

b) 600 slits/mm $\Rightarrow d = \dfrac{1}{6.00 \times 10^5 \text{ m}^{-1}} = 1.67 \times 10^{-6}$ m.

$\theta' = 0^\circ : m = 0 : \theta = \arcsin(0) = 0.$

$$m = 1 : \theta = \arcsin\left(\frac{\lambda}{d}\right) = \arcsin\left(\frac{6.50 \times 10^{-7}\ \text{m}}{1.67 \times 10^{-6}\ \text{m}}\right) = 22.9^\circ.$$

$$m = -1 : \theta = \arcsin\left(-\frac{\lambda}{d}\right) = \arcsin\left(-\frac{6.50 \times 10^{-7}\ \text{m}}{1.67 \times 10^{-6}\ \text{m}}\right) = -22.9^\circ.$$

$\theta' = 20.0^\circ : m = 0 : \theta = \arcsin(-\sin 20.0^\circ) = -20.0^\circ.$

$$m = 1 : \theta = \arcsin\left(\frac{6.50 \times 10^{-7}\ \text{m}}{1.67 \times 10^{-6}\ \text{m}} - \sin 20.0^\circ\right) = 2.71^\circ.$$

$$m = -1 : \theta = \arcsin\left(\frac{6.50 \times 10^{-7}\ \text{m}}{1.67 \times 10^{-6}\ \text{m}} - \sin 20.0^\circ\right) = -47.0^\circ.$$

36.65: For 650 slits/mm $\Rightarrow d = \dfrac{1}{6.50 \times 10^5\ \text{m}^{-1}} = 1.53 \times 10^{-6}\ \text{m}.$

We need $d \sin\theta = m\lambda \Rightarrow \sin\theta = \dfrac{m\lambda}{d} \le 1$, if the whole spectrum is to be seen.

$$\lambda_1 = 4.00 \times 10^{-7}\text{m} : m = 1 : \frac{\lambda_1}{d} = 0.26;\ m = 2 : \frac{2\lambda_1}{d} = 0.52;\ m = 3 : \frac{3\lambda_1}{d} = 0.78.$$

$$\lambda_2 = 7.00 \times 10^{-7}\text{m} : m = 1 : \frac{\lambda_2}{d} = 0.46;\ m = 2 : \frac{2\lambda_2}{d} = 0.92;\ m = 3 : \frac{3\lambda_2}{d} = 1.37.$$

So the third order does not contain the violet end of the spectrum, and therefore only the first and second diffraction patterns contain all colors of the spectrum.

36.66: a) $2d \sin\theta = m\lambda \Rightarrow \theta = \arcsin\left(\dfrac{m\lambda}{2d}\right) = \arcsin\left(m\dfrac{0.125\ \text{nm}}{2(0.282\ \text{nm})}\right)$

$$= \arcsin(0.2216\ m).$$

So for $m = 1 : \theta = 12.8°$, $m = 2 : \theta = 26.3°$, $m = 3 : \theta = 41.7°$, and $m = 4 : \theta =$ 62.4°. No larger m values yield answers.

b) If the separation $d = \dfrac{a}{\sqrt{2}}$, then $\theta = \arcsin\left(\dfrac{\sqrt{2}m\lambda}{2a}\right) = \arcsin(0.3134\ m).$

So for $m = 1 : \theta = 18.3°$, $m = 2 : \theta = 38.8°$, and $m = 3 : \theta = 70.1°$. No larger m values yield answers.

36.67: a) $d \sin\theta = m\lambda$. Place 1st maximum at ∞ or $\theta = 90^\circ$.

$d = \lambda$. If $d < \lambda$, this puts the first maximum "beyond ∞." Thus, if $d < \lambda$, there is only a single principal maximum.

b) $\Phi_{\text{path}} = 2\pi\left(\dfrac{d\sin\theta}{\lambda}\right)$. This just scales 2π radians by the fraction the wavelength is of the path difference between adjacent sources.

If we add a relative phase δ between sources, we still must maintain a total phase difference of zero to keep our principal maximum.

$$\Phi_{\text{path}} \pm \delta = 0 \Rightarrow \frac{2\pi d\sin\theta}{\lambda} = \pm\delta \text{ or } \theta = \sin^{-1}\left(\frac{\delta\lambda}{2\pi d}\right)$$

c) $d = \dfrac{0.280\text{ m}}{14} = 0.0200\text{ m}$ (count the number of spaces between 15 points).

Let $\theta = 45^\circ$. Also recall $f\lambda = c$, so

$$\delta_{\max} = \pm\frac{2\pi(0.0200\text{ m})(8.800\times10^9\text{ Hz})\sin 45^\circ}{(3.00\times10^8\text{ m/s})} = \pm2.61\text{ radians.}$$

36.68: $\sin\theta = 1.22\dfrac{\lambda}{D} \Rightarrow \theta = \arcsin\left(1.22\dfrac{\lambda}{D}\right)$. So for

a) Mauna Kea: $\theta = \arcsin\left(1.22\dfrac{(5.00\times10^{-7}\text{ m})}{(8.3\text{ m})}\right) = (4.21\times10^{-6})^\circ$.

b) Arecibo: $\theta = \arcsin\left(1.22\dfrac{(0.210\text{ m})}{(305\text{ m})}\right) = 0.0481^\circ$.

36.69: To resolve two objects, according to Rayleigh's criterion, one must be located at the first minimum of the other. In this case, knowing the equation for the angle to the first minimum, and also the objects' separation and distance away, the sine of the angle subtended by them is calculated to be:

$$\sin\theta = \frac{\lambda}{a} = \frac{\Delta x}{R} \Rightarrow R = \frac{a\Delta x}{\lambda} = \frac{(3.50\times10^{-4}\text{ m})(2.50\text{ m})}{6.00\times10^{-7}\text{ m}} = 1458\text{ m} = 1.46\text{ km.}$$

36.70: $\sin\theta = 1.22\dfrac{\lambda}{D} \approx \dfrac{\Delta x}{R} \Rightarrow \Delta x = \dfrac{1.22\,\lambda R}{D} = \dfrac{(1.22)cR}{Df}\ (\lambda f = c)$.

$$\Delta x = \frac{(1.22)(3.00\times10^5\text{ km/s})(7.2\times10^8\text{ ly})}{(77.000\times10^3\text{ km})(1.665\times10^9\text{ Hz})} = 2.06\text{ ly.}$$

$$\Rightarrow 9.41\times10^{12}\text{ km/ly}\cdot 2.06\text{ ly} = 1.94\times10^{13}\text{ km.}$$

36.71: Diffraction limited seeing and Rayleigh's criterion tell us:

$$\sin\theta = 1.22\frac{\lambda}{D} = \frac{(1.22)(5.00\times10^{-7}\text{ m})}{(4.00\times10^{-3}\text{ m})} = 1.53\times10^{-4}.$$

But now the altitude of the astronaut can be calculated from the angle (above) and the object separation (75 m). We have:

$$\frac{\Delta x}{h} = \tan\theta \Rightarrow h = \frac{\Delta x}{\tan\theta} \approx \frac{\Delta x}{\sin\theta} = \frac{75.0 \text{ m}}{1.53\times10^{-4}} = 4.90\times10^{5} \text{ m} = 490 \text{ km}.$$

36.72: a) $\sin\theta = 1.22\,\dfrac{\lambda}{D} \approx \dfrac{\Delta x}{R} \Rightarrow R = \dfrac{D\Delta x}{1.22\lambda} = \dfrac{(6.00\times10^{6} \text{ m})(2.50\times10^{5} \text{ m})}{(1.22)(1.0\times10^{-5} \text{ m})} =$

1.23×10^{17} m. But 9.41×10^{15} m/ly $\Rightarrow R = 13.1$ ly.

b) $\Delta x = \dfrac{1.22\,\lambda R}{D} = \dfrac{(1.22)(1.0\times10^{-5} \text{ m})(4.22 \text{ ly})(9.41\times10^{15} \text{ m/ly})}{1.0 \text{ m}} =$

$4.84\times10^{11} \text{ m} = 4.84\times10^{8}$ km.

$\approx 10{,}000$ times the diameter of the earth! Not enough resolution to see an earth-like planet!

≈ 3 times the distance from the earth to the sun.

c) $\Delta x = \dfrac{(1.22)(1.0\times10^{-5} \text{ m})(59 \text{ ly})(9.41\times10^{15} \text{ m/ly})}{6.00\times10^{6} \text{ m}} = 1.13\times10^{6} \text{ m} = 1130$ km.

$\dfrac{\Delta x}{D_{\text{planet}}} = \dfrac{1130 \text{ km}}{1.38\times10^{5} \text{ km}} = 8.19\times10^{-3}$; Δx is small compared to the size of the planet.

36.73: a) From the segment dy', the fraction of the amplitude of E_0 that gets through is $E_0\left(\dfrac{dy'}{a}\right) \Rightarrow dE = E_0\left(\dfrac{dy'}{a}\right)\sin(kx-\omega t)$.

b) The path difference between each little piece is $y'\sin\theta \Rightarrow kx = k(D - y'\sin\theta) \Rightarrow dE = \dfrac{E_0 dy'}{a}\sin(k(D-y'\sin\theta)-\omega t)$. This can be rewritten as $dE = \dfrac{E_0 dy'}{a}(\sin(kD-\omega t)\cos(ky'\sin\theta) + \sin(ky'\sin\theta)\cos(kD-\omega t))$.

c) So the total amplitude is given by the integral over the slit of the above.

$\Rightarrow E = \int_{-a/2}^{a/2} dE = \dfrac{E_0}{a}\int_{-a/2}^{a/2} dy'\,(\sin(kD-\omega t)\cos(ky'\sin\theta) + \sin(ky'\sin\theta)\times$ $\cos(kD-\omega t))$.

But the second term integrates to zero, so we have:

$$E = \frac{E_0}{a}\sin(kD-\omega t)\int_{-a/2}^{a/2} dy'\,(\cos(ky'\sin\theta)) = E_0\sin(kD-\omega t)\left[\left(\frac{\sin(ky'\sin\theta}{ka\sin\theta/2}\right)\right]_{-a/2}^{a/2}$$

$$\Rightarrow E = E_0\sin(kD-\omega t)\left(\frac{\sin(ka(\sin\theta)/2}{ka(\sin\theta)/2}\right) = E_0\sin(kD-\omega t)\left(\frac{\sin(\pi a(\sin\theta)/\lambda)}{\pi a(\sin\theta)/\lambda}\right).$$

At $\theta = 0$, $\dfrac{\sin[\ldots]}{[\ldots]} = 1 \Rightarrow E = E_0\sin(kD-\omega t)$.

d) Since $I = E^2 \Rightarrow I = I_0\left(\frac{\sin(ka(\sin\theta)/2)}{ka(\sin\theta)/2}\right)^2 = I_0\left(\frac{\sin(\beta/2)}{\beta/2}\right)^2$, where we have used $I_0 = E_0^2\sin^2(kx - \omega t)$.

36.74: a) Recall that the expression for the amplitude of a traveling wave is $\cos(kx - \omega t)$. Thus each source can be thought of as a traveling wave evaluated at $x = R$ with a maximum amplitude of E_0. However, each successive source will pick up an extra phase from its respective pathlength to point P. $\phi = 2\pi\left(\frac{d\sin\theta}{\lambda}\right)$ which is just 2π, the maximum phase, scaled by whatever fraction the path difference, $d\sin\theta$, is of the wavelength, λ. By adding up the contributions from each source (including the accumulating phase difference) this gives the expression provided.

b) $e^{i(kR-\omega t+n\phi)} = \cos(kR - \omega t + n\phi) + i\sin(kR - \omega t + n\phi)$.

The real part is just $\cos(kR - \omega t + n\phi)$.

So $\mathrm{Re}\left[\sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)}\right] = \sum_{n=0}^{N-1} E_0\cos(kR - \omega t + n\phi)$.

(Note: *Re* means "the real part of . . ..")

but this is just $E_0\cos(kR - \omega t + \phi) + E_0\cos(kR - \omega t + \phi) + E_0\cos(kR - \omega t + 2\phi) + \ldots + E_0\cos(kR - \omega t + (N-1)\phi)$.

c) $\sum_{n=0}^{N-1} E_0\, e^{i(kR-\omega t+n\phi)} = E_0\sum_{n=0}^{N-1} e^{-i\omega t}\, e^{+ikR} e^{in\phi} = E_0 e^{i(kR-\omega t)}\sum_{n=0}^{N-1} e^{in\phi}$.

but $\sum_{n=0}^{\infty} e^{in\phi} = \sum_{n=0}^{N-1}(e^{i\phi})^n$. But recall $\sum_{n=0}^{N-1} x^n = \frac{x^N - 1}{x - 1}$.

Let $x = e^{i\phi}$ so $\sum_{n=0}^{N-1}(e^{i\phi})^n = \frac{e^{iN\phi} - 1}{e^{i\phi-1}}$ (nice trick!). But $\frac{e^{iN\phi} - 1}{e^{i\phi} - 1} = \frac{e^{\frac{iN\phi}{2}}(e^{\frac{iN\phi}{2}} - e^{-\frac{iN\phi}{2}})}{e^{i\frac{\phi}{2}}(e^{i\frac{\phi}{2}} - e^{-i\frac{\phi}{2}})} =$

$$e^{i(N-1)\frac{\phi}{2}}\frac{(e^{\frac{iN\phi}{2}} - e^{-\frac{iN\phi}{2}})}{(e^{i\frac{\phi}{2}} - e^{-i\frac{\phi}{2}})}.$$

Putting everything together:

$$E = \sum_{n=0}^{N-1} E_0 e^{i(kR-\omega t+n\phi)} = E_0 e^{i(kR-\omega t+(N-1)\frac{\phi}{2})}\frac{(e^{iN\frac{\phi}{2}} - e^{-iN\frac{\phi}{2}})}{(e^{i\frac{\phi}{2}} - e^{-i\frac{\phi}{2}})}$$

$$= E_0\left[\cos(kR - \omega t + n\phi) + i\sin(kR - \omega t + n\phi)\right]\left[\frac{\cos\frac{N\phi}{2} + i\sin\frac{N\phi}{2} - \cos\frac{N\phi}{2} - i\sin\frac{N\phi}{2}}{\cos\frac{\phi}{2} + i\sin\frac{\phi}{2} - \cos\frac{\phi}{2} - i\sin\frac{\phi}{2}}\right]$$

Taking only the real part gives $\Rightarrow E_0\cos(kR - \omega t + n\phi)\frac{\sin(N\frac{\phi}{2})}{\sin\frac{\phi}{2}} = E$.

d) $I = \left|E\right|^2_{\text{ave}} = I_0 \dfrac{\sin^2 (N\frac{\phi}{2})}{\sin^2 (\frac{\phi}{2})}$. (The $\cos^2$ term goes to $\frac{1}{2}$ in the time average and is included in the definition of I_0.) $I_0 \propto \dfrac{E_0^2}{2}$.

e) $N = 2$. $I = I_0 \dfrac{\sin^2 (2\frac{\phi}{2})}{\sin^2 \frac{\phi}{2}} = \dfrac{I_0 (2 \sin\frac{\phi}{2} \cos\frac{\phi}{2})^2}{\sin^2 \frac{\phi}{2}} = 4I_0 \cos^2 \dfrac{\phi}{2}$. Looking at Eq. 37-10, $I_0' \propto 2E_0^2$ but for us $I_{0_{\text{us}}} \propto \dfrac{E_0^2}{2} = \dfrac{I_0'}{4}$.

36.75: a) $I = I_0 \dfrac{\sin^2 (N\frac{\phi}{2})}{\sin^2 \frac{\phi}{2}}$ $\lim\limits_{\phi \to 0} I \to \dfrac{0}{0}$.

$$\text{Use l'Hôpital's rule}: \lim_{\phi \to 0} \frac{\sin (N\frac{\phi}{2})}{\sin \frac{\phi}{2}} = \lim_{\phi \to 0} \left(\frac{N/2}{1/2}\right) \frac{\cos(N\frac{\phi}{2})}{\cos(\frac{\phi}{2})}.$$
$$= N$$

So $\lim\limits_{\phi \to 0} I = N^2 I_0$.

b) The location of the first minimum is when the numerator first goes to zero at $\dfrac{N}{2}\phi_{\min} = \pi$ or $\phi_{\min} = \dfrac{2\pi}{N}$. The width of the central maximum goes like $2\phi_{\min}$, so it is proportional to $\dfrac{1}{N}$.

c) Whenever $\dfrac{N\phi}{2} = n\pi$ where n is an integer, the numerator goes to zero, giving a minimum in intensity. That is, I is a minimum wherever $\phi = \dfrac{2n\pi}{N}$. This is true assuming that the denominator doesn't go to zero as well, which occurs when $\dfrac{\phi}{2} = m\pi$, where m is an integer. When both go to zero, using the result from part(a), there is a maximum. That is, if $\dfrac{n}{N}$ is an integer, there will be a maximum.

d) From part c), if $\dfrac{n}{N}$ is an integer we get a maximum. Thus, there will be $N-1$ minima. (Places where $\dfrac{n}{N}$ is not an integer for fixed N and integer n.) For example, $n = 0$ will be a maximum, but $n = 1, 2\ldots, N-1$ will be minima with another maximum at $n = N$.

e) Between maxima $\frac{\phi}{2}$ is a half-integer multiple of $\pi\left(\text{i.e.}\ \frac{\pi}{2}, \frac{3\pi}{2}, \text{etc.}\right)$ and if N is odd then $\frac{\sin^2(N\frac{\phi}{2})}{\sin^2\frac{\phi}{2}} \to 1$, so $I \to I_0$.

37.1: If O′ sees simultaneous flashes then O will see the A(A′) flash first since O would believe that the A′ flash must have traveled longer to reach O′, and hence started first.

37.2: a) $\gamma = \frac{1}{\sqrt{1-(0.9)^2}} = 2.29.$

$$t = \gamma\tau = (2.29)\,(2.20 \times 10^{-6}\,\text{s}) = 5.05 \times 10^{-6}\,\text{s}.$$

b) $d = vt = (0.900)\,(3.00 \times 10^8\,\text{m/s})\,(5.05 \times 10^{-6}\,\text{s}) = 1.36 \times 10^3\,\text{m} = 1.36\,\text{km}.$

37.3: $\sqrt{1-u^2/c^2} = (1-u^2/c^2)^{1/2} \approx 1 - \frac{u^2}{2c^2} + \cdots$

$$\Rightarrow (\Delta t - \Delta t_0) = (1-\sqrt{1-u^2/c^2})(\Delta t) = \frac{u^2}{2c^2}\Delta t = \frac{(250\,\text{m/s})^2\,(4\,\text{hrs})\cdot(3600)}{2(3.00\times 10^8\,\text{m/s})^2}$$

$$\Rightarrow (\Delta t - \Delta t_0) = 5.00 \times 10^{-9}\,\text{s}.$$

The clock on the plane shows the shorter elapsed time.

37.4: $\gamma = \frac{1}{\sqrt{1-(0.978)^2}} = 4.79.$

$$\gamma\Delta t = (4.79)\,(82.4 \times 10^{-6}\,\text{s}) = 3.95 \times 10^{-4}\,\text{s} = 0.395\,\text{ms}.$$

37.5: a) $\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} \Rightarrow 1 - \frac{u^2}{c^2} = \left(\frac{\Delta t_0}{\Delta t}\right)^2$

$$\Rightarrow u = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{2.6}{42}\right)^2}$$

$$\therefore u = 0.998c.$$

b) $\Delta x = u\Delta t = (0.998)\,(3.00 \times 10^8\,\text{m/s})\,(4.2 \times 10^{-7}\,\text{s}) = 126\,\text{m}.$

37.6: $\gamma = 1.667$ a) $\Delta t_0 = \frac{\Delta t}{\gamma} = \frac{1.20 \times 10^8\,\text{m}}{\gamma(0.800c)} = 0.300\,\text{s}.$

b) $(0.300\,\text{s})\,(0.800c) = 7.20 \times 10^7\,\text{m}.$

c) $\Delta t_0 = 0.300\ \text{s}/\gamma = 0.180\ \text{s}$. (This is what the *racer* measures *your* clock to read at that instant.) At *your* origin you read the original $\dfrac{1.20\times10^8\text{m}}{(0.800)\ (3\times10^8\ \text{m/s})} = 0.5\ \text{s}$.

Clearly the observes (you and the racer) will not agree on the order of events!

37.7: $\Delta t_0 = \sqrt{1-u^2/c^2}\,\Delta t = \sqrt{1-\left(\dfrac{4.80\times10^6\ \text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2}\ (1\ \text{yr})$

$\Rightarrow (\Delta t - \Delta t_0) = (1.28\times10^{-4})\ \text{yr} = 1.12\ \text{hrs}.$

The least time elapses on the rocket's clock because it had to be in two inertial frames whereas the earth was only in one.

37.8: a) The frame in which the source (the searchlight) is stationary is the spacecraft's frame, so 12.0 ms is the proper time. b) To three figures, $u = c$. Solving Eq. (37.7) for u/c in terms of γ,

$$\frac{u}{c} = \sqrt{1-(1/\gamma)^2} \approx 1 - \frac{1}{2\gamma^2}.$$

Using $1/\gamma = \Delta t_0/\Delta t = 12.0\ \text{ms}/190\ \text{ms}$ gives $u/c = 0.998$.

37.9: $\gamma = \dfrac{1}{\sqrt{1-(u/c)^2}} = \dfrac{1}{\sqrt{1-(0.9860)^2}} = 6.00$

a) $l = \dfrac{l_0}{\gamma} = \dfrac{55\ \text{km}}{6.00} = 9.17\ \text{km}.$

b) In muon's frame:

$$d = u\Delta t = (0.9860c)(2.20\times10^{-6}\ \text{s}) = 0.651\ \text{km}.$$

$$\Rightarrow \% = \frac{d}{h} = \frac{0.651}{9.17} = 0.071 = 7.1\%.$$

c) In earth's frame:

$$\Delta t = \Delta t_0\gamma = (2.2\times10^{-6}\ \text{s})(6.00) = 1.32\times10^{-5}\ \text{s}$$

$$\Rightarrow d' = u\Delta t = (0.9860c)(1.32\times10^{-5}\ \text{s}) = 3.90\ \text{km}$$

$$\Rightarrow \% = \frac{d'}{h'} = \frac{3.90\ \text{km}}{55.0\ \text{km}} = 7.1\%.$$

37.10: a) $t = \dfrac{4.50\times10^4\ \text{m}}{0.99540c} = 1.51\times10^{-4}\ \text{s}.$

$$b)\ \gamma = \frac{1}{\sqrt{1-(0.9954)^2}} = 10.44$$

$$h' = \frac{h}{\gamma} = \frac{45\ \text{km}}{10.44} = 4.31\ \text{km}.$$

c) $\dfrac{h'}{0.99540c} = 1.44\times10^{-5}$ s, and $\dfrac{t}{\gamma} = 1.44\times10^{-5}$ s; so the results agree but the particle's lifetime is dilated in the frame of the earth.

37.11: a) $l_0 = 3600$ m

$$\Rightarrow l = l_0\sqrt{1-\frac{u^2}{c^2}} = l_0(3600\ \text{m})\sqrt{1-\frac{(4.00\times10^7\ \text{m/s})^2}{(3.00\times10^8\ \text{m/s})^2}}$$

$$= (3600\ \text{m})(0.991) = 3568\ \text{m}.$$

$$b)\ \Delta t_0 = \frac{l_0}{u} = \frac{3600\ \text{m}}{4.00\times10^7\ \text{m/s}} = 9.00\times10^{-5}\ \text{s}.$$

$$\text{c)}\quad \Delta t = \frac{l}{u} = \frac{3568\ \text{m}}{4.00\times10^7\ \text{m/s}} = 8.92\times10^{-5}\ \text{s}.$$

37.12: $\gamma = 1/0.3048$, so $u = c\sqrt{1-(1/\gamma)^2} = 0.952c = 2.86\times10^8\ \text{m/s}.$

37.13: $l = l_0\sqrt{1-\dfrac{u^2}{c^2}} \Rightarrow l_0 = \dfrac{l}{\sqrt{1-u^2/c^2}}$

$$\Rightarrow l_0 = \frac{74.0\ \text{m}}{\sqrt{1-\left(\frac{0.600c}{c}\right)^2}} = 92.5\,\text{m}.$$

37.14: Multiplying the last equation of (37.21) by u and adding to the first to eliminate t gives

$$x' + ut' = \gamma x\left(1-\frac{u^2}{c^2}\right) = \frac{1}{\gamma}x,$$

and multiplying the first by $\dfrac{u}{c^2}$ and adding to the last to eliminate x gives

$$t' + \frac{u}{c^2}x' = \gamma t\left(1-\frac{u^2}{c^2}\right) = \frac{1}{\gamma}t,$$

so $x = \gamma(x' + ut')$ and $t = \gamma(t' + ux'/c^2)$,

which is indeed the same as Eq. (37.21) with the primed coordinates replacing the unprimed, and a change of sign of u.

37.15: a) $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.400c + 0.600c}{1 + (0.400)\,(0.600)} = 0.806c$

b) $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.900c + 0.600c}{1 + (0.900)(0.600)} = 0.974c$

c) $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.990c + 0.600c}{1 + (0.990)(0.600)} = 0.997c.$

37.16: $\gamma = 1.667 (\gamma = 5/3$ if $u = (4/5)c)$. a) In Mavis's frame the event "light on" has space-time coordinates $x' = 0$ and $t' = 5.00$ s, so from the result of Exercise 37.14 or Example 37.7, $x = \gamma(x' + ut')$ and $t = \gamma\left(t' + \dfrac{ux'}{c^2}\right) \Rightarrow x = \gamma ut' = 2.00 \times 10^9$ m, $t = \gamma t' =$ 8.33 s.

b) The 5.00-s interval in Mavis's frame is the proper time Δt_0 in Eq. (37.6), so $\Delta t = \gamma \Delta t_0 = 8.33$ s, as in part (a).

c) $(8.33 \text{ s})\,(0.800c) = 2.00 \times 10^9$ m, which is the distance x found in part (a).

37.17: Eq. (37.18): $x' = \dfrac{x - ut}{\sqrt{1 - u^2/c^2}}$ Eq. (37.19): $x' = -ut' + x\sqrt{1 - u^2/c^2}$

Equate: $(x - ut)\,\gamma = -ut' + \dfrac{x}{\gamma}$

$$\Rightarrow t' = \left(-\frac{x\gamma}{u} + t\gamma + \frac{x}{u\gamma}\right) = t\gamma + \frac{x}{u}\left(\frac{1}{\gamma} - \gamma\right)$$

$$\frac{1}{\gamma} - \gamma = \sqrt{1 - (u/c)^2} - \frac{1}{\sqrt{1 - (u/c)^2}} = \frac{1 - (u/c)^2 - 1}{\sqrt{1 - (u/c)^2}} = \frac{-u^2/c^2}{\sqrt{1 - (u/c)^2}} = \gamma u^2/c^2$$

$$\Rightarrow t' = t\gamma + \frac{xu\gamma}{c^2} = \frac{t - ux/c^2}{\sqrt{1 - (u/c)^2}}.$$

37.18: Starting from Eq. (37.22),

$$v' = \frac{v - u}{1 - uv/c^2}$$

$$v'(1 - uv/c^2) = v - u$$

$$v' + u = v + v'uv/c^2$$

$$= v(1 + uv'/c^2)$$

from which Eq. (37.23) follows. This is the same as switching the primed and unprimed coordinates and changing the sign of u.

37.19: Let the unprimed frame be Tatooine and let the primed frame be the pursuit ship. We want the velocity v' of the cruiser knowing the velocity of the primed frame u and the velocity of the cruiser v in the unprimed frame (Tatooine).

$$v' = \frac{v-u}{1-\frac{uv}{c^2}} = \frac{0.600c - 0.800c}{1-(0.600)\,(0.800)} = -0.385c$$

$\Rightarrow$ the cruiser is moving toward the pursuit ship at $0.385c$.

37.20: In the frame of one of the particles, u and v are both $0.9520c$ but with opposite sign.

$$v' = \frac{-v-(u)}{1-(u)\,(-v)/c^2} = \frac{-0.9520c - 0.9520c}{1-(0.9520)(-0.9520)} = -0.9988c.$$

Thus, one particle moves at a speed $0.9988c$ toward the other in the other particle's frame.

37.21: $v = \dfrac{v'+u}{1+\frac{uv'}{c^2}} = \dfrac{-0.950c + 0.650c}{1+(-0.950)(0.650)} = -0.784c.$

37.22: a) In Eq.(39-24), $u = 0.400c,\ v' = 0.700c \Rightarrow v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.700c + 0.400c}{1+(0.700)\,(0.400)}$ $= 0.859c.$

b) $\dfrac{\Delta x}{v} = \dfrac{8.00\times10^9\ \text{m}}{0.859c} = 31.0\ \text{s}.$

37.23: $v' = \dfrac{v-u}{1-uv/c^2} \Rightarrow v' - \dfrac{uvv'}{c^2} = v-u$

$$\Rightarrow u\left(1-\frac{vv'}{c^2}\right) = v - v' \Rightarrow u = \frac{v-v'}{(1-vv'/c^2)}$$

$$\Rightarrow u = \frac{0.360c - 0.920c}{(1-(0.360)(0.920))} = -0.837c$$

$\Rightarrow$ moving opposite the rocket, i.e., away from Arrakis.

37.24: Solving Eq. (37.25) for u/c, (see solution to Exercise 37.25)

$$\frac{u}{c} = \frac{1-(f/f_0)^2}{1+(f/f_0)^2},$$

and so (a) if $f/f_0 = 0.98,\ (u/c) = 0.0202,$ the source and observer are moving away from each other. b) if $f/f_0 = 4,\ (u/c) = -0.882,$ they are moving toward each other.

37.25: a) $f = \sqrt{\dfrac{c+u}{c-u}}\,f_0 \Rightarrow (c-u)f^2 = (c+u)f_0^2$

$$\Rightarrow u = \frac{c(f^2 - f_0^2)}{f_0^2 + f^2} = \frac{c((f/f_0)^2 - 1)}{(f/f_0)^2 + 1} = \frac{c((\lambda_0/\lambda)^2 - 1)}{((\lambda_0/\lambda)^2 + 1)}$$

$$\therefore u = c\frac{((675/575)^2 - 1)}{((675/575)^2 + 1)} = 0.159c = 4.77 \times 10^7 \text{ m/s} = 4.77 \times 10^4 \text{ km/s} = 1.72 \times 10^8 \text{ km/h}.$$

b) $(1.72 \times 10^8 \text{ km/h} - 90 \text{ km/h})\ (\$1.00) = \$172$ million dollars!

37.26: Using $u = -0.600c = -(3/5)c$ in Eq. (37.25) gives

$$f = \sqrt{\frac{1-(3/5)}{1+(3/5)}}f_0 = \sqrt{\frac{2/5}{8/5}}f_0 = f_0/2.$$

37.27: a) $F = \dfrac{dp}{dt} = \dfrac{d}{dt}\left(\dfrac{mv}{\sqrt{1 - v^2/c^2}}\right) = \dfrac{ma}{\sqrt{1 - v^2/c^2}} - \dfrac{\frac{1}{2}mv \cdot \frac{1}{2}(2va/c^2)}{(1 - v^2/c^2)^{3/2}}$

$$= ma\left(\frac{(1 - v^2/c^2) + v^2/c^2}{(1 - v^2/c^2)^{3/2}}\right) = ma/(1 - v^2/c^2)^{3/2}$$

$$\Rightarrow a = \frac{F}{m}\left(1 - \frac{v^2}{c^2}\right)^{3/2}.$$

b) If the force is perpendicular to velocity then denominator is constant $\Rightarrow F = \dfrac{dp}{dt} = \dfrac{m\,dv/dt}{\sqrt{1 - v^2/c^2}} \Rightarrow a = \dfrac{F}{m}\sqrt{1 - v^2/c^2}.$

37.28: The force is found from Eq. (37.32) or Eq. (37.33). (a) Indistinguishable from $F = ma = 0.145\text{ N}$. b) $\gamma^3 ma = 1.75\text{ N}$. c) $\gamma^3 ma = 51.7\text{ N}$. d) $\gamma ma = 0.145\text{ N}$, 0.333 N, 1.03 N.

37.29: a) $p = \dfrac{mv}{\sqrt{1 - v^2/c^2}} = 2mv$

$$\Rightarrow 1 = 2\sqrt{1 - v^2/c^2} \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2}$$

$$\Rightarrow v^2 = \frac{3}{4}c^2 \Rightarrow v = \frac{\sqrt{3}}{2}c = 0.866c.$$

b) $F = \gamma^3 ma = 2ma \Rightarrow \gamma^3 = 2 \Rightarrow \gamma = (2)^{1/3}$ so $\dfrac{1}{1 - \dfrac{v^2}{c^2}} = 2^{2/3} \Rightarrow \dfrac{v}{c}$

$$= \sqrt{1 - 2^{-2/3}} = 0.608.$$

37.30: a) $\gamma = 1.01$, so $(v/c) = 0.140$ and $v = 4.21 \times 10^8$ m/s. b) The relativistic expression is *always* larger in magnitude than the non-relativistic expression.

37.31: a) $K = \dfrac{mc^2}{\sqrt{1 - v^2/c^2}} - mc^2 = mc^2$

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{3}{4}} = 0.866c.$$

b) $K = 5mc^2 \Rightarrow \dfrac{1}{\sqrt{1-v^2/c^2}} = 6 \Rightarrow \dfrac{1}{36} = 1 - \dfrac{v^2}{c^2} \Rightarrow v = \sqrt{\dfrac{35}{36}}c = 0.986c.$

37.32: $E = 2mc^2 = 2(1.67 \times 10^{-27}\text{ kg})(3.00 \times 10^8\text{ m/s})^2 = 3.01 \times 10^{-10}\text{ J} = 1.88 \times 10^9\text{ eV}.$

37.33: $K = (\gamma - 1)mc^2 \approx \dfrac{1}{2}mv^2 + \dfrac{3}{8}\dfrac{mv^4}{c^2} + \cdots$

if $K - K_0 = 1.02\dfrac{1}{2}mv^2 \Rightarrow \dfrac{3}{8}\dfrac{v^4}{c^2} = \dfrac{0.02}{2}v^2$

$\Rightarrow \dfrac{150}{4}v^2 = c^2 \Rightarrow v = \sqrt{\dfrac{4}{150}}c = 0.163c = 4.89 \times 10^7$ m/s.

37.34: a) $W = \Delta K = (\gamma_f - 1)mc^2 = (4.07 \times 10^{-3})\, mc^2$. b) $(\gamma_f - \gamma_i)mc^2 = 4.79mc^2$.
c) The result of part (b) is far larger than that of part (a).

37.35: a) Your total energy E increases because your potential energy increases;
$\Delta E = mg\Delta y$

$\Delta E = (\Delta m)c^2$ so $\Delta m = \Delta E/c^2 = mg(\Delta y)/c^2$

$\Delta m/m = (g\Delta y)/c^2 = (9.80\text{ m/s}^2)(30\text{ m})/(2.998 \times 10^8\text{ m/s})^2 = 3.3 \times 10^{-13}\ \%$

This increase is much, much too small to be noticed.

b) $\Delta E = \Delta U = \frac{1}{2}kx^2 = \frac{1}{2}(2.00 \times 10^2\text{ N/m})(0.060\text{ m})^2 = 36.0\text{ J}$

$\Delta m = (\Delta E)/c^2 = 4.0 \times 10^{-16}$ kg

Energy increases so mass increases. The mass increase is much, much too small to be noticed.

37.36: a) $E_0 = m_0c^2$

$$2E = mc^2 = 2m_0c^2$$

$$\therefore m = 2m_0 \rightarrow \frac{m_0}{\sqrt{1 - v^2/c^2}} = 2m_0$$

$$\frac{1}{4}=1-\frac{v^2}{c^2}\rightarrow\frac{v^2}{c^2}=\frac{3}{4}\rightarrow v=c\sqrt{3/4}$$

$$v=0.866\ c=2.60\times10^8\ \text{m/s}$$

b) $$10\,m_0c^2=mc^2=\frac{m_0}{\sqrt{1-v^2/c^2}}c^2$$

$$1-\frac{v^2}{c^2}=\frac{1}{100}\rightarrow\frac{v^2}{c^2}=\frac{99}{100}$$

$$v=c\sqrt{\frac{99}{100}}=0.995\ c=2.98\times10^8\ \text{m/s}$$

37.37: a) $E=mc^2+K$, so $E=4.00mc^2$ means $K=3.00mc^2=4.50\times10^{-10}\ \text{J}$

b) $E^2=(mc^2)^2+(pc)^2$; $E=4.00mc^2$, so $15.0(mc^2)^2=(pc)^2$

$p=\sqrt{15}mc=1.94\times10^{-18}\text{kg}\cdot\text{m/s}$

c) $E=mc^2/\sqrt{1-v^2/c^2}$

$E=4.00mc^2$ gives $1-v^2/c^2=1/16$ and $v=\sqrt{15/16c}=0.968c$

37.38: The work that must be done is the kinetic energy of the proton.

a) $$K=(\gamma-1)m_0c^2=\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right)m_0c^2$$

$$=(1.67\times10^{-27}\ \text{kg})(3.00\times10^8\ \text{m/s})^2\left(\frac{1}{\sqrt{1-\left(\frac{0.1c}{c}\right)^2}}-1\right)$$

$$=(1.50\times10^{-10}\ \text{J})\left(\frac{1}{\sqrt{1-0.01}}-1\right)$$

$$=7.56\times10^{-13}\ \text{J}$$

b) $$K=(1.50\times10^{-10}\text{J})\left(\frac{1}{\sqrt{1-(0.5)^2}}-1\right)$$

$$=2.32\times10^{-11}\ \text{J}$$

c) $$K=(1.50\times10^{-10}\ \text{J})\left(\frac{1}{\sqrt{1-(0.9)^2}}-1\right)$$

$$=1.94\times10^{-10}\ \text{J}$$

37.39: $(m=6.64\times10^{-27}\ \text{kg},\quad p=2.10\times10^{-18}\ \text{kg}\cdot\text{m/s})$

a) $$E=\sqrt{(mc^2)^2+(pc)^2}$$

$$=8.68\times10^{-10}\ \text{J.}$$

b) $K=E-mc^2=8.68\times10^{-10}-(6.64\times10^{-27}\ \text{kg})c^2=2.70\times10^{-10}\ \text{J.}$

c) $\dfrac{K}{mc^2} = \dfrac{2.70\times10^{-10}\ \text{J}}{(6.64\times10^{-27}\ \text{kg})c^2} = 0.452.$

37.40: $E = (m^2c^4 + p^2c^2)^{1/2} = mc^2\left(1+\left(\dfrac{p}{mc}\right)^2\right)^{1/2}$

$$\approx mc^2\left(1+\frac{1}{2}\frac{p^2}{m^2c^2}\right) = mc^2 + \frac{p^2}{2m} = mc^2 + \frac{1}{2}mv^2$$

the sum of the rest mass energy and the classical kinetic energy.

37.41: a) $v = 8\times10^7\ \text{m/s} \Rightarrow \gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = 1.0376$

$$m = m_p \qquad K_0 = \frac{1}{2}mv^2 = 5.34\times10^{-12}\ \text{J}$$

$$K = (\gamma-1)mc^2 = 5.65\times10^{-12}\ \text{J} \therefore \frac{K}{K_0} = 1.06.$$

b) $v = 2.85\times10^8\ \text{m/s} \therefore \gamma = 3.203$

$K_0 = \dfrac{1}{2}mv^2 = 6.78\times10^{-11}\ \text{J}$

$K = (\gamma-1)mc^2 = 3.31\times10^{-10}\ \text{J} \qquad K/K_0 = 4.88.$

37.42: $(5.52\times10^{-27}\ \text{kg})(3.00\times10^8\ \text{m/s})^2 = 4.97\times10^{-10}\ \text{J} = 3105\ \text{MeV}.$

37.43: a) $K = q\Delta V = e\Delta V$

$K = mc^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right) = 4.025mc^2 = 3.295\times10^{-13}\ \text{J} = 2.06\ \text{MeV}$

$\Delta V = K/e = 2.06\times10^6\ \text{V}$

b) From part (a), $K = 3.30\times10^{-13}\ \text{J} = 2.06\ \text{MeV}$

37.44: a) According to Eq. 37.38 and conservation of mass-energy

$$2Mc^2 + mc^2 = \gamma 2Mc^2 \Rightarrow \gamma = 1+\frac{m}{2M} = 1+\frac{9.75}{2(16.7)} = 1.292.$$

Note that since $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$, we have that

$$\frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{(1.292)^2}} = 0.6331.$$

b) According to Eq. 37.36, the kinetic energy of each proton is

$$K = (\gamma - 1)Mc^2 = (1.292 - 1)(1.67 \times 10^{-27}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\frac{1.00\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}} \right) =$$

274 MeV.

c) The rest energy of η^0 is $mc^2 = (9.75 \times 10^{-28}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 \left(\frac{1.00\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}} \right) =$
548 MeV.

d) The kinetic energy lost by the protons is the energy that produces the η^0,

$$548\ \text{MeV} = 2(274\ \text{MeV}).$$

37.45: a) $E = 0.420\ \text{MeV} = 4.20 \times 10^5\ \text{eV}.$

b) $$E = K + mc^2 = 4.20 \times 10^5\ \text{eV} + \frac{(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2}{1.6 \times 10^{-19}\ \text{J/eV}}$$

$$= 4.20 \times 10^5\ \text{eV} + 5.11 \times 10^5 \text{eV} = 9.32 \times 10^5\ \text{eV}.$$

c) $$E = \frac{mc^2}{\sqrt{1 - v^2/c^2}} \Rightarrow v = c\sqrt{1 - \left(\frac{mc^2}{E} \right)^2}$$

$$= c\sqrt{1 - \left(\frac{5.11 \times 10^5\ \text{eV}}{9.32 \times 10^5\ \text{eV}} \right)^2} = 0.836c = 2.51 \times 10^8\ \text{m/s}.$$

d) Nonrel: $$K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4.20 \times 10^5\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})}{9.11 \times 10^{-31}\ \text{kg}}}$$

$$= 3.84 \times 10^8\ \text{m/s}.$$

37.46: a) The fraction of the initial mass that becomes energy is

$1 - \dfrac{(4.0015\ \text{u})}{2(2.0136\ \text{u})} = 6.382 \times 10^{-3}$, and so the energy released per kilogram is

$(6.382 \times 10^{-3})(1.00\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 = 5.74 \times 10^{14}\ \text{J}.$

b) $$\frac{1.0 \times 10^{19}\ \text{J}}{5.74 \times 10^{14} \text{J/kg}} = 1.7 \times 10^4\ \text{kg}.$$

37.47: a) $E = mc^2, m = E/c^2 = (3.8 \times 10^{26}\ \text{J})/(2.998 \times 10^8\ \text{m/s})^2 = 4.2 \times 10^9\ \text{kg}$

1 kg is equivalent to 2.2 lbs, so $m = 4.6 \times 10^6$ tons

b) The current mass of the sun is 1.99×10^{30} kg, so it would take it
$(1.99 \times 10^{30}\ \text{kg})/(4.2 \times 10^9\ \text{kg/s}) = 4.7 \times 10^{20} \text{s} = 1.5 \times 10^{13}$ y to use up all its mass.

37.48: a) Using the classical work-energy theorem we obtain

$$\Delta x = \frac{m(v^2 - v^2{}_0)}{2F} = \frac{(2.00 \times 10^{-12}\ \text{kg})[(0.920)(3.00 \times 10^8\ \text{m/s})]^2}{2(4.20 \times 10^4\ \text{N})} = 1.81\ \text{m}.$$

b) Using the relativistic work-energy theorem for a constant force (Eq. 37.35) we obtain

$$\Delta x = \frac{(\gamma - 1)mc^2}{F}.$$

For the given speed, $\gamma = \frac{1}{\sqrt{1-0.920^2}} = 2.55$, thus

$$\Delta x = \frac{(2.55-1)(2.00\times10^{-12}\ \text{kg})(3.00\times10^8\ \text{m/s})^2}{(4.20\times10^4\ \text{N})} = 6.65\ \text{m}.$$

c) According to Eq. 37.30,

$$a = \frac{F}{m}\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} = \frac{(4.20\times10^4\ \text{N})}{(2.00\times10^{-12}\ \text{kg})}\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} = (2.10\times10^{16}\ \text{m/s}^2)\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}},$$

which yields, i) $a = 2.07\times10^{16}\ \text{m/s}^2$ $(\beta = 0.100)$

ii) $a = 1.46\times10^{16}\ \text{m/s}^2$ $(\beta = 0.462)$

iii) $a = 0.126\times10^{16}\ \text{m/s}^2$ $(\beta = 0.920)$.

37.49: a) $d = c\Delta t \Rightarrow \Delta t = \frac{d}{c} = \frac{1200\ \text{m}}{3\times10^8\ \text{m/s}} = 4.00\times10^{-6}\ \text{s}$ (v is very close to c).

But $\frac{\Delta t_0}{\Delta t} = \sqrt{1-\frac{u^2}{c^2}} \Rightarrow \left(\frac{u}{c}\right)^2 = 1-\left(\frac{\Delta t_0}{\Delta t}\right)^2$

$$\Rightarrow (1-\Delta)^2 = 1-\left(\frac{\Delta t_0}{\Delta t}\right)^2$$

$$\Rightarrow \Delta = 1-\left[1-\left(\frac{\Delta t_0}{\Delta t}\right)^2\right]^{1/2} = 1-\left[1-\left(\frac{2.6\times10^{-8}}{4.00\times10^{-6}}\right)^2\right]^{1/2} = 2.11\times10^{-5}.$$

b) $E = \gamma mc^2 = \left(\frac{\Delta t}{\Delta t_0}\right)\text{mc}^2 = \left(\frac{4.00\times10^{-6}}{2.6\times10^{-8}}\right)139.6\ \text{MeV}$

$$\Rightarrow E = 2.15\times10^4\ \text{MeV}.$$

37.50: One dimension of the cube appears contracted by a factor of $\frac{1}{\gamma}$, so the volume in S' is $a^3/\gamma = a^3\sqrt{1-(u/c)^2}$.

37.51: Need $a = b \Rightarrow l_0 = a, l = b$

$$\therefore \frac{l}{l_0} = \frac{b}{a} = \frac{b}{1.40b} = \sqrt{1-u^2/c^2}$$

$$\Rightarrow u = c\sqrt{1-\left(\frac{b}{a}\right)^2} = c\sqrt{1-\left(\frac{1}{1.40}\right)^2} = 0.700c.$$

$$= 2.10\times10^8\ \text{m/s}.$$

37.52: The change in the astronaut's biological age is Δt_0 in Eq. (37.6), and Δt is the distance to the star as measured from earth, divided by the speed. Combining, the astronaut's biological age is

$$19\text{ yr} + \frac{42.2\text{ yr } c}{\gamma u} = 19\text{ yr} + \frac{42.2\text{ yr}}{\gamma(0.9910)} = 24.7\text{ yr}.$$

37.53: a) $E = \gamma mc^2$ and $\gamma = 10 = \dfrac{1}{\sqrt{1-(v/c)^2}} \Rightarrow \dfrac{v}{c} = \sqrt{\dfrac{\gamma^2 - 1}{\gamma^2}} \Rightarrow \dfrac{v}{c} = c\sqrt{\dfrac{99}{100}}$

$= 0.995.$

b) $(pc)^2 = m^2v^2\gamma^2c^2,\ E^2 = m^2c^4\left(\left(\dfrac{v}{c}\right)^2\gamma^2 + 1\right)$

$$\Rightarrow \frac{E^2 - (pc)^2}{E^2} = \frac{1}{1+\gamma^2\left(\dfrac{v}{c}\right)^2} = \frac{1}{1+(10/(0.995))^2} = 0.01 = 1\%.$$

37.54: a) Note that the initial velocity is parallel to the x-axis. Thus, according to Eqn. 37.30,

$$a_x = \frac{F_x}{m}\left(1-\frac{v^2}{c^2}\right)^{\frac{3}{2}} = \frac{(-3.00\times10^{-12}\text{ N})}{(1.67\times10^{-27}\text{kg})}(1-0.900^2)^{\frac{3}{2}} = -1.49\times10^{14}\text{ m/s}^2.$$

Now note that the initial velocity is perpendicular to the y-axis. Thus, according to Eqn. 37.33,

$$a_y = \frac{F_y}{m}\left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}} = \frac{(5.00\times10^{-12}\text{ N})}{(1.67\times10^{-27}\text{ kg})}(1-0.900^2)^{\frac{1}{2}} = 1.31\times10^{15}\text{ m/s}^2.$$

b) The angle between the force and acceleration is given by

$$\cos\theta = \frac{F_xa_x+F_ya_y}{Fa} = \frac{(-3.00\times10^{-12}\text{ N})(-1.49\times10^{14}\text{ m/s}^2)+(5.00\times10^{-12}\text{ N})(1.31\times10^{15}\text{ m/s}^2)}{\sqrt{(-3.00\times10^{-12}\text{ N})^2+(5.00\times10^{-12}\text{ N})^2}\sqrt{(-1.49\times10^{14}\text{ m/s}^2)^2+(1.31\times10^{15}\text{ m/s}^2)^2}} \Rightarrow$$

$\theta = 24.5°.$

37.55: a) $K = 20\times10^{12}\text{eV} = 3.204\times10^{-6}\text{ J}$

$$K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right),\text{ so } \frac{1}{\sqrt{1-v^2/c^2}} = 2.131\times10^4$$

$$1-\frac{v^2}{c^2} = \frac{1}{(2.131\times10^4)^2};\ 1-\frac{v^2}{c^2} = \frac{(c+v)(c-v)}{c^2} = \frac{2(c-v)}{c}\ \text{ since } c+v \approx 2c$$

$v = (1-\Delta)c$ so $1 - v^2/c^2 = 2\Delta$ and $\Delta = 1.1\times10^{-9}$

b) $m_{\text{rel}} = \dfrac{m}{\sqrt{1-v^2/c^2}} = \dfrac{m}{\sqrt{2\Delta}} = (2.1\times10^4)m$

37.56: a) $(8.00\ \text{kg})(1.00\times10^{-4})(3.00\times10^{8}\ \text{m/s})^2 = 7.20\times10^{13}\ \text{J}.$

b) $(\Delta E/\Delta t) = (7.20\times10^{13}\ \text{J})/(4.00\times10^{-6}\ \text{s}) = 1.80\times10^{19}\ \text{W}.$

c) $M = \dfrac{\Delta E}{gh} = \dfrac{(7.20\times10^{13}\ \text{J})}{(9.80\ \text{m/s}^2)(1.00\times10^{3}\ \text{m})} = 7.35\times10^{9}\ \text{kg}.$

37.57: Heat in $Q = mL_f = (4.00\ \text{kg})(3.34\times10^{5}\ \text{J/kg}) = 1.34\times10^{6}\ \text{J}$

$$\Rightarrow \Delta m = \frac{Q}{c^2} = \frac{(1.34\times10^{6}\ \text{J})}{(3.00\times10^{8}\ \text{m/s})^2} = 1.49\times10^{-11}\ \text{kg}.$$

37.58: a) $v = \dfrac{p}{m} = \dfrac{(E/c)}{m} = \dfrac{E}{mc}$, where the atom and the photon have the same magnitude of momentum, E/c. b) $v = \dfrac{E}{mc} = << c$, so $E << mc^2$.

37.59: Speed in glass $v = \dfrac{c}{n} = \dfrac{c}{1.52} = 1.97\times10^{8}\ \text{m/s}$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = 1.326$$

$$\Rightarrow K = (\gamma-1)mc^2 = (0.326)(0.511\ \text{MeV}) = 0.167\ \text{MeV} = 1.67\times10^{5}\ \text{eV}.$$

37.60: a) 80.0 m/s is non-relativistic, and $K = \dfrac{1}{2}mv^2 = 186\ \text{J}.$

b) $(\gamma-1)mc^2 = 1.31\times10^{15}\ \text{J}.$

c) In Eq. (37.23),
$v' = 2.20\times10^{8}\ \text{m/s}$, $u = -1.80\times10^{8}\ \text{m/s}$, and so $v = 7.14\times10^{7}\text{m/s}$.

d) $\dfrac{20.0\ \text{m}}{\gamma} = 13.6\ \text{m}.$

e) $\dfrac{20.0\ \text{m}}{2.20\times10^{8}\ \text{m/s}} = 9.09\times10^{-8}\ \text{s}.$

f) $t' = \dfrac{t}{\gamma} = 6.18\times10^{-8}\ \text{s}$, or $t' = \dfrac{13.6\ \text{m}}{2.20\times10^{8}\ \text{m/s}} = 6.18\times10^{-8}\ \text{s}.$

37.61: $x'^2 = c^2t'^2$

$$\Rightarrow (x-ut)^2\gamma^2 = c^2\gamma^2\left(t - ux/c^2\right)^2$$

$$\Rightarrow x - ut = c(t - ux/c^2)$$

$$\Rightarrow x\left(1+\frac{u}{c}\right) = \frac{1}{c}x(u+c) = t(u+c) \Rightarrow x = ct$$

$$\Rightarrow x^2 = c^2t^2.$$

37.62: a) From Eq. (37.37),

$$K - \frac{1}{2}mv^2 = \frac{3}{8}m\frac{v^4}{c^2} = \frac{3}{8}(90.0\text{ kg})\frac{(3.00\times10^4\text{ m/s})^4}{(3.00\times10^8\text{ m/s})^2} = 304\text{ J}.$$

b) $\dfrac{(3/8)mv^4/c^2}{(1/2)mv^2} = \dfrac{3}{4}\left(\dfrac{v}{c}\right)^2 = 7.50\times10^{-9}.$

37.63: $a = \dfrac{dv}{dt} = \dfrac{F}{m}(1 - v^2/c^2)^{3/2}$

$$\Rightarrow \int_0^v \frac{dv}{(1-(v^2/c^2))^{3/2}} = \frac{F}{m}\int_0^t dt = \frac{F}{m}t$$

$$\Rightarrow c\int_0^{v/c}\frac{dx}{(1-x^2)^{3/2}} = c\left.\frac{x}{\sqrt{1-x^2}}\right|_0^{v/c} = \frac{v}{\sqrt{1-(v/c)^2}} = \frac{F}{m}t$$

$$\Rightarrow v^2 = \left(\frac{Ft}{m}\right)^2\left(1-\left(\frac{v}{c}\right)^2\right) = \left(\frac{Ft}{m}\right)^2 - v^2\left(\frac{Ft}{mc}\right)^2$$

$$\Rightarrow v = \frac{Ft/m}{\sqrt{1+(Ft/mc)^2}}.$$ So as $t\to\infty, v\to c$.

37.64: Setting $x = 0$ in Eq. (37.21), the first equation becomes $x' = -\gamma ut$ and the last, upon multiplication by c, becomes $ct' = \gamma ct$. Squaring and subtracting gives

$$c^2t'^2 - x'^2 = \gamma^2(c^2t^2 - u^2t^2) = c^2t^2,$$

or $x' = c\sqrt{t'^2 - t^2} = 4.53\times10^8$ m.

37.65: a) Want $\Delta t' = t_2' = t_1'$

$$x_1' = (x_1 - ut_1)\gamma = x_2' = (x_2 - ut_2)\gamma \Rightarrow u = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

And $\Delta t' = \gamma\left(t_2 - t_1 - u\dfrac{(x_2 - x_1)}{c_2}\right)$. Since $u = \Delta x/\Delta t$,

$$\Delta t' = \gamma\left(\Delta t - \frac{u\Delta x}{c^2}\right) = \gamma\left(\Delta t - \frac{\Delta x^2}{c^2\Delta t}\right) \Rightarrow \Delta t' = \Delta t\sqrt{1-\left(\tfrac{\Delta x}{\Delta t}\right)^2/c^2} = \sqrt{\Delta t^2 - \frac{\Delta x^2}{c^2}}$$

There's no physical solution for $\Delta t < \dfrac{\Delta x}{c} \Rightarrow \Delta x \geq c\Delta t.$

b) Simultaneously $\Rightarrow \Delta t' = 0 \;\therefore t_1 - \frac{ux_1}{c^2} = t_2 - \frac{ux_2}{c^2}$

$$\Rightarrow \Delta t = \frac{u}{c^2}\Delta x \Rightarrow u = \frac{c^2 \Delta t}{\Delta t}.$$

Also

$$\Delta x' = x_2' - x_1' = \frac{1}{\sqrt{1-(u/c)^2}}(\Delta x - u\Delta t) = \frac{1}{\sqrt{1-c^2\Delta t^2/\Delta x^2}}\left(\Delta x - c^2\frac{\Delta t^2}{\Delta x}\right)$$

$$= \Delta x\sqrt{1-\frac{c^2\Delta t^2}{\Delta x^2}} \Rightarrow \Delta x' = \sqrt{\Delta x^2 - c^2\Delta t^2}.$$

c) Part (b): $\Delta t = \frac{1}{c}\sqrt{(\Delta x)^2 - (\Delta x')^2} = \frac{1}{c}\sqrt{(5.00\text{ m})^2 - (2.50\text{ m})^2} = 1.44\times 10^{-8}\text{s}.$

37.66: a) $(100\text{ s})(0.600)(3.00\times 10^8\text{ m/s}) = 1.80\times 10^{10}\text{ m}$. b) In Sebulbas frame, the relative speed of the tachyons and the ship is $3.40c$, and so the time $t_2 = 100\text{ s} + \frac{1.80\times 10^{10}\text{ m}}{3.4c} = 118\text{ s}$. At t_2 Sebulba measures that Watto is a distance from him of $(118\text{ s})(0.600)(3.00\times 10^8\text{ m/s}) = 2.12\times 10^{10}\text{ m}$. c) From Eq. (37.23), with $v' = -4.00c$ and $u = 0.600c$, $v = +2.43c$, with the plus sign indicating a direction in the same direction as Watto's motion (that is, away from Sebulba). d) As the result of part (c) suggests, Sebulba would see the tachyons moving toward Watto and hence t_3 is the time they would have left Sebulba in order to reach Watto at the distance found in part (b), or $118\text{ s} - \frac{2.12\times 10^{10}\text{ m}}{2.43c} = 89\text{ s}$, and so Sebulba receives Watto's message before even sending it! Tachyons seem to violate causality.

37.67: Longer wavelength (redshift) implies recession. (The emitting atoms are moving away.) Using the result of Ex. 37.26: $u = c\frac{(\lambda_0/\lambda)^2 - 1}{(\lambda_0/\lambda) + 1}$

$$\Rightarrow u = c\left[\frac{(656.3/953.4)^2 - 1}{(656.3/953.4)^2 + 1}\right] = -0.3570c = -1.071\times 10^8\text{ m/s}$$

37.68: The baseball had better be moving non-relativistically, so the Doppler shift formula (Eq. (37.25)) becomes $f \cong f_0(1-(u/c))$. In the baseball's frame, this is the frequency with which the radar waves strike the baseball, and the baseball reradiates at f. But in the coach's frame, the reflected waves are Doppler shifted again, so the detected frequency is $f(1-(u/c)) = f_0(1-(u/c))^2 \approx f_0(1-2(u/c))$, so $\Delta f = 2f_0(u/c)$ and the fractional frequency shift is $\frac{\Delta f}{f_0} = 2(u/c)$. In this case,

$$u = \frac{\Delta f}{2 f_0} c = \frac{(2.86 \times 10^{-7})}{2} (3.00 \times 10^8 \text{ m})$$
$$= 42.9 \text{ m/s} = 154 \text{ km/h}$$
$$= 92.5 \text{ mi/h}.$$

37.69: a) Since the two triangles are similar:

$$H = A\gamma = mc^2\gamma = E.$$

b) $O = \sqrt{H^2 - A^2} = \sqrt{E^2 - (mc^2)^2} = pc.$

c) $K = E - mc^2.$

The kinetic energy can be obtained by the difference between the hypoteneuse and adjacent edge lengths.

37.70: a) As in the hint, both the sender and the receiver measure the same distance. However, in our frame, the ship has moved between emission of successive wavefronts, and we can use the time $T = 1/f$ as the proper time, with the result that $f = \gamma f_0 > f_0$.

b) Toward: $f = f_0 \sqrt{\dfrac{c+u}{c-u}} = 345 \text{ MHz} \left(\dfrac{1+0.758}{1-0.758}\right)^{1/2} = 930 \text{ MHz}$

$$f - f_0 = 930 \text{ MHz} - 345 \text{ MHz} = 585 \text{ MHz}.$$

Away:

$$f = f_0 \sqrt{\frac{c-u}{c+u}} = 345 \text{ MHz} \left(\frac{1-0.758}{1+0.758}\right)^{1/2} = 128 \text{ MHz}$$

$f - f_0 = -217$ MHz.

c) $\gamma f_0 = 1.53 f_0 = 528$ MHz, $f - f_0 = 183$ MHz. The shift is still bigger than f_0, but not as large as the approaching frequency.

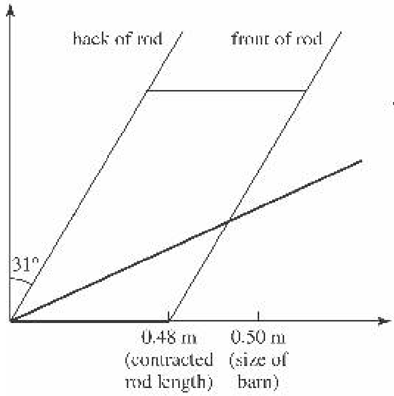
37.71: The crux of this problem is the question of simultaneity. To be "in the barn at one time" for the runner is different than for a stationary observer in the barn.

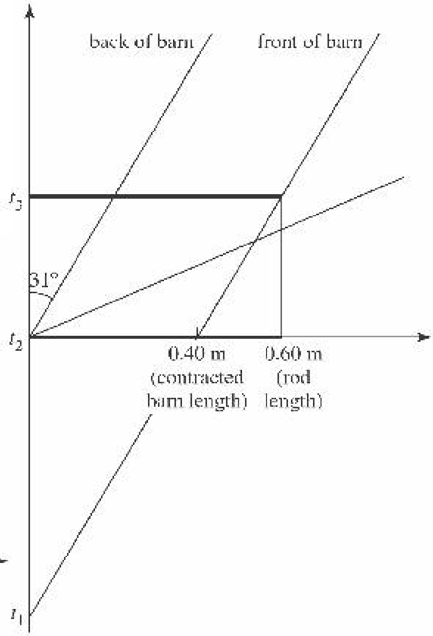
The diagram below, at left, shows the rod fitting into the barn at time $t = 0$, according to the stationary observer.

The diagram below, at right, is in the runner's frame of reference.

The front of the rod enters the barn at time t_1 and leaves the back of the barn at time t_2.

However, the back of the rod does not enter the front of the barn until the later time t_3.





37.72: In Eq. (37.23), $u = V, v' = (c/n),$ and so

$$v = \frac{(c/n)+V}{1+\frac{cV}{nc^2}} = \frac{(c/n)+V}{1+(V/nc)}.$$

For V non-relativistic, this is

$$\begin{aligned} v &\approx ((c/n)+V)(1-(V/nc)) \\ &= (c/n)+V-(V/n^2)-(V^2/nc) \\ &\approx \frac{c}{n}+\left(1-\frac{1}{n^2}\right)V \end{aligned}$$

so $k = \left(1-\frac{1}{n^2}\right)$. For water, $n = 1.333$ and $k = 0.437$.

37.73: a) $a' = \dfrac{dv}{dt'} \qquad dt' = \gamma(dt - udx/c^2)$

$$dv' = \frac{dv}{(1-uv/c^2)} + \frac{v-u}{(1-uv/c^2)^2}\frac{u}{c^2}dv$$

$$\frac{dv'}{dv} = \frac{1}{1-uv/c^2} + \frac{v-u}{(1-uv/c^2)^2}\cdot\left(\frac{u}{c^2}\right)$$

$$\therefore dv' = dv\left(\frac{1}{1-uv/c^2} + \frac{(v-u)u/c^2}{(1-uv/c^2)^2}\right) = dv\left(\frac{1-u^2/c^2}{(1-uv/c^2)^2}\right)$$

$$\therefore a' = \frac{dv \dfrac{(1-u^2/c^2)}{(1-uv/c^2)^2}}{\gamma dt - u\gamma dx/c^2} = \frac{dv}{dt} \cdot \frac{(1-u^2/c^2)}{(1-uv/c^2)^2} \cdot \frac{1}{\gamma(1-uv/c^2)}$$
$$= a(1-u^2/c^2)^{3/2}\,(1-uv/c^2)^{-3}.$$

b) Changing frames from $S' \to S$ just involves changing

$$a \to a',\ v \to -v' \Rightarrow a = a'(1-u^2/c^2)^{3/2}\left(1+\frac{uv'}{c^2}\right)^{-3}.$$

37.74: a) The speed v' is measured relative to the rocket, and so for the rocket and its occupant, $v' = 0$. The acceleration as seen in the rocket is given to be $a' = g$, and so the acceleration as measured on the earth is

$$a = \frac{du}{dt} = g\left(1-\frac{u^2}{c^2}\right)^{3/2}.$$

b) With $v_1 = 0$ when $t = 0$,

$$dt = \frac{1}{g}\frac{du}{(1-u^2/c^2)^{3/2}}$$
$$\int_0^{t_1} dt = \frac{1}{g}\int_0^{v_1}\frac{du}{(1-u^2/c^2)^{3/2}}$$
$$t_1 = \frac{v_1}{g\sqrt{1-v_1^2/c^2}}.$$

c) $dt' = \gamma dt = dt/\sqrt{1-u^2/c^2}$, so the relation in part (b) between dt and du, expressed in terms of dt' and du, is

$$dt' = \gamma dt = \frac{1}{\sqrt{1-u^2/c^2}}\frac{du}{g(1-u^2/c^2)^{3/2}} = \frac{1}{g}\frac{du}{(1-u^2/c^2)^2}.$$

Integrating as above (perhaps using the substitution $z = u/c$) gives

$$t_1' = \frac{c}{g}\operatorname{arctanh}\left(\frac{v_1}{c}\right).$$

For those who wish to avoid inverse hyperbolic functions, the above integral may be done by the method of partial fractions;

$$gdt' = \frac{du}{(1+u/c)(1-u/c)} = \frac{1}{2}\left[\frac{du}{1+u/c} + \frac{du}{1-uc}\right],$$

which integrates to

$$t_1' = \frac{c}{2g}\ln\left(\frac{c+v_1}{c-v_1}\right).$$

d) Solving the expression from part (c) for v_1 in terms of t_1, $(v_1/c) = \tanh(gt_1'/c)$, so that $\sqrt{1-(v_1/c)^2} = 1/\cosh(gt_1'/c)$, using the appropriate indentities for hyperbolic functions. Using this in the expression found in part (b),

$$t_1 = \frac{c}{g}\frac{\tanh(gt_1'/c)}{1/\cosh(gt_1'/c)} = \frac{c}{g}\sinh(gt_1'/c),$$

which may be rearranged slightly as

$$\frac{gt_1}{c} = \sinh\left(\frac{gt_1'}{c}\right).$$

If hyperbolic functions are not used, v_1 in terms of t_1' is found to be

$$\frac{v_1}{c} = \frac{e^{gt_1'/c} - e^{-gt_1'/c_1}}{e^{gt_1'/c} + e^{-gt_1'/c'}}$$

which is the same as $\tanh(gt_1'/c)$. Inserting this expression into the result of part (b) gives, after much algebra,

$$t_1 = \frac{c}{2g}(e^{gt_1'/c} - e^{-gt_1'/c}),$$

which is equivalent to the expression found using hyperbolic functions.

e) After the first acceleration period (of 5 years by Stella's clock), the elapsed time on earth is

$$t_1' = \frac{c}{g}\sinh(gt_1'/c) = 2.65\times10^9 \text{ s} = 84.0 \text{ yr}.$$

The elapsed time will be the same for each of the four parts of the voyage, so when Stella has returned, Terra has aged 336 yr and the year is 2436. (Keeping more precision than is given in the problem gives February 7 of that year.)

37.75: a) $f_0 = 4.568110\times10^{14}$ Hz $f_+ = 4.568910\times10^{14}$ Hz $f_- = 4.567710\times10^{14}$ Hz

$$\left.\begin{aligned} f_+ &= \sqrt{\frac{c+(u+v)}{c-(u+v)}}\cdot f_0 \\ f_- &= \sqrt{\frac{c+(u-v)}{c-(u-v)}}\cdot f_0 \end{aligned}\right\} \Rightarrow \begin{aligned} f_+^2(c-(u+v)) &= f_0^2\ (c+(u+v)) \\ f_-^2(c-(u-v)) &= f_0^2(c+(u-v)) \end{aligned}$$

where u is the velocity of the center of mass and v is the orbital velocity.

$$\Rightarrow (u+v) = \frac{(f_+/f_0)^2-1}{(f_+/f_0)^2+1}c \text{ and } (u-v) = \frac{(f_-^2/f_0^2)-1}{(f_-^2/f_0^2)+1}c$$

$$\Rightarrow u+v = 5.25\times10^4 \text{ m/s} \quad u-v = -2.63\times10^4 \text{ m/s}$$

$$\Rightarrow u = +1.31\times10^4 \text{ m/s} \Rightarrow \text{moving toward at } 13.1 \text{ km/s}.$$

b) $v = 3.94\times10^4$ m/s $T = 11.0$ days.

$$\Rightarrow 2\pi R = vt \Rightarrow R = \frac{(3.94\times10^4 \text{ m/s})(11.0 \text{ days})(24 \text{ hrs/day})(3600 \text{ sec/hr})}{2\pi} =$$

5.96×10^9 m $\approx$ 0.040 earth - sun distance.

Also the gravitational force between them (a distance of $2R$) must equal the centripetal force from the center of mass:

$$\frac{\partial^2 E}{\partial x^2} = \gamma^2 \frac{\partial^2 E}{\partial x'^2} + \gamma^2 \frac{v^2}{c^4} \frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 \frac{v}{c^2} \frac{\partial^2 E}{\partial x' \partial t'}$$
$$\frac{\partial^2 E}{\partial t^2} = \gamma^2 v^2 \frac{\partial^2 E}{\partial x'^2} + \gamma^2 \frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 v \frac{\partial^2 E}{\partial x' \partial t'}.$$

Substituting into the wave equation and combining terms (note that the mixed partials cancel),

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \gamma^2\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 E}{\partial x'^2} + \gamma^2\left(\frac{v^2}{c^4} - \frac{1}{c^2}\right)\frac{\partial^2 E}{\partial t'^2}$$
$$= \frac{\partial^2 E}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t'^2}$$
$$= 0.$$

37.77: a) In the center of momentum frame, the two protons approach each other with equal velocities (since the protons have the same mass). After the collision, the two protons are at rest—but now there are kaons as well. In this situation the kinetic energy of the protons must equal the total rest energy of the two kaons $\Rightarrow 2(\gamma_{\text{cm}} - 1)m_p c^2 = 2m_k c^2 \Rightarrow$

$\gamma_{\text{cm}} = 1 + \dfrac{m_k}{m_p} = 1.526.$ The velocity of a proton in the center of momentum frame is then

$v_{\text{cm}} = c\sqrt{\dfrac{\gamma_{\text{cm}}^2 - 1}{\gamma_{\text{cm}}^2}} = 0.7554c.$

To get the velocity of this proton in the lab frame, we must use the Lorentz velocity transformations. This is the same as "hopping" into the proton that will be our target and asking what the velocity of the projectile proton is. Taking the lab frame to be the unprimed frame moving to the left, $u = v_{\text{cm}}$ and $v' = v_{\text{cm}}$ (the velocity of the projectile proton in the center of momentum frame).

$$v_{\text{lab}} = \frac{v' + u}{1 + \frac{uv'}{c^2}} = \frac{2v_{\text{cm}}}{1 + \frac{v_{\text{cm}}^2}{c^2}} = 0.9619c$$
$$\Rightarrow \gamma_{\text{lab}} = \frac{1}{\sqrt{1 - \frac{v_{\text{lab}}^2}{c^2}}} = 3.658$$
$$\Rightarrow K_{\text{lab}} = (\gamma_{\text{lab}} - 1)m_p c^2 = 2494 \text{ MeV}.$$

b) $\dfrac{K_{\text{lab}}}{2m_k} = \dfrac{2494 \text{ MeV}}{2(493.7 \text{ MeV})} = 2.526.$

c) The center of momentum case considered in part (a) is the same as this situation. Thus, the kinetic energy required *is* just twice the rest mass energy of the kaons. $K_{\text{cm}} = 2(493.7 \text{ MeV}) = 987.4 \text{ MeV}$. This offers a substantial advantage over the fixed target experiment in part (b). It takes less energy to create two kaons in the proton center of momentum frame.